

Experimental verification and comparison of the rubber V-belt continuously variable transmission models

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Abstract. The paper includes the analysis of the rubber V-belt cooperation with the CVT transmission pulleys. The analysis of the forces and torques acting in the CVT transmission was conducted basing on calculated characteristics of the centrifugal regulator and the torque regulator. The accurate estimation of the regulator surface curvature allowed for calculation of the relation between the driving wheel axial force, the engine rotational speed and the gear ratio of the CVT transmission. Simplified analytical models of the rubber V-belt- pulley cooperation are based on three basic approaches. The Dittrich model assumes two contact regions on the driven and driving wheel. The Kim-Kim model considers, in addition to the previous model, also the radial friction. The radial friction results in the lack of the developed friction area on the driving pulley. The third approach, formulated in the Cammalleri model, assumes variable sliding angle along the wrap arch and describes it as a result the belt longitudinal and cross flexibility. Theoretical torque on the driven and driving wheel was calculated on the basis of the known regulators characteristics. The calculated torque was compared to the measured loading torque. The best accordance, referring to the centrifugal regulator range of work, was obtained for the Kim-Kim model.

1. Introduction

Description of the rubber V-belt cooperation with the bevel gears assemblies is a very complex issue. The complexity is a result of the belt transverse and longitudinal flexibility, the rubber hysteresis, the V-shape cross section of the belt and changes of the forces acting on the belt in one driving cycle. Furthermore, the changes of the forces acting on the moving vehicle influence on the transmitted power and gear ratio of the CVT transmission. Further difficulties arise from the internal combustion engine vibrations (usually with one cylinder), which is a common source of motive power in this kind of vehicles.

The existing concepts of the mathematical description of the considered phenomena can be divided into three basic approaches. The first approach is the complex theoretical model, developed mostly by Göran and Gerbert [2,3]. The model considered the belt slip in radial and longitudinal direction and its elastic properties. This theory, because of its complexity, has not found wide application in practice.

Another approach is the simplified semi- experimental model, which was highly modified over the years. In the work of Dittrich [7] the two zones of the belt-bevel gear contact were assumed. The first zone was involved with the undeveloped friction area when the belt enters on the bevel gears assembly. The second zone referred to developed friction area when the changes of the belt tension forces occur. Development of this approach, introduced by Worley [5], was the introduction of the radial component of the friction force. The formulated development allowed for the assumption of the constant belt force on the whole wrap angle of the driving wheel (the single zone of contact). This assumption was



explained by the occurrence of the self-locking phenomenon, preventing the radial displacement of the relaxing part of the belt. The practical applications mostly use the Kim model [5]. The model differs from the Worley approach only in the friction coefficient description. The Kim model doesn't include the difference the radial and longitudinal friction coefficient. Above conceptions assume the constant slip angle. The slip angle is defined as an angle between the friction vector and the radial direction. The value of the slip angle changes in the real CVT assembly, which is considered in the Cammalleri model [1].

The last of the three approaches is involved with the numerical models. Julió and Plante [4] introduced the discretized belt model. The model allowed for the prediction of the belt reaction according to the changes of the following parameters: load of the transmission (vehicle), angular velocity and axial force of the driving wheel. The prediction was feasible in the steady-state and transient working conditions.

The gear ratio change in the CVT transmission usually is a result of the centrifugal regulator operation. The pressing force is generated by the rollers of the regulator. The force mostly depends on the engine angular velocity and the profile of ramp curvature. In the case of the driven wheel of the transmission, the axial force is exerted by the coil spring and the torque regulator. Usually in the papers dealing with CVT operation the axial force resulting from the belt-bevel gears cooperation is not associated with the force resulting from the regulator assembly.

2. Test stand

The experiments were carried out using complete drivetrain of the TGB 101S scooter (1), mounted on the test stand frame (figure 1). The drivetrain included the CVT transmission with the centrifugal regulator. The centrifugal clutch was eliminated to allow the possibility of loading the drivetrain in the full range of the engine speed. To provide the load to the drivetrain, the water brake (2) was used. The flywheel was used to simulate the vehicle starting process.

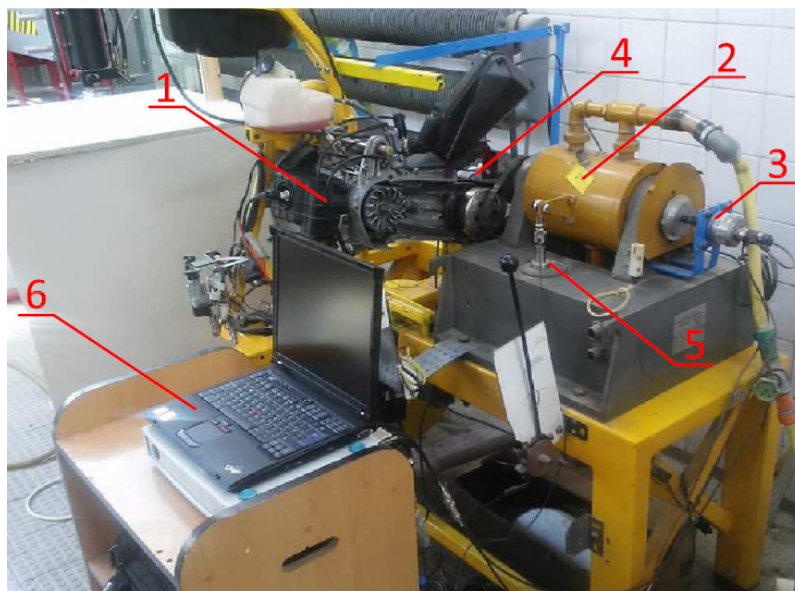


Figure 1. Test stand.

The following parameters were measured: rotational velocity of the driving and driven pulley of the CVT transmission (3), the force exerted on the water brake (5) and the driven wheel winding radius (4). The winding radius was measured using the optical sensor. The sensors signals were recorded with the frequency of 100 Hz, using the analog-to-digital converter coupled to the PC (6). Two experimental scenarios were conducted. The first was involved with maintaining constant throttle angle and changing the load. The second scenario referred to changes of the throttle angle and maintaining constant load on the drivetrain assembly.

3. The characteristics of the regulator assembly

3.1 The centrifugal regulator

The main elements of the centrifugal regulator are: coupling plate fixed in relation to the engine crankshaft, sliding bevel gear and rollers forcing the axial movement of the bevel gear. In the range of the regulator operation, the movement of the bevel gear and thus the gear ratio change is the results of the equilibrium of the axial forces of the rollers and belt. Variable curvature of the rollers ramp provokes the difficulties in determining the relationship between the sliding movement of the bevel gear and the radial movement of the roller. On the other side, determining the actual position of the rollers is important for the regulator axial force calculation. To solve this problem, a imprint of the sliding track was made. The imprint allowed for the graphical estimation of the sliding ramp shape (curvature). Subsequently, using the CAD software, the positions of the roller for several sliding bevel gear positions were calculated (figure 2). The obtained relationships between the angle of the line tangent to the ramp (ε) and the radius of the roller centre (y), and with the axial movement of the bevel gear (x) is illustrated in figure 3. These characteristics were estimated by the third and second-order polynomial. The estimation allowed for determination of the regulator axial force for each position of the sliding bevel gear.

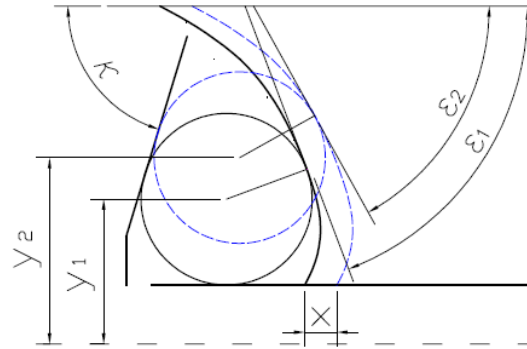


Figure 2. The centrifugal regulator scheme and example of the graphical rollers position estimation for the determined position of the sliding pulley.

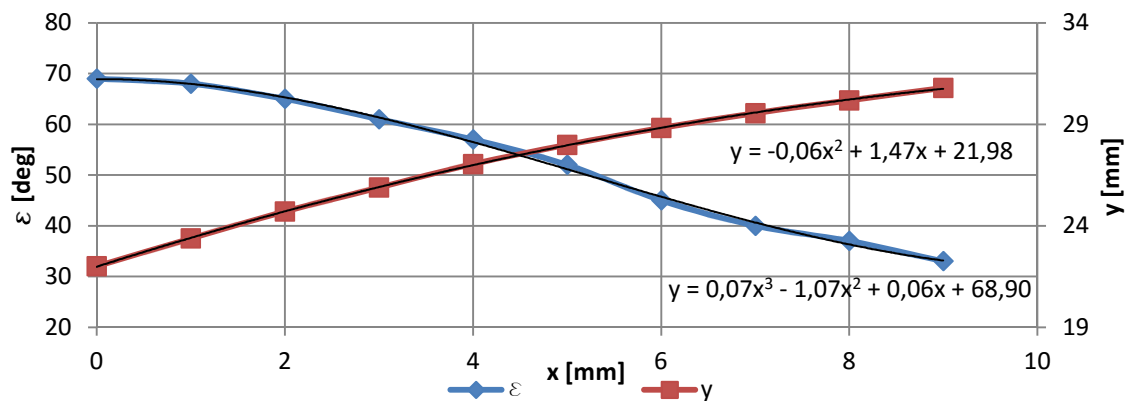


Figure 3. Change of the ε and y against axial shift of the bevel gear

The forces arrangement acting on the roller and the relationship describing the axial force are shown in [1]. The axial force is expressed as:

$$F_{zg} = \frac{zm\omega_g^2 y}{ctg(\kappa - \varphi_z) + ctg(\varepsilon - \varphi_b)}, \quad (1)$$

where:

z – number of the rollers, m – mass of the single roller, ω_g – engine angular speed, φ_z – sliding friction angle between the roller and the coupling plate, φ_b – rolling friction angle between the roller and the ramp. In the considered case, the angle of the coupling plate (κ) is always bigger than the angle of the

line tangent to the ramp (ε), thus in the entire regulator working range the roller will roll on the ramp. The tangential force between the rollers and the coupling plate will have the character of the sliding friction force. By examining the next positions of the pulley with different angular velocities, the centrifugal regulator characteristics was established. The characteristics was formulated as the relation between the axial force, engine angular velocity and the transmission ratio. The roller position, involved with the ratio, has tremendous impact on the generated axial force. Wide range of the ε angle provokes that the increasing of the axial force while reducing the engine speed is possible. The increase results from the ε angle increase during the gear ratio change. This situation may occur, for example, when the load exerted on transmission increases but transmission works with low gear ratio.

3.2 The torque regulator

The driven wheel of the belt-drive CVT transmission is commonly equipped with the torque regulator of the axial force. The regulator is shown schematically in figure 4. Sliding bevel gear moves along the orienting groove, milled at a specified angle in relation with the pulley rotation axis. Thus, the axial force, acting on the belt, depends on the transmitted torque. The value of the torque regulator axial force can be calculated as follows:

$$F_{zn} = k_n(x_o + x) + \frac{2}{d_k} \left(\frac{M}{2} + k_t \xi \right) tg(\delta), \quad (2)$$

where: k_n – positioning spring stiffness, x_o – initial deformation of the spring, k_t – torsional stiffness of the spring, ξ – angle of the torsional deflection (depending on the bevel gear sliding displacement), M – torque transmitted by the CVT transmission driven wheel, d_k – diameter of the pulley guiding bushing, δ – guiding groove angle.

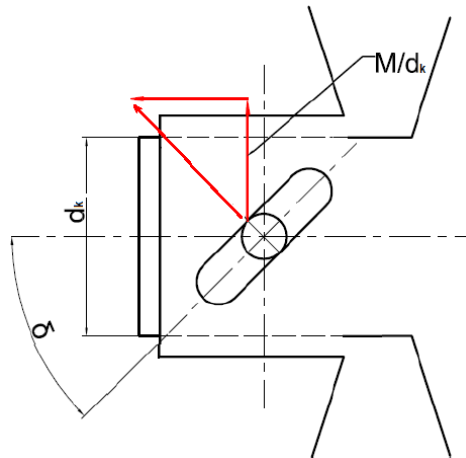


Figure 4. Torque regulator scheme [1].

3.3 General characteristics of the CVT transmission

Formulation of the general characteristics of the CVT transmission as a the torque possible to transmit in the function of the engine angular speed and transmission ratio change requires calculation of the relation between driving and driven wheel axial force and the belt force. This calculation is usually conducted using the belt cooperation model, presented in, among others, work [5] and [6]. The correctness of this approach will be justified further in this work. The sliding wheels axial forces can be expressed as:

$$F_{zg} = \frac{F_1}{2} \theta_g \frac{1 - \mu tg \frac{\alpha}{2}}{\mu + tg \frac{\alpha}{2}}, \quad (3)$$

$$F_{zn} = \frac{F_2}{2} (\theta_n - \theta_{na}) \frac{1 - \mu tg \frac{\alpha}{2}}{\mu + tg \frac{\alpha}{2}} + \frac{F_1 - F_2}{2\mu} \cos \frac{\alpha}{2}, \quad (4)$$

where F_{zg} and F_{zn} – axial forces of the driving (zg) and driven (zn) wheel, F_1 and F_2 – forces in active (1) and passive (2) part of the belt, θ_g i θ_n – wrap angle of the driving (g) and driven (n) wheel, θ_{na} – active wrap angle of the driven wheel, μ - friction coefficient between the belt and the pulley, $\alpha/2$ – belt wedge angle. The active wrap angle can be expressed as

$$\theta_{na} = \frac{1}{\mu} \left(\ln \frac{F_1}{F_2} \right) \sin \frac{\alpha}{2}. \quad (5)$$

When the characteristics of the centrifugal regulator is known, it is possible (basing on the equation 3) to calculate the belt force in the active part of the belt (F_1). The force F_1 can be determined in every point of the CVT transmission operation, within the range of the centrifugal regulator characteristics. Subsequently, using the equations (2) and (4) the force in the belt passive side (F_2) can be calculated. The torque of the CVT transmission driven wheel (M_n) can be calculated as follows:

$$M_n = (F_1 - F_2) \frac{d_n}{2}, \quad (6)$$

where: d_n is the belt pitch diameter in the driven wheel. The characteristics, obtained in this way is illustrated in figure 5. In addition, the curves of the constant torque value are marked. The curves were obtained by rectangular projection of the spatial characteristics on the ω_i plane. The experimentally determined parameters of CVT transmission and the assumed coefficients are established as follows: $z = 6$, $m = 0,006\text{kg}$, $\alpha/2 = 13^\circ$, $\kappa = 73^\circ$, $\mu = 0,4$, $\mu_z = \text{tg} \varphi_z = 0,05$, $f_b = \text{tg} \varphi_b = 0,01$, $k_n = 4,8\text{N/mm}$, $x_o = 47,5\text{mm}$, $d_k = 43\text{mm}$, $k_t = 63\text{Nmm}/^\circ$, $\delta = 45^\circ$.

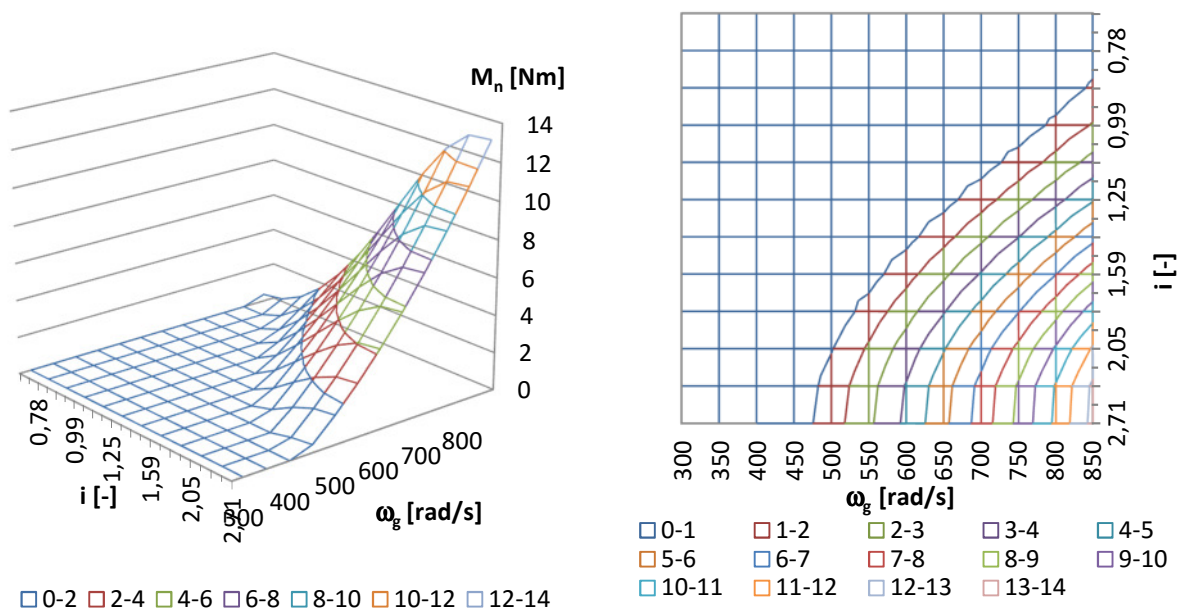


Figure 5. General characteristics of the examined CVT transmission and the constant torque curves; ω_g – engine speed, i – transmission ratio, M_n – driven pulley torque.

As it is shown in figure 5, the construction of the centrifugal regulator assembly provides that the torque transfer ability (in the regulator working range) is a function of the gear ratio. Whereas, the gear ratio is involved with the relative position between the rollers and the ramp. The influence of the ε angle increases with the increase of the engine rotational speed (figure 5). Hence, the ramp curvature is the essential parameter in the design of the drivetrains based on the CVT transmissions. The momentary transmission ratio value is the function the rotational velocity and the load exerted on the drivetrain. The position of the sliding driving wheel results from the equilibrium of the belt and rollers forces. Thus, the gear ratio change continues until the equilibrium of the forces described by equations (1) and (3) occurs. For the maximal ratio value, the CVT transmission works as a transmission with constant ratio. In this state, the total axial force is a combination of the regulator rollers force and the reaction force, acting on

the sliding pulley. The determination of this force requires additional measurement equipment. Therefore the torque regulator, applied in the driven wheel of the transmission, allows for the extension of the torque transfer capability in the range of lower engine speeds.

4. The models of the rubber belt and bevel gear cooperation

4.1 The Dittrich model [7]

The belt circumferential forces differentiation occurs as a result of the transmitted torque. The belt force increases on the driven pulley while it decreases on the driving pulley. These changes occur only on some part of the wrap circumference, called the active wrap angle. The equilibrium conditions on the belt element on the driving wheel is shown in [7]. The conditions refer to the active wrap angle. The friction force acts only in the circumferential direction. The friction force doesn't exist in the passive area where the belt tension is constant. Hence, the axial force takes different value in this area. The total axial force is the sum of the axial forces from two belt-bevel gear contact areas and can be expressed as:

$$F_{zg} = F_{zg}^m + F_{zg}^a = \frac{F_1}{2} (\theta_g - \theta_{ga}) \operatorname{ctg} \frac{\alpha}{2} + \frac{F_1 - F_2}{2\mu} \cos \frac{\alpha}{2}, \quad (7)$$

$$F_{zn} = F_{zn}^m + F_{zn}^a = \frac{F_2}{2} (\theta_n - \theta_{na}) \operatorname{ctg} \frac{\alpha}{2} + \frac{F_1 - F_2}{2\mu} \cos \frac{\alpha}{2}, \quad (8)$$

where subscripts (^a) and (^m) refer to the active and passive zone (contact area). Because the belt force in the active area (F_1) is always bigger than the belt force in the passive area (F_2), the driving wheel axial force will take bigger values with the same wrap angles.

On the base of the known regulator characteristics and the registered parameters, it is possible to estimate the belt forces. Comparison of the equations (1) to (7) and (2) to (8) provides to the set of the two nonlinear equations with unknown variables F_1 and F_2 . In order to verification of the obtained values, the theoretical driven wheel torque (described by equation 6) was compared with the measured load torque.

4.2 The Kim-Kim model [5]

According to the Dittrich approach, the winding radius on each of the wheels must undergo the changes as a result of the belt force differentiation. The experimental research results, described in the work [5], indicate that such belt behavior occurs only for the driven wheel. In the case of the driving wheel, the winding radius remains constant on the whole wrap angle. This behavior is explained by the self-locking phenomenon occurrence. As a consequence, it is assumed that the belt force is constant. Thus, the circumferential friction component is neglected. In the case of the active pulley (driving wheel), only the radial static friction is assumed. Thus, single contact area is considered. The total axial force for the driving pulley is expressed by equation (3). In the case of the driven pulley the two contact areas are assumed. In the passive contact area (at the entrance), the belt tension is constant. In this area the belt element forces distribution is similar to the belt element forces distribution on the driving wheel. The kinetic force in radial and circumferential direction is considered in the active contact area. The total axial force for the driven pulley is expressed by equation (4).

4.3 The Cammalleri model [1]

The main difference between the previously described models and the Cammalleri model [1] lies in the consideration of the belt flexibility and the changes of the slip angle γ along the pulley wrap angle. The axial forces of the pulleys are described in [1].

4.4 The models comparison

This chapter includes the selected results of the experimental researches for two way of the CVT transmission loading. The first loading scenario refers to the load increase with maintaining the constant, full throttle opening. The figure 6 illustrates the registered angular velocities of the pulleys and changes of the winding radiuses. In the first working stage, the CVT transmission works in the unloaded state,

which corresponds with the engine speed about 8000rpm. The winding radiuses on the driving and driven wheel are then 43 and 33mm respectively. Thus the gear ratio obtained as the quotient of the radiuses has the minimal value 0,75 (figure 8). In this state, the centrifugal regulator axial force is about 400N, which corresponds to the minimal limit value of the angle ε . The real load torque and the theoretical torques obtained basing on the described approaches are shown in figure 7. The negligible theoretical capability of the torque transfer in this phase results mainly from the gear ratio value. The small driven pulley winding radii entails the large belt force value, even in the light load conditions. Furthermore, the increased wrap angle of the driving wheel provokes the necessity for increased axial force of the centrifugal regulator (equation 3). In the described CVT working area, the theoretical torque values according the Kim-Kim model are about 0,1Nm bigger than the real torque value. Similar torque difference is noticed for the Cammalleri model, but the obtained values are smaller than the real torque. In the application of the Dittrich model, the obtained torque values are about 0,3Nm smaller from the real torque values. The increase of the load (which started about 5th s.) results in the decrease of the engine speed (firstly) and change of the transmission ratio (subsequently) (figure 6 and 8). These phenomena result in the temporary loss of the CVT transmission driving capacity. This phenomenon may result from the delay operation of the regulation system. Its effect is a temporary slip increase. From the 6th to 8th second of the experiment, the decrease of the engine angular speed occurs simultaneously with an increase of the regulator axial force. The axial force increase is provoked by the gear ratio increase and thus the increase of the ε angle. Furthermore, the change of the driven wheel winding radius (associated with the belt force change) and the change of the driving wheel wrap angle influence on the reduction of the axial force, required from the regulator. Consequently, relatively small axial force change provokes the significant improvement of the theoretical torque. The best compatibility in this area was obtained for the first of the analysed models, wherein the theoretical torque is bigger than the measured torque but the difference doesn't exceed 0,8Nm. The torque values obtained using the another models are understated and the maximal difference is 2Nm for the Cammaleri model and 3,6Nm for the Dittrich model. The torque gradient obtained from the Dittrich model is twice lower than the torque gradient calculated basing on the measurements. Because the axial forces were calculated basing on the regulators characteristics, the obtained torque diagrams apply only to the centrifugal regulator working range. As it is shown in figure 6 and 8, in the range from 8th to 21th s. the sliding driving pulley was in the initial position. Therefore, the real axial force includes also the reaction force component. The reaction force determination requires additional measurement equipment. Hence, the belt forces and torques, obtained in the considered time range, achieve understate values. The forces and torques decrease (at a fixed gear ratio) is a result of the engine rotational speed decrease (figure 6). From the 21th second, the engine rotational speed increases as a result of the reduced load exerted by the water brake. During next second (to 22th s.) the gear ratio doesn't change its maximal value, which corresponds to the sudden growth of the theoretical drive capacity. Sudden decrease of the gear ratio, provoked by the centrifugal regulator operation forces the sudden decrease of the axial force (the ε angle change)

(figure 8). These changes cause the theoretical torque change (figure 7), which is also provoked by wrap angles change and the change of the belt force in the belt active part. As the result, in the range from 24th to 25th s., the theoretical drive capacity disappears which corresponds to the slip increase. From the 25th s., the CVT transmission returns to the base state.

The second loading scenario refers to the sudden throttle open at the constant load. The figure 9 shows the registered angular velocities of the pulleys and changes of the winding radiuses. In the initial stage, the CVT transmission works in the fixed load state at the partly open throttle which corresponds with the engine speed about 3000rpm. In the following conditions, the driving and the driven wheel winding radiuses are correspondingly 21mm and 55mm. The gear ratio i_r calculated as the quotient of radiuses has maximal value 2,7 (figure 11). The real loading torque and the theoretical torque obtained using the three described approaches are illustrated in figure 10. The small engine

The constant full throttle and increasing load

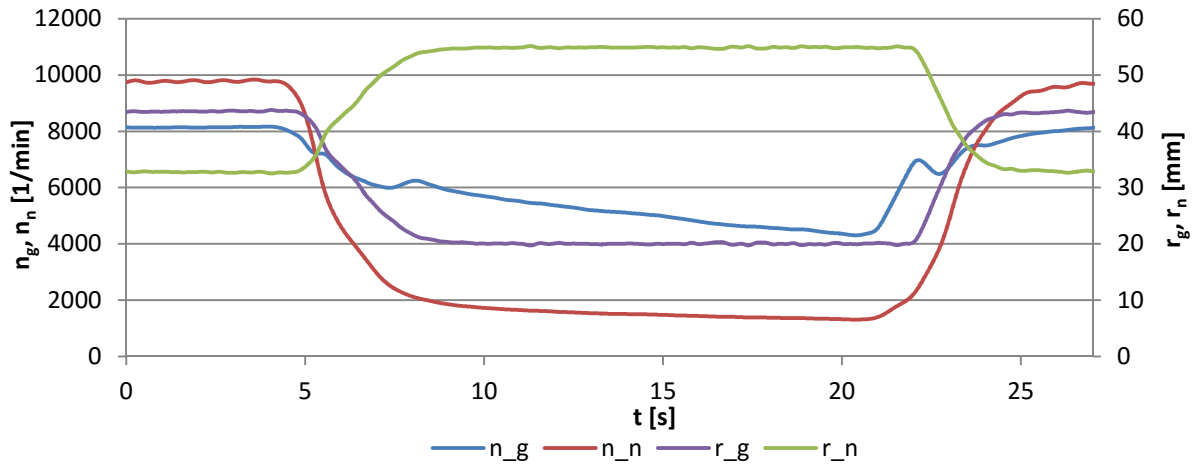


Figure 6. Change of the pulley angular speed (n) and winding radii (r) against time; subscript (g) – driving pulley, subscript (n) – driven pulley.

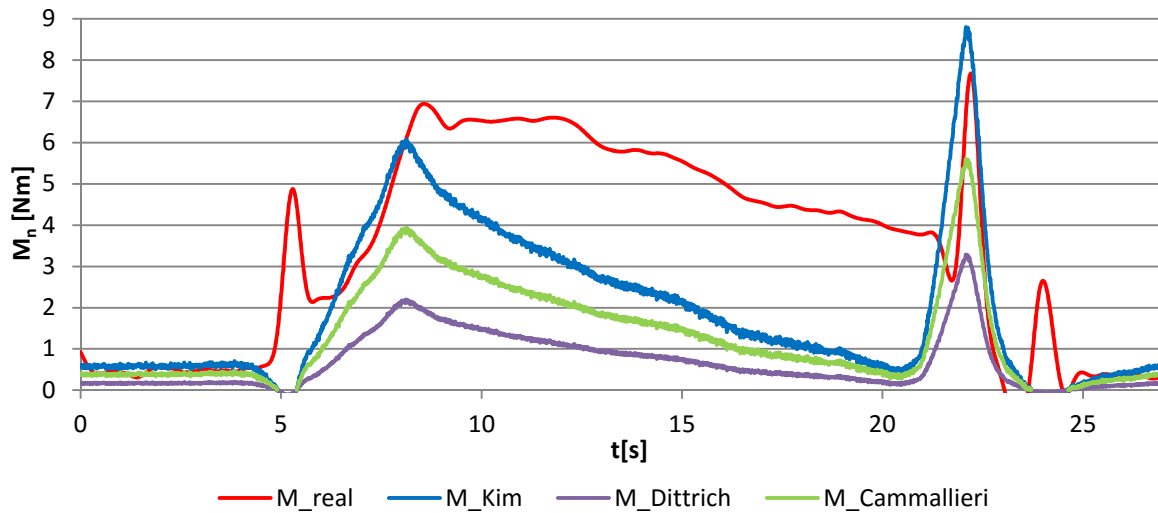


Figure 7. Real and theoretical driven pulley torque against time.

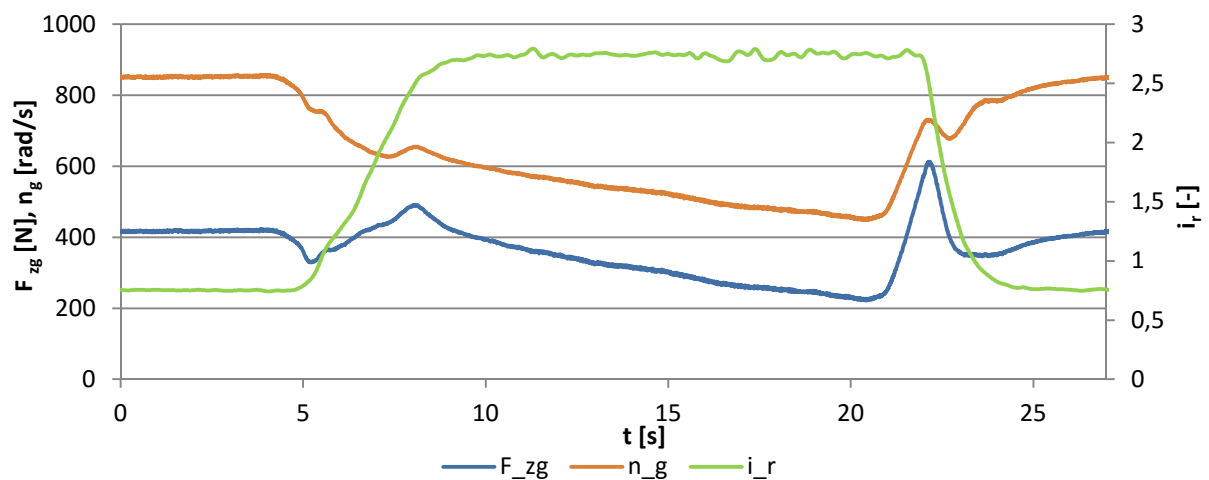


Figure 8. Centrifugal regulator axial force (F_{zg}) and transmission ratio (i_r) against time.

The constant load and rapid throttle opening

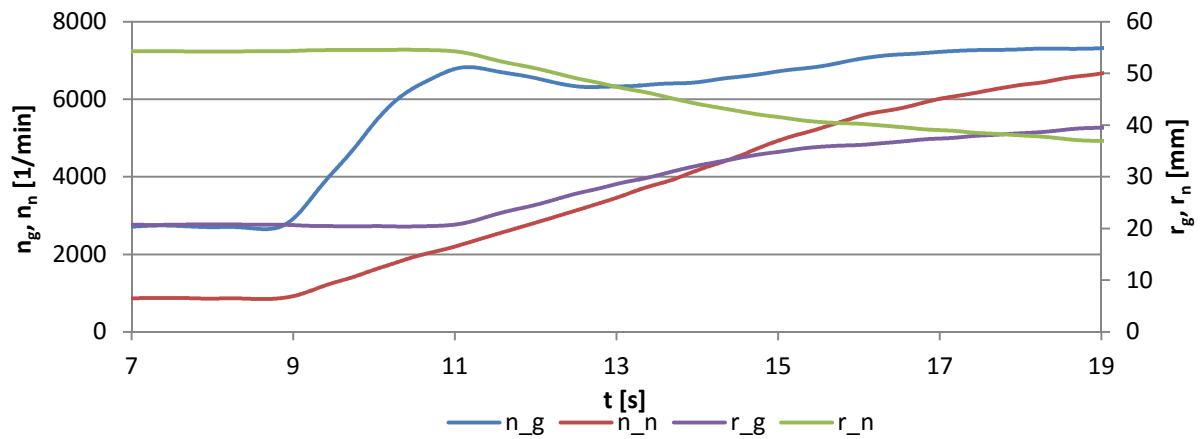


Figure 9. Change of the pulley angular speed (n) and winding radii (r) against time; subscript (g) – driving pulley, subscript (n) – driven pulley.

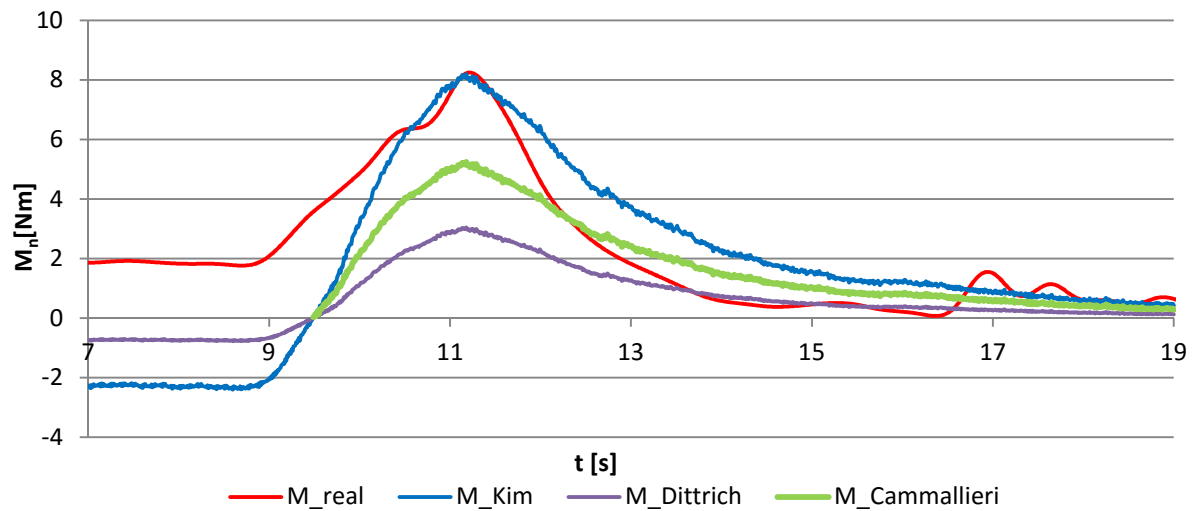


Figure 10. Real and theoretical driven pulley torque against time.

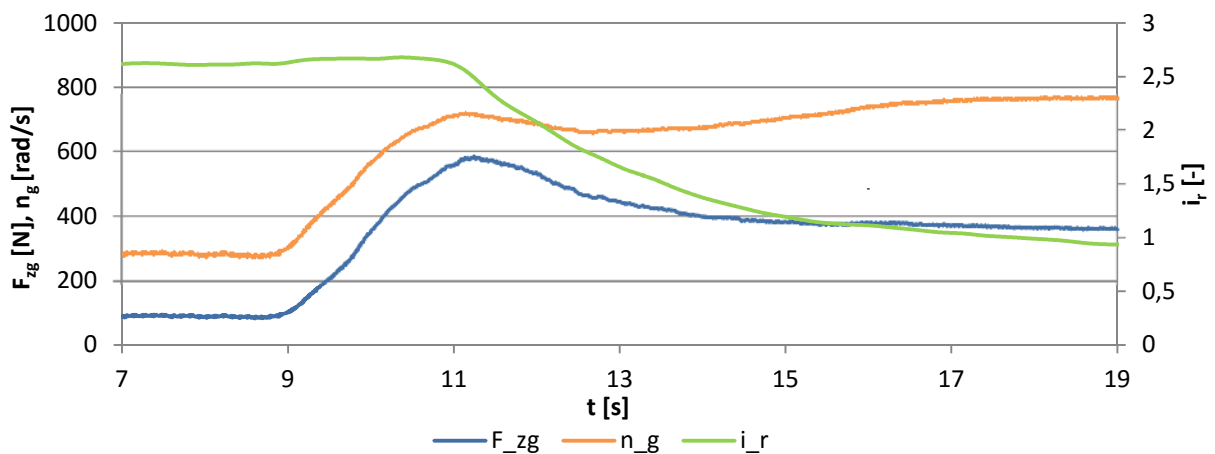


Figure 11. Centrifugal regulator axial force (F_{zg}) and transmission ratio (i_r) against time.

speed causes that the drive capacity results from the reaction force, acting on the sliding wheel of the regulator. Therefore, the theoretical torques take the negative values. The torque values in the time range

from 10th to 19th s. have the physical importance. The gear ratio change begins at 11th s., at the engine speed 7000rpm (figure 10 and 11). The gear ratio change corresponds with progressive decrease of the regulator axial force and the CVT transmission drive capacity, at almost fixed engine speed. As previously the best torque compliance was obtained for the Kim-Kim model. The theoretical torque (Kim-Kim model) exceeds the real torque load and the maximal torque difference in the analysed situation is about 2Nm.

Summarizing, the best compliance between the real and modelled torque was obtained using the Kim- Kim model. The Cammalleri model, despite its complexity, gives understated torque values. However, the general trend of changes is provided. The main difficulty in the Cammalleri model is the necessity of the determination of the belt material properties, which can be the reason for the differences. Hence, the Kim-Kim model seems to be the most suitable for the practical applications. The model is characterized by simplicity and uncomplicated form. The results obtained by the Dittrich model are substantially understated, but the trend of changes is similar to another analysed models. Application of the identical belt cooperation model for the driven and driving wheel leads to values, which are not similar to the reality.

5. Conclusions

1. The experimentally obtained results indicate that the commonly cited The Kim-Kim model describes the CVT transmission working accurately.
2. The prominent difference between the real values and the parameters obtained basing on the Dittrich model justifies the occurrence of the single belt-pulley contact area on the driving wheel.
3. The Cammalleri model, including the belt elastic properties, requires the consideration of the belt material parameters. The complex form of this model limits its engineering applications.
4. The shape of the centrifugal regulator ramp provides that the temporary ε angle value and subsequently the temporary value of the transmission ratio have significant influence on the generated axial force. The engine angular speed decrease may occur correspondingly to the regulator axial force increase, provoked by the increase of the gear ratio (ε angle change).
5. The torque regulator can extend the torque transferring ability for the maximal gear ratio value, when the CVT assembly works as a transmission with a constant gear ratio.
6. The determination of the belt forces, basing on the regulators characteristics is possible only in the regulators operation ranges (excluding the maximal and minimal transmission ratio).

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