Simplified planar model of a car steering system with rack and pinion and McPherson suspension

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Abstract. The paper presents the analysis and optimization of steering system with rack and pinion and McPherson suspension using spatial model and equivalent simplified planar model. The dimension of the steering linkage that give minimum steering error can be estimated using planar model. The steering error is defined as the difference between the actual angle made by the outer front wheel during steering manoeuvers and the calculated angle for the same wheel based on the Ackerman principle. For a given linear rack displacement, a specified steering arms angular displacements are determined while simultaneously ensuring best transmission angle characteristics (i) without and (ii) with imposing linear correlation between input and output. Numerical examples are used to illustrate the proposed method.

1. Introduction; review of the literature dealing with car steering system

Real steering mechanisms are complex spatial linkages. However, the kingpin inclination and caster angles that provide compliance to the steering linkage with the suspension system have little influence on the motion transmission of the steering linkage. As a result, the real rack and pinion steering linkage, which is spatial in nature, can be modelled as an equivalent planar linkage for the investigation of Ackermann condition. Therefore, by neglected the kingpin inclination and caster angle, in this study, the steering linkage of a vehicle is considered as a planar linkage with six revolute joints (*R*) and one prismatic joint (*P*) forming the kinematic chain (*R-R-R*)-*P*-(*R-R-R*) with two closed loops. Such a simplification of the steering system has been also used by other researchers [1, 2, 7].
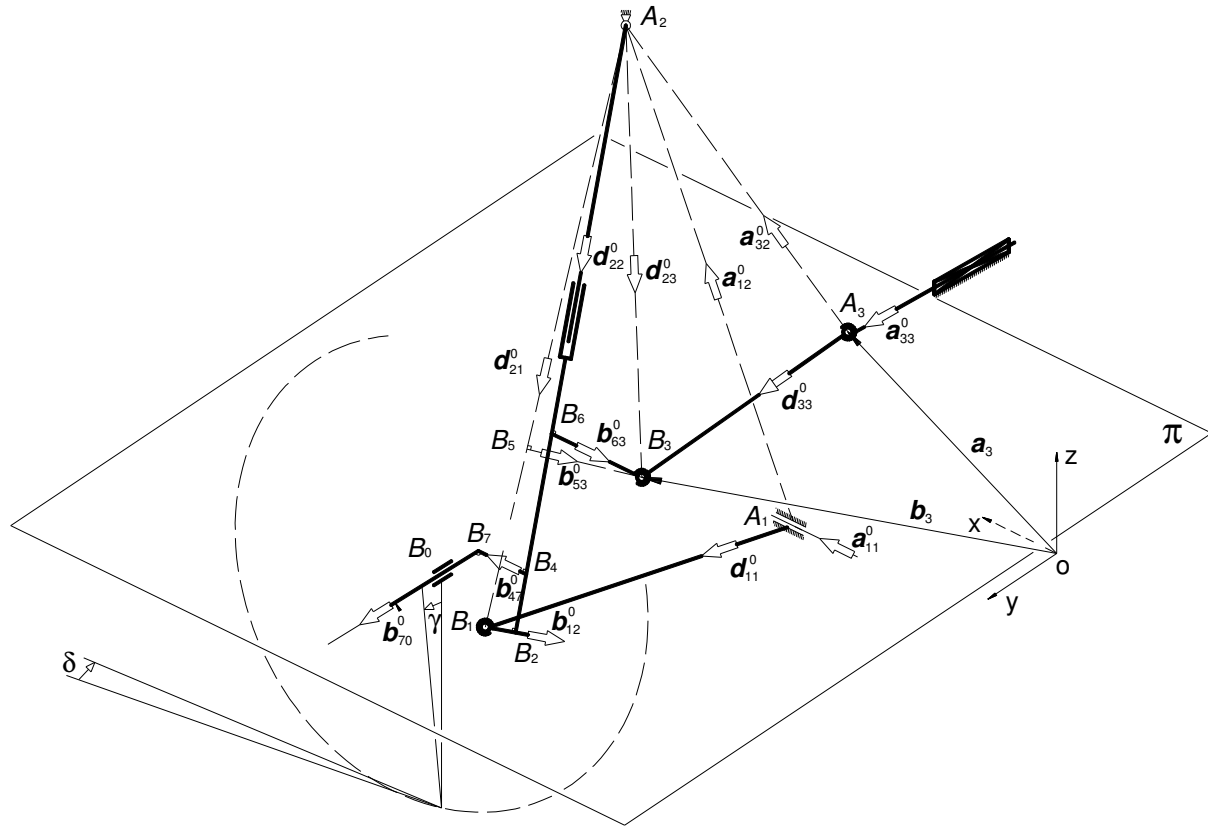
The purpose of this research is to analyse the rack and pinion steering linkage with McPherson suspension. From this analysis, the dimension of the planar linkage model and the position of the projection plane of the steering linkage that give minimum steering error can be obtained. The steering error is defined as the difference between the actual angle made by the outer front wheel during steering manoeuvers and the calculated angle for the same wheel based on the Ackerman principle [1].
The considered slider-rocker mechanism (*P-R-R-R*) can generate, for a given slider (rack) displacement, a specified maximum rocker (steering arm) swing while simultaneously ensuring best transmission angle characteristics (i) without and (ii) with imposing linear correlation between input and output. For case (i), the multi-dimensional space of the optimization problem is inspected using partial-global-minimum plots, and some interesting properties are revealed [1]. Case (ii) is formulated as a multi-objective optimization problem that is solved using a modified Normal Boundary Intersection method. To assist practicing engineers with their designs, parametric design charts accompanied by transmission angle, input–output linearity error, and torque-to-force multiplication factor diagrams are also provided in [1].
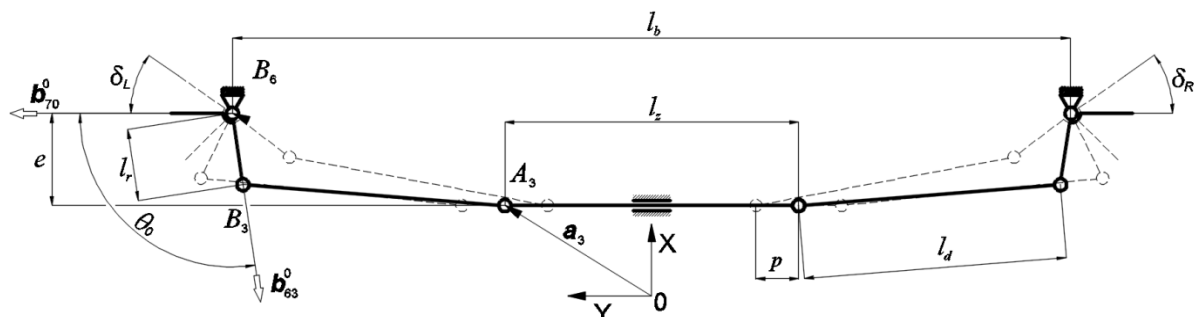
The synthesis of the slider-rocker (*P-R-R-R*) mechanism for given limit positions can be performed graphically. However, satisfying good motion transmitting characteristics is not guaranteed, and a trial-

and-error search must be performed until acceptable mechanical-advantage or transmission angle properties are satisfied. To avoid overloading the links and joints, it is recommended that the transmission angle remains between 45°and 135°[2, 5].

Figure 1. The scheme of the steering system with rack and pinion gear and with McPherson suspension. Notations: A_1 , B_0 – revolute joints; A_2, A_3, B_1, B_3 – spherical joints; B_6 – cylindrical joint; $a_{ij}^{\,o}$ – unit vector of the line connecting points A_i and A_j ; $d_{ij}^{\,o}$ – unit vector of the line connecting points A_i and B_j ; $b_{ij}^{\,o}$ – unit vector of the line connecting points B_i and B_j ; δ – wheel steer (toe) angle; γ – wheel camber angle; a_3 – position vector of the point A_3; b_3 – position vector of the point B_3 with respect to the origin of the body reference system $Oxyz$; π – plane parallel to Oxy.

Figure 2. The simplified planar model of the steering linkage. Notations: $(A^l_3 A^p_3 = l_z)$ – the rack as input; $(A_3 B_3 = l_d)$ – the tie rod; $(B_3 B_6 = l_r)$ – the steering arm; p – displacement of the rack; δ_L – steer angle of the left wheel; δ_R – steer angle of the right wheel; e – the distance from and the rack placement to the steering axis (off-center of the rack with respect to the steering axis).

2. Objective
The work was aimed at the simplification for modelling and optimization of steering system with rack and pinion and McPherson suspension. The simplified planar model is used for analysis and design of slider-rocker (rack-steering arm) mechanisms for function generation, with transmission angle taking into account.

3. The simplified planar model
The planar model of the six-bar steering linkage (Fig.2) consists of the rack, two tie rods and two steering arms of the wheel knuckle. This side take-off configuration is more common in passenger cars. The analytical method used to solve the nonlinear problem for position and displacement analysis of this mechanism is described as follows: 1. For the assumed value of δ_L the corresponding value of p is calculated by using formulae (1)-(3); 2. To evaluate corresponding angle δ_R the formulae (4)-(11) are used.

$$\theta_1 = \pi - (\alpha_2 + \delta_L) \tag{1}$$

$$\Delta_1 = l_d^2 - (l_r sin\theta_1 + e)^2 \tag{2}$$

$$p = \sqrt{\Delta_1} + l_r cos\theta_1 - \frac{1}{2}(l_b - l_z) \tag{3}$$

$$A = l_r[p - \frac{1}{2}(l_b - l_z)] \tag{4}$$

$$B = l_r e \tag{5}$$

$$D = \frac{1}{2}\left[l_d^2 - e^2 - \left(\frac{1}{2}(l_b - l_z) - p\right)^2\right] \tag{6}$$

$$Acos\theta_2 + Bsin\theta_2 = D \tag{7}$$

$$\theta_2 = 2 \cdot \text{arctg}\left[\left(-B \pm \sqrt{A^2 + B^2 - D^2}\right)/(A + D)\right] \tag{8}$$

$$\delta_R = (\alpha_2 + \theta_2) - \pi \tag{9}$$

$$\Delta_2 = l_d^2 - (l_r sin\theta_2 + e)^2 \tag{10}$$

$$p = \frac{1}{2}(l_b - l_z) - \sqrt{\Delta_2} - l_r cos\theta_2 \tag{11}$$

Note that + and – signs in equation (8) correspond to the two different assembly positions of the mechanism, where only + sign is applicable for the steering linkage.
The steering axis is determined by the centers of two spherical joints (A_3 and B_3) and its orientation is described by using: σ - kingpin inclination angle and τ - caster angle, using the formulae [3, 4, 5]
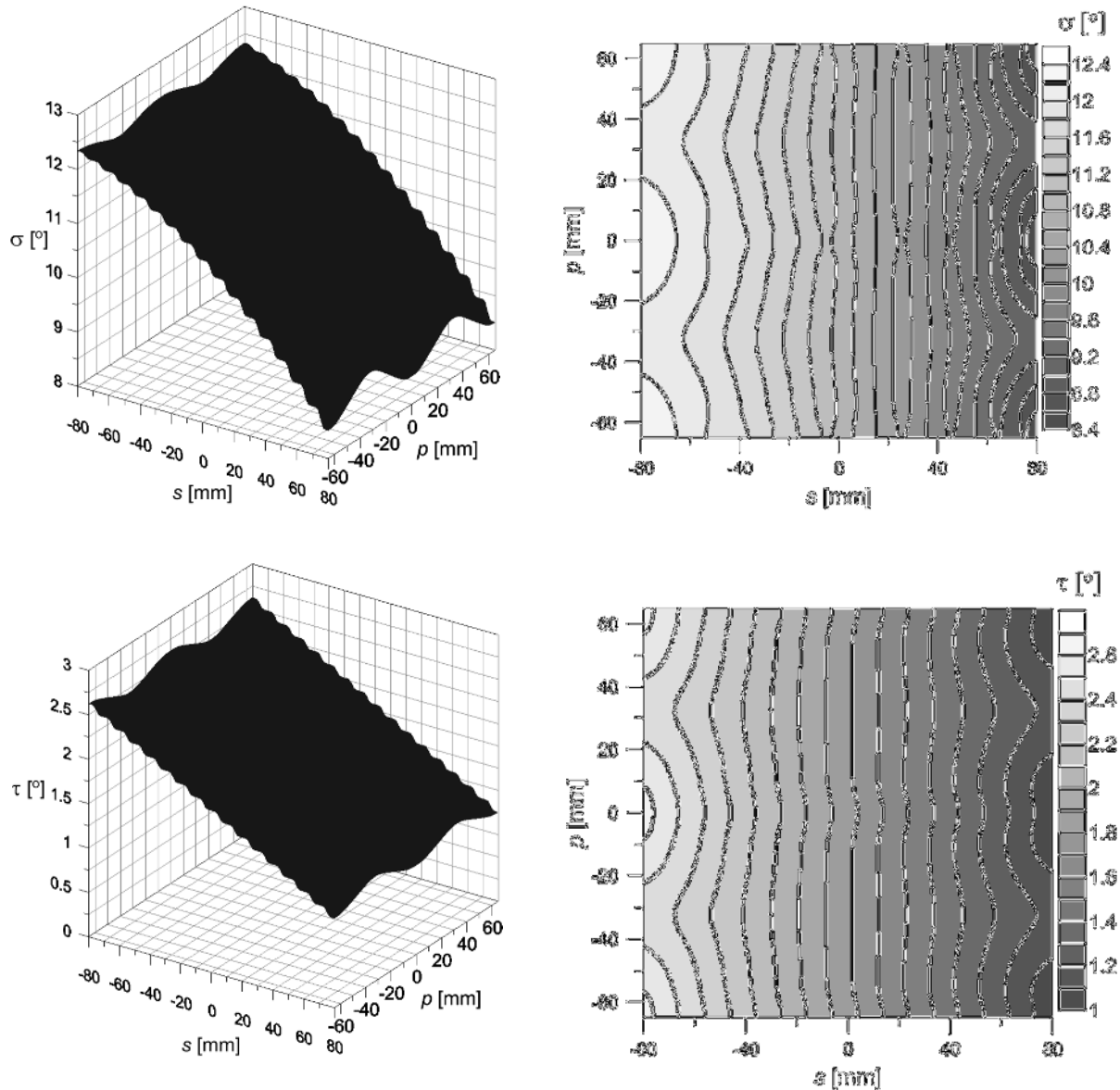
$$\sigma = arctg\left(-d_{21y}^o/d_{21z}^o\right) \tag{12}$$

$$\tau = arctg(-d_{21x}^o/d_{21z}^o) \tag{13}$$

where $\boldsymbol{d}_{21}{}^o = [d_{21x}^o \quad d_{21y}^o \quad d_{21z}^o]^T$ – unit vector of the steering axis, its coordinates are dependent on two variables: s – spring deflection of the suspension strut and p – rack displacement.
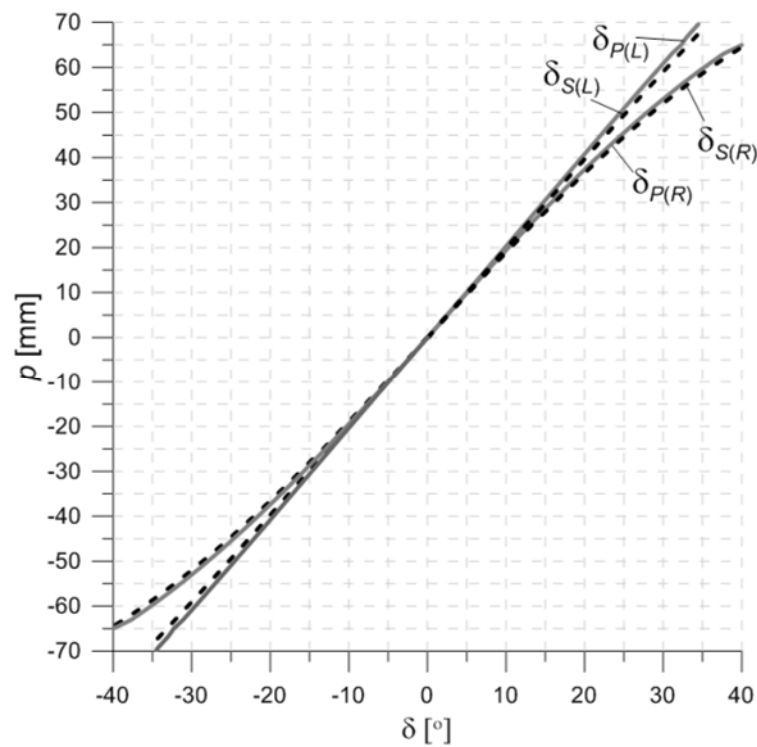
4. Numerical example
Spatial mechanism of the steering system (VW Golf) is dimensioned as follow: d_{11}= 395,8; b_{63}=126,0; d_{33}=435,7; b_{12}=57,3; b_{13}=124,4; b_{26}=51,5; $\boldsymbol{a}_1$ = [245,7; 308,5; 22,0]^T; $\boldsymbol{a}_2$ = [271,9; 578; 616,7]^T; $\boldsymbol{a}_3$ = [147,0; 239,9; 51,0]^T [mm], α_1= $arc\ cos$(-$\boldsymbol{b}_{70}^o \cdot \boldsymbol{d}_{22}^o$) = 81,5°; α_2= arc cos($\boldsymbol{b}_{70}^o \cdot \boldsymbol{b}_{63}^o$) = 98,5°;
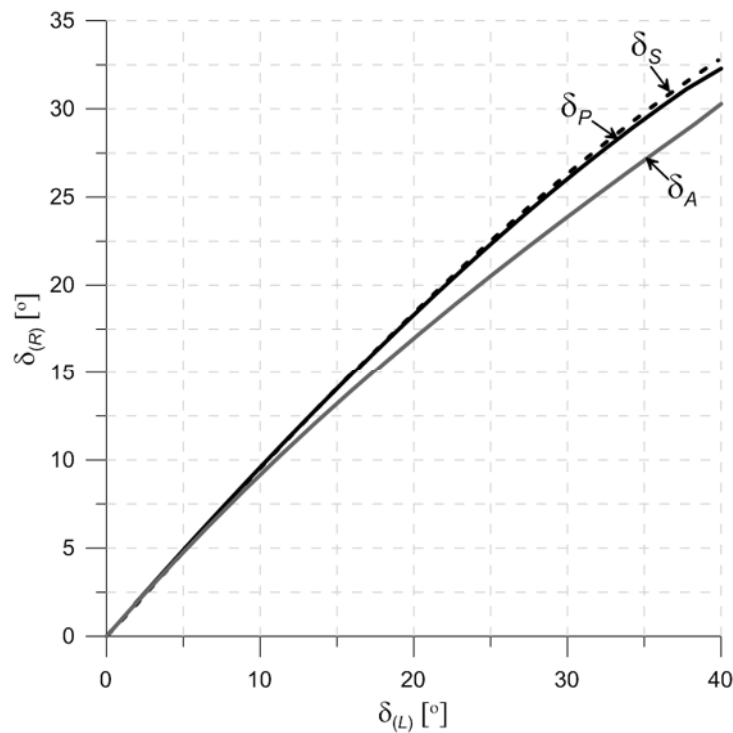
Planar mechanism considered as projection of the spatial steering linkage on the horizontal plane (π) passing through point A_3 is described by using the following dimensions: $a_3 = [147; -239]^T$; $b_3 = [179; -674]^T$; $b_6 = [296; -684]^T$ [mm]; $\alpha_2 = arc\ cos(b_{70}^o \cdot b_{63}^o) = 98,5^o$; $e = 150$; $l_b = 1370$; $l_d = 435$; $l_r = 117$; $l_z = 480$; [mm]
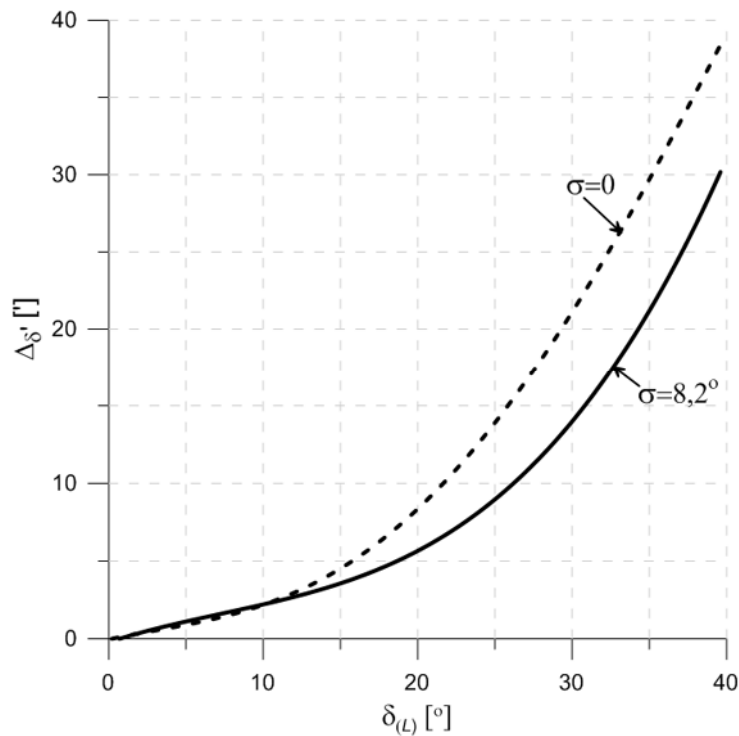

Figure 3. The surfaces describing the relations: a) kingpin inclination σ (s, p); b) caster angle τ (s, p) obtained for McPherson suspension according to data for example (VW Golf).
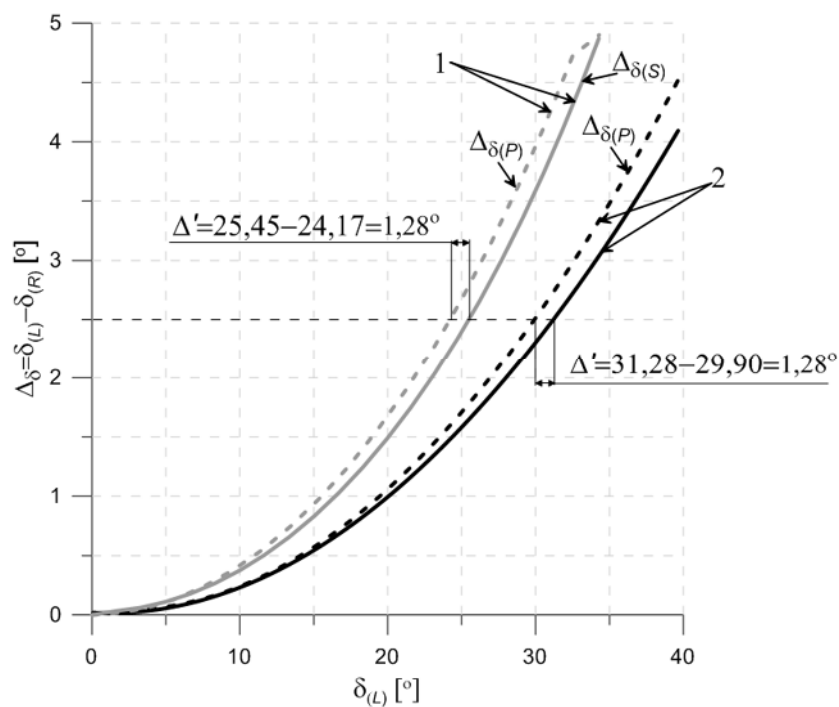
Figure 4. The relations between p - the rack displacement and the wheel steer angles: δ_L - left side and δ_R - right side, obtained using data for example 1 (VW Golf): δ_S – spatial model; δ_P – planar model.
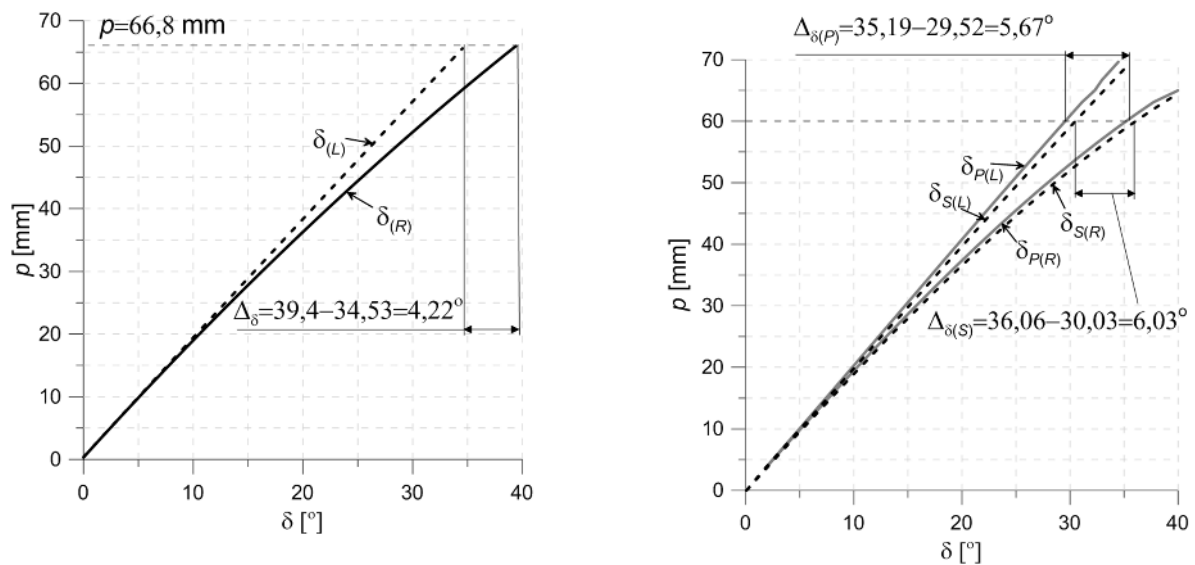

Figure 5. The relations between the wheel steer angles: δ_L - left side and δ_R - right side, obtained using data for example 1 (VW Golf): δ_S – spatial model; δ_P – planar model; δ_A – according to Ackermann.

Figure 6. The relations: $\varDelta_\delta$ - difference between the calculated and measured right wheel steer angle, obtained using data for example 2 (Ford), with inclination angle: $\sigma = 8,2^\circ$ – for the spatial model and $\sigma = 0$ – for the planar model as functions of: δ_L - wheel steer angle of the left side [6].

Figure 7. The relations: $\varDelta_\delta$ - difference of the right and left wheel steer angles, obtained using data for example 1 (VW Golf) and 2 (Ford), calculated for: $\varDelta_{\delta(S)}$ – spatial model; $\varDelta_{\delta(P)}$ – planar model as functions of δ_L - wheel steer angle of the left side, $\varDelta' = \varDelta_{\delta(S)} - \varDelta_{\delta(P)}$.

Figure 8. The relations: p - rack displacement as functions of $\delta_{(L)}$ - wheel steer angle of the left side, obtained using data for example 1 (VW Golf), calculated using: δ_S − spatial model; δ_P − planar model, $\Delta_\delta = \delta_{(L)} - \delta_{(R)}$ - difference of the right and left wheel steer angles

5. Analysis results

The differences between the results obtained for planar and spatial models arise, above all, from the relations describing the position and displacement of the wheel knuckle in planar model as functions of the rack displacement only.

For the model parameters, characteristics and the conditions as specified for planar model, the minimum differences with respect to spatial model are taken as the selection criteria for the position of projection plane.

6. Conclusions

The optimum position of the projection plane for planar model is situated near to the vertical coordinate of the rack spherical joints. The lengths of the tie rod and the steering arm can be selected as the lengths of the respective projections.

In the method presented herein, some different solutions have been obtained using a spatial model and a planar model for the calculations. In the first step, the steering linkage can be optimized using the simplified planar model. Finally, the three-dimensional model, best representing the properties of a real motor vehicle, should be used for the verifying calculations.

References

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