

Method of planning a reference trajectory of a single lane change manoeuvre with Bezier curve

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Abstract. For a comprehensive simulation of vehicle steering process it is vital to model the decision process of planning a trajectory shape and process of the selected trajectory. A single lane change manoeuvre is only slightly restricted by the road geometry. There are also other requirements of a possible trajectory of movement, such as the continuity of change (derivative) of curvature, maximizing the passenger's comfort measured with appropriate indicators based on variables of motion dynamics or parameters of motion trajectory which influence that dynamic. This article presents a suggested method of automatic generation of trajectory of single lane change manoeuvre. The proposed method can be used as an integral part of driver models and is based on a combination of two symmetrical Bezier curves optionally supplemented with a straight lane connector. The method meets the requirements of a trajectory shape, which results from optimizing the value of parameters controlling Bezier curve based on minimizing the curvature and the resulting lateral acceleration while preserving the continuity of curvature derivative of the planned trajectory.

1. Introduction

The process of driving a vehicle in terms of lateral dynamics can be defined as a task of choosing and following the trajectory of movement which does not exceed the area available for the manoeuvre and ensures the safety and desired comfort in terms of forces resulting from the performed manoeuvre. A mathematical description of the processes realized during this tasks leads to two separate driver models:

- trajectory planning,
- following of planned trajectory.

The latter group can be further divided into compensating and anticipating models [1]. In both cases the knowledge of planned trajectory is required. This leads to the necessity to define it arbitrarily or with trajectory planning models.

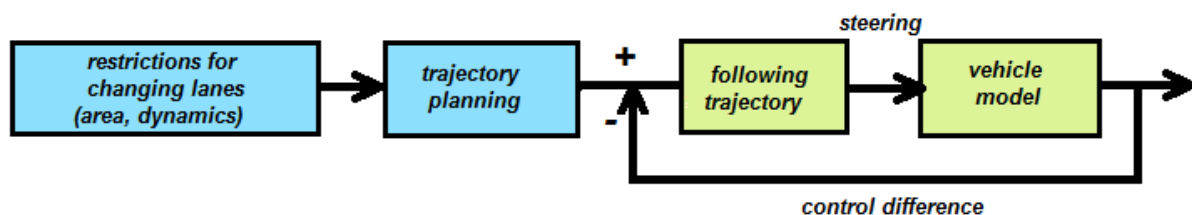


Figure 1. Simplified diagram of steering of autonomous vehicle [own study based on 1].

Besides being used in computational simulations, such models have been recently important in designing the steering systems for autonomous vehicles which perform driver tasks. Unlike

computational simulations, the calculations for such vehicles have to be accurate and fast, and realized in real time.

A single line change is one of the most common elementary manoeuvre in road traffic. But this does not mean that it is simple and safe one. Statistics clearly show that many collisions and accidents occur during line change manoeuvres [2]. Additional difficulty in trajectory generation for this manoeuvre is result of practically infinite variants of its realization. Geometric restrictions in form of lane width and manoeuvre distance are not significantly decreasing number of available movement paths.

2. Analysis of methods of generating trajectories presented in literature

The literature includes many concepts and methods to solve the problem of planning a trajectory for a single line change. The way of performing this manoeuvre may be considered from two aspects. One results from an observation, according to which drivers performing this manoeuvre turn the steering wheel in such a way that its angle changes in a way resembling a sine function [3]. Based on this observation the trajectory can be planned assuming a shape which resulting from a sinusoidal steering input. This method was examined in dissertation [4]. The author tested a steering input in the shape of a sine wave with different amplitudes and frequencies obtaining a range of vehicle motion trajectory, which were subjected to the optimization process to choose one with favourable lateral acceleration and geometry not exceeding the available area .

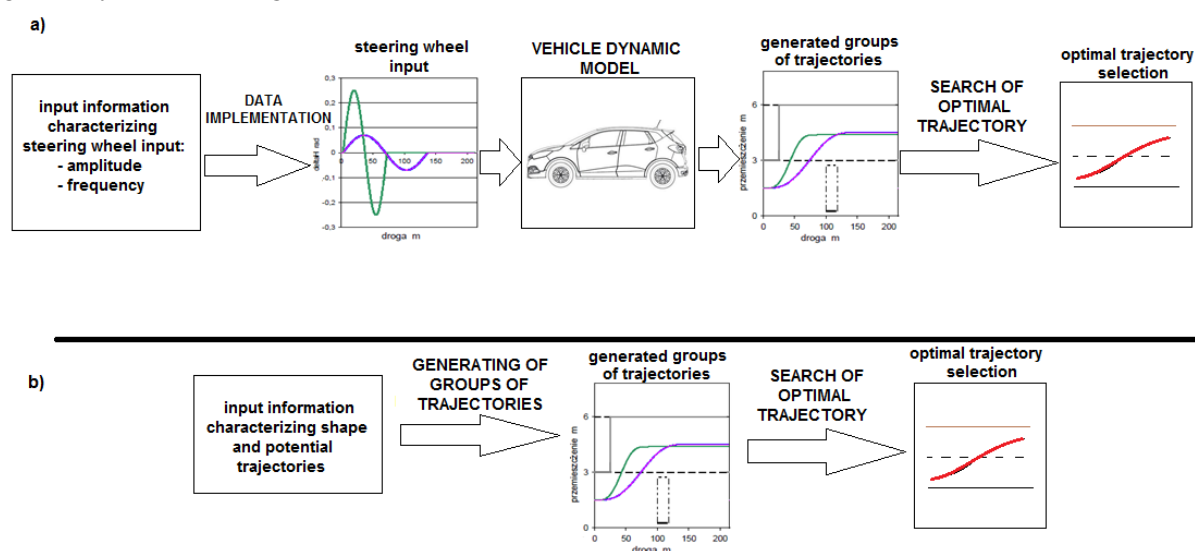


Figure 2. Methods of generating and optimizing the trajectory of a single lane change [own study].

The transition from a steering input to a trajectory requires an additional element, namely a dynamic model of vehicle (figure (2a)). Changing steering input parameters, performing calculations and simulation and finally optimizing the achieved trajectories may prolong the time of manoeuvre performance to such an extent that the method cannot be used in real time steering.

The other group includes the methods which involve a generation of potential trajectories first and then optimizing without the use of dynamic vehicle models (figure (2b)). While optimizing the condition of not exceeding the designated area is met already when the trajectory is generated and lateral acceleration is calculated based on the vehicle speed and the trajectory curvature.

Since the method proposed here belongs to the latter group, the next part focuses on presenting the literature concepts which cover only the methods beginning with generation of potential trajectories.

a) circular trajectory [3]

- **method description** – vehicle trajectory is composed of two arcs of constant radii connected with a line segment;
- **number of sections** – three;
- **advantages** – a simple division into sections, utilizing only two mathematical functions, necessity of a relatively short computational time and shortest time of physical realization;
- **disadvantages** – rapid changes in momentary turning radii at points connecting the sections of the trajectory, discontinuities of the curvature, large and rapid changes of lateral acceleration values, low comfort level.

b) composition of a few arcs of different radii [3]

- **method description** – development of the previous method, both curve segments are made up by several arcs with the radii adjusted to smoother transitions and more control over the trajectory shape;
- **number of sections** – several (4-9);
- **advantages** – using only two mathematical functions, easy implementation in algorithms generating movement trajectory, smoother transition between sections than in the previous method;
- **disadvantages** – discontinuities of curvature between sections, difficult to modify the trajectory shape using only one parameter, rapid changes in lateral acceleration – lower comfort level.

c) constant radii arcs connected with clothoids [5]

- **method description** – this method involves the use of clothoid curve to connect two segments with different curvature created with other methods. The use of clothoid curve (as transition curve) helps to create smooth transition between the straight line and the arc section ;
- **number of sections** – $2n - 1$, where n is the number of sections created with the basic method;
- **advantages** – smooth changes in lateral acceleration, continuous curvature;
- **disadvantages** – additional trajectory components prolong the calculation time, necessity to numerically calculate Fresnel integral, potential problems with computing time in real time calculations.

d) 5th order polynomial trajectory [3]

- **method description** – polynomial curve of the 5th order with properly selected parameters was proposed , as a function that relatively well describes the trajectory of a single lane change maneuver. What is important, the whole trajectory is created with only one curve;
- **number of sections** – one;
- **advantages** – one equation describing the whole trajectory, ensured continuity of curvature, short computational time;
- **disadvantages** – limited possibilities to modify the trajectory shape (none of input parameters is directly related to the curve shape), a long time of physical performance compared with the previous methods.

e) clothoids obtained by polynomial approximation [10]

- **method description** – a single lane change trajectory consisting of four symmetrically connected clothoids. The use of polynomial approximation helps to shorten the computational time to define curve shape ;
- **number of sections** – four;
- **advantages** – lower computational costs to determine clothoids, continuity of curvature;

- **disadvantages** – no significant disadvantages.

The review of several methods used to determine the reference trajectory of a single lane change manoeuvre shows that there is no single accepted solution. The advantages and disadvantages of different concepts should be considered with consideration of the chosen optimization parameters and the purpose of performed manoeuvre. A large number of possibilities in the choice of optimization parameters was indicated by G. Prokop [7]. For a single lane change manoeuvre some of them are minimization of time needed for manoeuvre execution, achieving the highest comfort of passengers and the related minimized lateral acceleration and minimized use of brakes or throttle controls.

The authors of this article suggest their own method to plan the trajectory based on generating a range of possible trajectories. And, in comparison with other methods, it combines a low computational complexity with the ensured continuity of curvature and possibility to optimize for a desired level of comfort measured with lateral acceleration values. The suggested method was verified in terms of achieving the assumed goals.

3. Method of determining a single lane change trajectory with symmetrically connected Bezier curves

3.1. General structure of single lane change trajectory planning

Based on an analysis of available methods the authors identified a general structure of actions useful to develop a method for trajectory planning. It assumes that the following subtasks should be performed while planning and optimizing the manoeuvre trajectory:

1. **goal definition** directly related to the conditions of manoeuvre execution. For example an emergency lane change to avoid collision or slow paced lane change with care for comfort,
 - definition of **optimization criteria**, constituting the basis for selecting final trajectory;
2. selecting a **method to designate a virtual desired trajectory**, which will help to achieve the defined goal most efficiently;
3. **Optimization** of the created solutions in order to indicate one movement path which best meets the defined criteria.

3.1. Description of a method to designate trajectory and desired vehicle speed

The suggested method to designate the virtual desired trajectory of the single lane change manoeuvre is based on an assumption that a driving line can be described with two Bezier curves symmetrical in relation to the centre of the road section where the manoeuvre takes place. The curves' shape is determined by the location of the so-called control points. Two control points from first curve, the initial and the final one, are synonymous with the beginning point of the manoeuvre and its middle. By analogy in the case of the other curve the initial and final control points are synonymous with the middle and final point of manoeuvre (figure 3). In accordance with the figures below the lane change manoeuvre is divided into two parts (II and III) in the form of Bezier curves and straight line sections before and after the manoeuvre (I and IV).

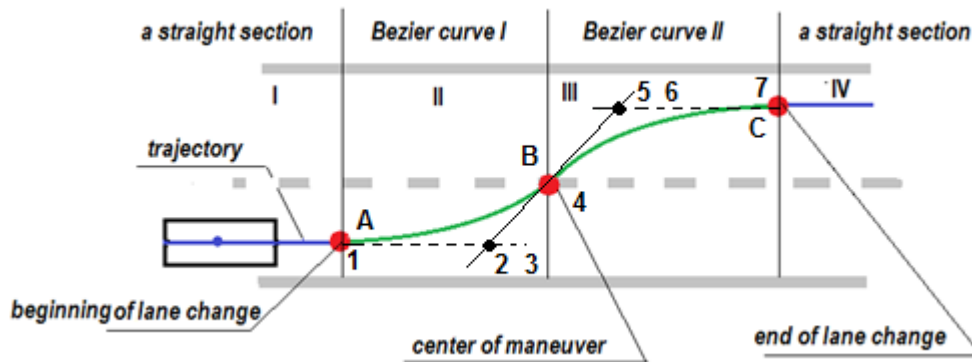


Figure 3. Trajectory of single lane change manoeuvre in proposed method [own research].

Bezier curves are a type of parametric curves whose shape depends on the position of control points. The curve interpolates the beginning and ending point and approximate intermediate points [8]. An increasing number of control points can lead to an undesired shape, so it is common to use the curves of up to 3rd order combined into a spline. The coordinates of curve points are described with two mathematical functions ($B_x(t), B_y(t)$) of auxiliary parameter t with values range of 0-1 (domain of polynomials describing Bezier curve). Bezier curve $B(t)$ with any number of control points is described with the following equation [8]:

$$B(t) = \sum_{i=1}^n \binom{n}{i} (1-t)^{n-1} t^{i-1} P_i \quad (1)$$

where:

$B(t)$ – coordinates of Bezier curve points,

t – auxiliary parameter with values range of [0, 1],

P_i – control point,

n – number of control points,

$n - 1$ – order of Bezier curve,

$\binom{n}{i}$ – binomial expansion:

$$\binom{n}{i} = \frac{n!}{i!(n-i)!} \quad (2)$$

The suggested method is based on the 3rd order Bezier curves defined by 4 control points and trajectory is assumed to be two-dimensional. If control points are defined as: $P_1 = (x_1, y_1)$, $P_2 = (x_2, y_2)$, $P_3 = (x_3, y_3)$ and $P_4 = (x_4, y_4)$, then the trajectory is described by the equations:

$$\begin{cases} B_x(t) = x_1(1-t)^3 + 3x_2t(1-t)^2 + 3x_3t^2(1-t) + x_4t^3 \\ B_y(t) = y_1(1-t)^3 + 3y_2t(1-t)^2 + 3y_3t^2(1-t) + y_4t^3 \end{cases} \quad (3)$$

for $0 \leq t \leq 1$

This method is shown in a graphic form in figure 4. Its realization is based on the following steps:

1. Definition of input data:

- restrictions of available area – indication of the positions of points A, B and C (figure 3) defining the beginning, centre and ending of the lane change manoeuvre,
- definition of a range of the alpha parameter, described as an angle between the vehicle trajectory at the centre point and the axis of the road. The values of the alpha influence the shape and curvature of the trajectory by positioning control points.

2. **Determining of acceptable trajectories** – generating Bezier curves for the assumed restrictions and alpha values and rejecting the trajectories with too small alpha value result in control points positioned before the starting point of the trajectory.
3. **Selecting the optimal trajectory:**
 - calculating the parameters which describe optimization criteria, i.e. the value of maximum curvature for all the acceptable trajectories,
 - selecting the optimal trajectory: finding the trajectory with the lowest value of maximum curvature.
4. Calculating the maximum vehicle speed restricted by comfort and safety parameters for defined manoeuvre.

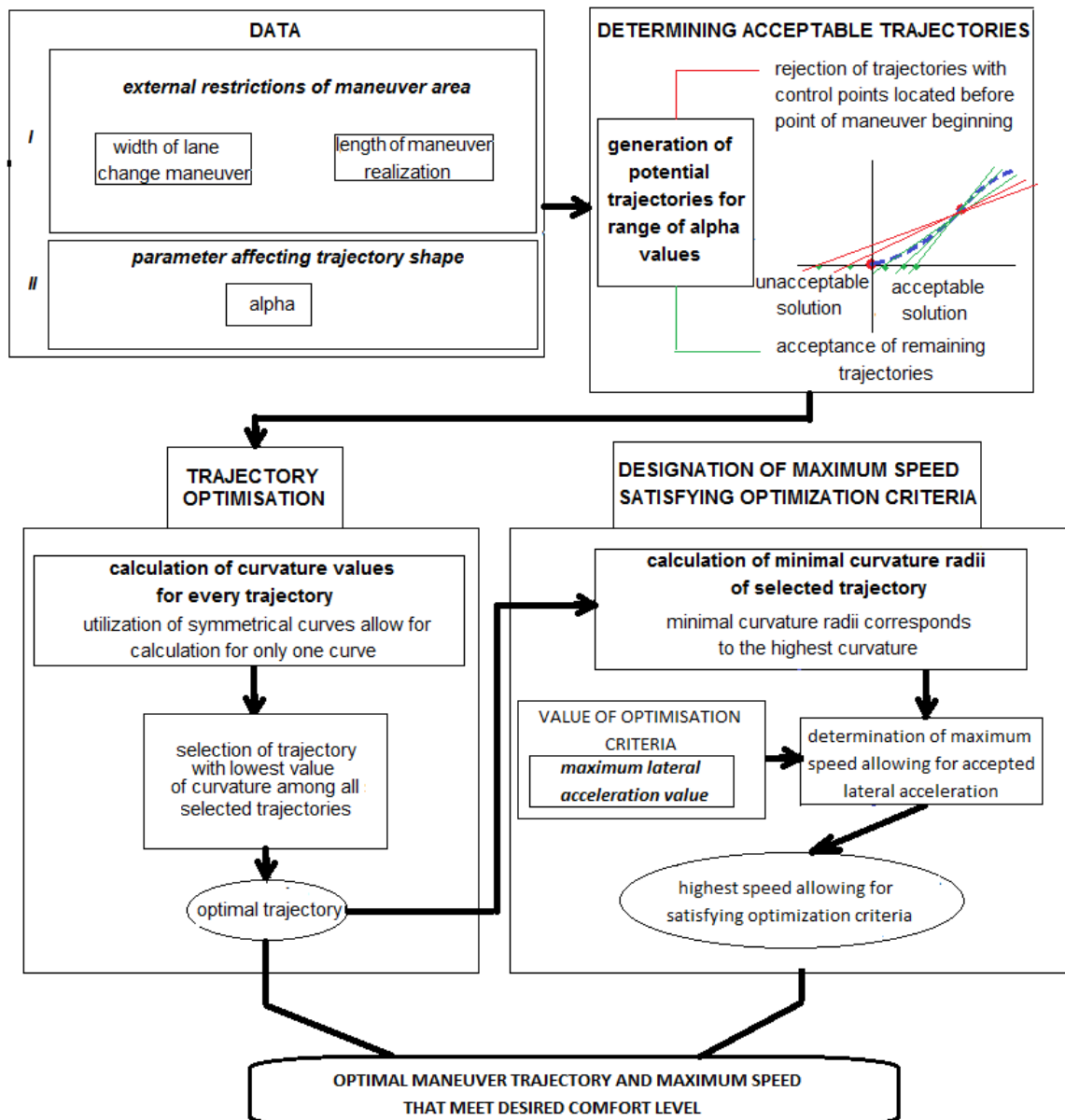


Figure 4. General idea of proposed method [own research].

4. Method implementation

The proposed method was implemented in the MATLAB environment with specifically developed procedure for determining possible trajectories and performing optimization process.

4.1. Definition of input data

The data which describe the restriction of the area for the manoeuvre is composed of the width of lane change and the length of the route section between the beginning and end of the manoeuvre. The first value results from road lane width and is easy to define. It must be noted that the generated trajectory describes the motion of gravity centre and a strict width of lane is not directly used as input information. The length of manoeuvre area can be set in two ways. The first directly as the length of road available for lane change, which is results from the distance to the obstacle, for example; this approach is used in this article. The other one indirectly assumes some vehicle speed and average time of manoeuvre execution, depending on conditions (emergency avoidance of an obstacle or a comfortable lane change). In the literature the average time for a single lane change manoeuvre in extra urban traffic is around 5–7 s [9,10].

Alpha parameter, which influences the trajectory shape, is set as range of possible values. While verifying the method the assumed range was 100 values from 0 to 15°. This was based on a definition of this parameter as an angle between the trajectory centre and the road axis. Data from literature suggest that this angle should not exceed 45° [11].

4.2. Method of generating potential trajectories

Every generated manoeuvre trajectory passes through three defined control points which geometrically restrict its shape. The position of other points (approximated by curve) define shape of trajectory need to be set during path generation (figure 3). Their location is determined by the alpha parameter value. One of the most important conditions applying to their position is the requirement to provide the continuity of trajectory curvature between all the manoeuvre sections. Parametric continuity between two Bezier curves is ensured if tangential vectors of two sections are opposite and of equal length. The way to achieve the continuity of curvature and an example of the number and positions of control points are shown in figure 3.

A distinctive element of the method proposed by the authors is the overlapping of two control points for both curves. In figure 5 these points are indicated with the numbers 2, 3 and 5, 6. Permissible location of these control points is on an intersection of extension of the movement line before the manoeuvre and line crossing centre point at an angle equal to alpha. This narrows possible trajectories and simplifies the procedure to generate trajectories.

It should be noted that low values of alpha can lead to control points placed outside the area to execute the manoeuvre, i.e. left of point 1 and right of point 7 (figure 3). In this situation the trajectory is significantly deformed and should be removed as unacceptable. Examples of acceptable trajectories with 4 different values of alpha are shown in figure 5.

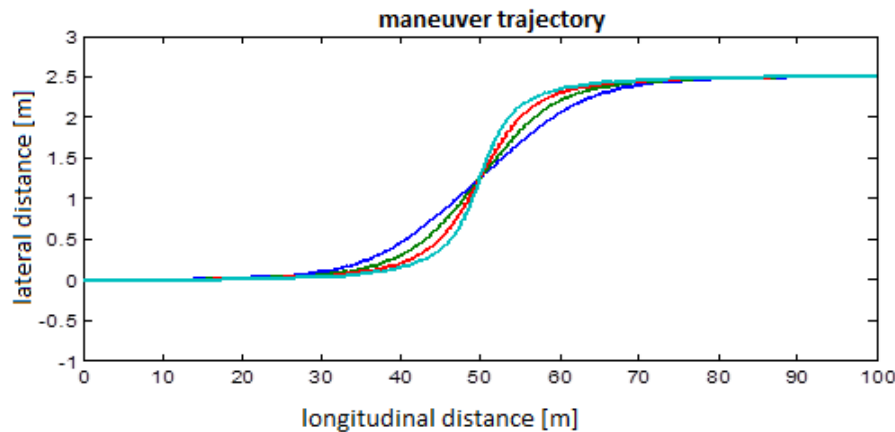


Figure 5. Example of trajectories of lane change manoeuver.

4.3. Trajectory optimization

The optimization of trajectory should lead to denoting one solution as best in terms of the chosen optimization criteria. In this article the criteria include the minimizing of the maximum trajectory curvature. Since the trajectory is described by parametric equations it is possible to calculate its curvature with the following formula [8]:

$$k = \frac{|y''_0 x'_0 - x''_0 y'_0|}{(x'^2_0 + y'^2_0)^{\frac{3}{2}}} \quad (4)$$

While optimizing the trajectory the stage accurate lateral acceleration value representing comfort threshold (optimization parameter). The course of curvature changes of the used Bezier curves is characterized by only one global maximum value denoting the greatest value of curvature. It is the value used to choose the optimal trajectory. The highest curvature reflects the turning radius and consequently the highest lateral acceleration. In further calculations only the maximum values of curvature of generated trajectories were taken into account. The optimal trajectory found in this way means that the goal of selecting trajectory with lowest lateral acceleration was achieved.

4.4. Determination of highest allowable speed for adopted indicators

The highest acceptable vehicle speed for the following selected trajectory should not lead to exceeding the threshold of passengers comfort or threshold of safely performing manoeuver. Determining the value of such speed means achieving the next step, i.e. ensuring the fastest executing of manoeuver while maintaining comfort and safety. The procedure designating the highest acceptable speed begins with finding the minimal turning radii, which is the inverse of trajectory curvature.

To designate the speed a threshold of lateral acceleration must be determined in order to achieve the desired level of comfort or safety. Research performed on 2014 [12] indicates that while changing lanes in ideal weather conditions 85% of all lateral acceleration values were close to 1 m/s² with highest values around 3,6 m/s². The authors of this article assumed a comfort threshold of 1.6 m/s², an average comfort 3,6 m/s² and a threshold of discomfort at 5 m/s². These values correspond to other research results [15], where the comfort threshold was considered at around 2 m/s². Converting the formula for lateral acceleration allow for direct calculation of desired value of maximum speed during manoeuver realization for assumed value of acceleration $a_{y\ max}$ and turning radii r :

$$v_{\ max} < \sqrt{a_{y\ max} r} \quad (5)$$

5. Verification of the proposed method

The correctness of the proposed method with regard to achieving the set objectives was verified in simulation, performed in MATLAB environment with the use of own procedures. The goal of experiment was to perform the process of planning movement trajectory with the use of the proposed method for a single lane change manoeuvre and designating the highest acceptable speed with lateral acceleration less than threshold of comfort set at 1.6 m/s^2 . The article presents the results of tests for three events with different manoeuvre lengths. Input information with assumed parameters alpha and results of optimization are presented in table 1.

Table 1. Input information and results of optimization for three events with different manoeuvre lengths.

No.	Lane change width (m)	manoeuvre distance (m)	comfort criteria $a_{\max}(\text{m/s}^2)$	range of alpha (deg)	optimal alpha (deg)	highest accepted speed (km/h)
1	2.5	30	1.6	0 – 15	7.4	28
2	2.5	50	1.6	0 – 15	4.6	47
3	2.5	100	1.6	0 – 15	2.2	94

The acceptable trajectories from the latter case are shown in figure (6a) with the optimal trajectory highlighted in red. Figure (6b) presents the curvature values (for the first curve of every trajectory) which allow to perform optimization process.

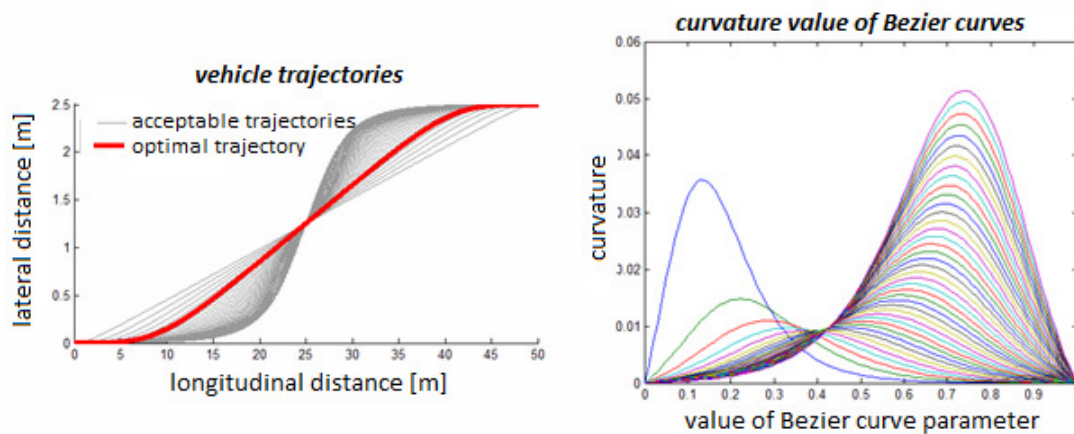


Figure 6. a) acceptable trajectories of single lane change manoeuvre, and optimal trajectory highlighted in red. b) values of curvature (for first Bezier curve) of acceptable trajectories.

Based on the achieved results it can be stated that the proposed method allows to determine one optimal trajectory for the considered manoeuvre. The quantitative results in the form of the highest acceptable speed and angle of vehicle deflection are within the range proposed in another research [13] and consistent with the values anticipated by authors. In the method [11] the maximum vehicle deviation (equivalent to alpha parameter in the method described in this article) for the speed around 70 km/h is around 4° , which is close to the values presented in table 1. An increase in manoeuvre length with a constant level of comfort threshold (lateral acceleration) results in a decrease of alpha parameter and an increase of permissible speed to perform a lane change manoeuvre.

6. Conclusions

The proposed method for generating the planned trajectory of a single lane change manoeuvre was implemented and verified in MATLAB environment and proved its effectiveness for choosing the optimal trajectory for the chosen criteria. The proposed method is characterized by:

- direct determination of the planned trajectory of the manoeuvre without the need for dynamic vehicle models,
- lack of need for test if the trajectory is inside the designated area due to it being initial assumption,
- single parameter optimization,
- trajectories with continuous change of curvature, consistent with physical capabilities of changing steering input and smooth trajectory,
- possibility of optimization of trajectory based on comfort or safety in terms of vehicle lateral dynamics.

With respect to other methods, presented in part 2 of this article, the proposed method is relatively computationally simple and allows to achieve a trajectory which fulfils the basic requirements of the generated trajectory, i.e. the smoothness of curvature changes and possibility to minimize curvature.

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