

Contact point of bush – sprocket tooth depending on pitch differences of bush chain transmissions

R Velicu¹, R Saulescu¹ and L Jurj¹

¹Product Design and Environment Department, Transilvania University of Brasov, Brasov, Romania

E-mail: rvelicu@unitbv.ro

Abstract. Chain transmissions have a large use in industrial applications. The study of kinematics and dynamics of this kind of transmissions has a lot of space to cover due to different type of chains, different geometries, standardized or not, or even different ISO and AGMA standards. This paper is referring to the bush chain transmission based on ISO standard for geometry and dimensions. The pitch of the chain is bigger than the pitch measured on the sprocket and the difference must be in a certain range. Depending on this difference, the contact point between bush and the tooth varies. It also varies depending on the position around the sprocket. All these are influencing the dynamic of the chain transmission and the friction losses. The paper analyses the kinematics of chain link depending on the rotation angle of the sprocket (θ), establishing the contact point between bush and tooth, for a given tooth profile dimension and pitch differences. An important result is the point of entering in contact between bush and sprocket tooth with an influence on the normal and transversal forces in the chain. The results of numerical simulations are the base of drawing conclusions and recommendations on the pitch differences considering criteria like minimum friction and minimizing the transversal vibration.

1. Introduction

A main branch of industry, which uses chain transmissions with bushes and short links, besides the agricultural and marine one, is the machine building industry, in which the chain wear implies the appearance of inevitable malfunctions produced by the induced vibrations.

A main objective to overcome these shortcomings is the optimization of the sprocket geometry, by optimization of the sprocket tooth profile, optimization proposed in [1]. This optimization must be based on geometry analysis of the bush – sprocket contact, analysis proposed in this paper.

Recommendations regarding the performance improvements of this kind of transmissions are given in [2], a study which had, as main objective, the investigation of the chain transmissions wear, which limits the operating capacity and within wear resistance which is stated to be influenced by many factors, among which the friction surfaces' geometry and chain tensioning can be considered. The wear of the bush and sprocket is studied in [3], in which it is concluded that, due to the structural characteristics, direct measurement of the wear is impossible, being necessary to know the contact surfaces' characteristics, for which there is stated that the bush is more worn out at the centre than at the ends (with approx. 30%), fact explained by the direct contact between the sprocket and the central part of the bush.



A laborious model for the contact between the bush and sprocket is presented in [4], in which there is done a comparative analysis between a circular profile and a real profile of the tooth shape. For these types of sprockets, the dynamics of such a transmission is presented, which, then, can be compared to the experimental tests, after which it is affirmed that the impact of the bushes with the sprocket leads to the development of chain vibrations. An interesting conclusion, proved approx. 35 years ago [5], refers to the operating capacity of the bush chain transmissions, which is judged in function of the bush – sprocket contact wear resistance.

Another step in the analysis of the bush – sprocket contact is performed in [6], where a comparative analysis of two sprocket tooth profiles (ISO 1395-1977 and ANSI ASA B29.1-1950) is presented, for which there is graphically represented the evolution of the contact angle for the rotation of the sprocket with different angles.

The present work is a continuation of the study presented in [6] and, in the same time, a development of those presented before by introducing an analytical model of the contact angle between the sprocket and the bush.

2. Problem statement

The dynamics of a chain transmission is laborious, having a complex behaviour regarding the impact between the bush and the sprocket due to the speeds and forces acting on the bushes and sprocket while they are in contact, produces some transversal and longitudinal vibrations and so, the flexibility of the links plays an important role.

An important aspect for improving these transmissions is to increase the reliability of the transmission as well as the improvement of the vibro-acoustic parameters. During the use of such transmissions, there are affected, not just the dimensions of the elements in contact (by wear), but also the general configuration, such as specific elongation of the chain, which is the main fact that leads to its replacement.

As it is known in the transmission operation, due to the specific elongation, the bushes have an angular movement against the tooth of the sprocket (at the fillet radius between the tooth profile and the dedendum circle of the sprocket), which creates friction and wear on the two contact surfaces. The contact surfaces are even bigger as the angle between the theoretic and real contact points is bigger, named hereinafter the contact angle (noted α_i). In this work, a generalized algorithm for contact angle determination is proposed, for a standardized ISO bush chain, which takes into account the geometry of the sprocket tooth with the assumption that, in the gearing process, the contact angle cannot exceed half value of the roller gap angle, which usually is limited to 120° . On the basis of this algorithm, there are followed the influence of the sprocket rotation, as well as the specific elongation of the chain over the behaviour of the contact angle.

Based on this algorithm, a numerical example for a chain transmission with bushes and short links is presented, for a transmission ratio equal to 1 and sprockets with 16 teeth. As a result, there is considered that is sufficient to determine the contact angle for the first four bushes in contact.

3. Geometric modeling of the transmission

The aim of this modelling is the determination of the contact point between the bush and the sprocket. Because of this, the main geometric parameters are considered to be: the pitch of the chain (p), the number of teeth of the sprocket (z) and the diameter of the bush (d_B). On the basis of these parameters, there can be determined the pitch radius of the sprocket (R_A), the radius of the dedendum circle of the sprocket, (r_A), the radius of the bush (r_B) and the angular pitch of the sprocket (τ) [5]. According to figure 1, the following notations can be made

$$A_0B_0 = r_A - r_B \quad (1)$$

for which $A_0B_0 = A_1B_1 = A_2B_2 = \dots = A_kB_{k+1}$

$$B_0B_{01} = p \cong d_0 \quad (2)$$

in which p is the standardized pitch of the sprocket, and d_0 can be determined as

$$d_0 = 2OB_0 \sin\left(\frac{\tau}{2}\right) = 2(R_A + r_B - r_A) \sin\left(\frac{\tau}{2}\right). \quad (3)$$

Considering that the chain during operation elongates up to a maximum value (x = approx. 2% of p) of its entire length, the elongated pitch can be determined, noted with $B_k B_{k+1}$ (figure 2)

$$B_k B_{k+1} = B_0 B_1 = d_0 + x. \quad (4)$$

In order to determine the elongated pitch of the chain transmission, the diagram from figure 1 is used, dimensions being exaggerated relative with the real case, in order to make the difference between the standard and elongated pitch.

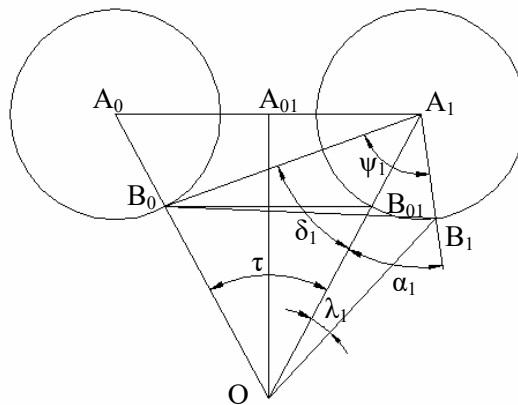


Figure 1. Diagram for main geometric parameters of the bush-sprocket contact.

The definition of the geometric parameters is done starting from the initial assembled position, in which the contact angle is zero ($\alpha_0=0$) (see figure 2.) and the considered first contact between the bush and the sprocket ($i=1$) will have the index 1.

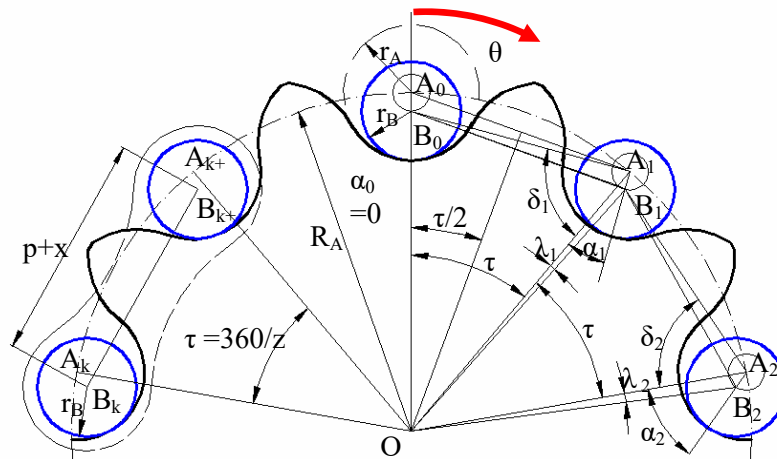


Figure 2. Chain with sprocket, geometric and kinematic parameter.

Based on figure 1, the contact angle α_1 can be determined following the next steps:

- a) Determination of angle δ_1
From $\triangle OB_0A_1$

$$B_0A_1^2 = OB_0^2 + OA_1^2 - 2OB_0OA_1 \cos \tau, \quad (5)$$

$$B_0 A_1 = \sqrt{(R_A - r_B + r_A)^2 + R_A^2 - 2R_A(R_A - r_B + r_A)\cos\tau}. \quad (6)$$

Applying the sin theorem $\frac{OB_0}{\sin\delta_1} = \frac{B_0 A_1}{\sin\tau}$, it results

$$\delta_1 = \arcsin \frac{OB_0 \sin\tau}{B_0 A_1}. \quad (7)$$

- b) Determination of angle ψ_1
From $\Delta B_0 B_1 A_1$

$$B_0 B_1^2 = B_0 A_1^2 + A_1 B_1^2 - 2B_0 A_1 A_1 B_1 \cos\delta_1, \quad (8)$$

$$\psi_1 = \arcsin \frac{B_0 A_1^2 + A_1 B_1^2 - B_0 B_1^2}{2B_0 A_1 A_1 B_1}. \quad (9)$$

- c) Determination of angle α_1

$$\alpha_1 = \psi_1 - \delta_1. \quad (10)$$

As the determination of the other geometric parameters depends on the ones presented before, due to the elongation of the chain (see figure 2), hereinafter the following steps will be done to compute the contact angle α_2 (for $i=2$):

- d) Determination of angle λ_1
From $\Delta O A_1 B_1$

$$OB_1^2 = OA_1^2 + A_1 B_1^2 - 2OA_1 A_1 B_1 \cos\alpha_1, \quad (11)$$

$$\lambda_1 = \arcsin \frac{A_1 B_1 \sin\alpha_1}{OB_1}. \quad (12)$$

- e) Determination of angle δ_2
From $\Delta O B_1 A_2$

$$B_1 A_2 = \sqrt{OB_1^2 + OA_2^2 - 2OB_1 OA_2 \cos(\tau - \lambda_1)}, \quad (13)$$

$$OB_1^2 = B_1 A_2^2 + OA_2^2 - 2B_1 A_2 OA_2 \cos\delta_2, \quad (14)$$

$$\delta_2 = \arccos \frac{B_1 A_2^2 + OA_2^2 - OB_1^2}{2B_1 A_2 OA_2}. \quad (15)$$

- f) Determination of angle ψ_2
From $\Delta B_1 A_2 B_2$

$$B_1 B_2^2 = B_1 A_2^2 + A_2 B_2^2 - 2B_1 A_2 A_2 B_2 \cos\psi_2, \quad (16)$$

$$\psi_2 = \arcsin \frac{B_1 A_2^2 + A_2 B_2^2 - B_1 B_2^2}{2B_1 A_2 A_2 B_2}. \quad (17)$$

- g) Determination of angle α_2

$$\alpha_2 = \psi_2 - \delta_2. \quad (18)$$

As it can be seen after determining the geometric parameters of the first two bushes being in contact with the chain ($i=1, 2$), a generalized calculus is proposed for the contact angle determination, in which it is considered that i is the contact number between the bush and chain; for $i=0$, in assembled position, $\alpha_0=0$, $\lambda_0=0$ and $OB_0 = R_A - r_B + r_A$

$$B_{i-1}A_i = \sqrt{OB_{i-1}^2 + OA_i^2 - 2OB_{i-1}OA_i \cos(\tau - \lambda_{i-1})}, \quad (19)$$

$$\delta_i = \arccos \frac{B_{i-1}A_i^2 + OA_i^2 - OB_{i-1}^2}{2B_{i-1}A_i \cdot OA_i}, \quad (20)$$

$$\psi_i = \arccos \frac{B_{i-1}A_i^2 + A_iB_i^2 - B_{i-1}B_i^2}{2B_{i-1}A_i \cdot A_iB_i}, \quad (21)$$

$$\alpha_i = \psi_i - \delta_i \quad (22)$$

$$OB_i = \sqrt{OA_i^2 + A_iB_i^2 - 2OA_i \cdot A_iB_i \cos \alpha_i}, \quad (23)$$

$$\lambda_i = \arcsin \frac{A_iB_i \sin \alpha_i}{OB_i}. \quad (24)$$

4. Case study

In the case study, two objectives of this work are proposed to be reached. The first is to show the contact point for a real case and the second follows the influence of the chain elongation on the contact point.

According to ISO 606:2004 [7], for: $p=9.525$ mm, we have $d_B=5.08$ mm. Considering $z=16$, the following values are obtained [8]:

-Angular pitch

$$\tau = \frac{360}{z} = 22.5 \text{ deg}; \quad (25)$$

-Deddendum diameter

$$D_f = p \left(\sin \frac{\tau}{2} \right)^{-1} - d_B = 43.74354 \text{ mm}; \quad (26)$$

-Fillet radius at the dedendum diameter of the sprocket

$$r_A = 0.505 \cdot d_B + 0.069 \frac{d_B}{3} = 2.684014 \text{ mm}; \quad (27)$$

-Pitch radius

$$R_A = \frac{D_f}{2} + r_A = 24.55578 \text{ mm}; \quad (28)$$

-Radius of the bush

$$r_B = \frac{d_B}{2} = 2.54 \text{ mm}. \quad (29)$$

Applying the algorithm proposed before, graphs from figure 3 are obtained for the contact angle (α_i) and the deviation angle of the segment B_iO against line A_iO (λ_i), for the specific chain elongation $x = 0.2\%$.

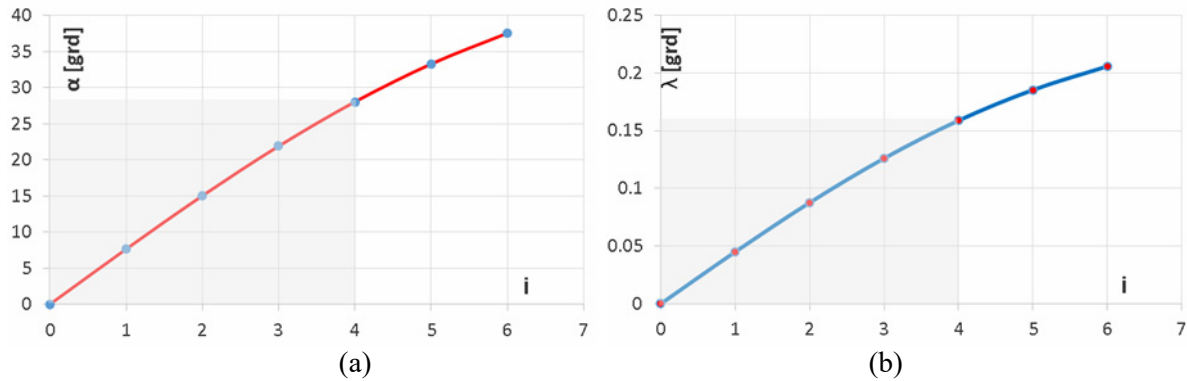


Figure 3. The representative geometric parameters in the bush-sprocket modeling: (a) contact angle; (b) the deviation angle caused due to specific chain elongation.

The influence of the sprocket rotation (θ) on the contact angle is presented in figure 4, for which the rotation angle can vary from zero to a corresponding pitch angle (τ):

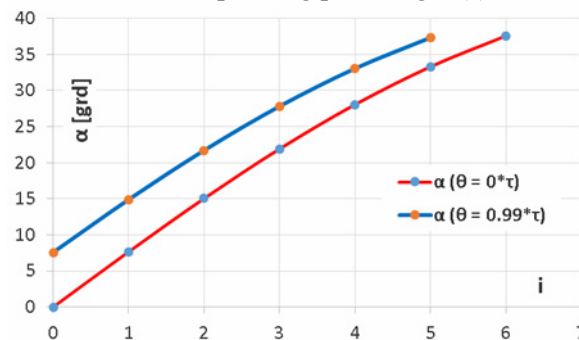


Figure 4. The influence of the sprocket rotation over the contact angle.

In order to present the influence of the specific chain elongation (x) on the induced contact angle, the algorithm presented before is applied for several values of the pitch difference (figure 5).

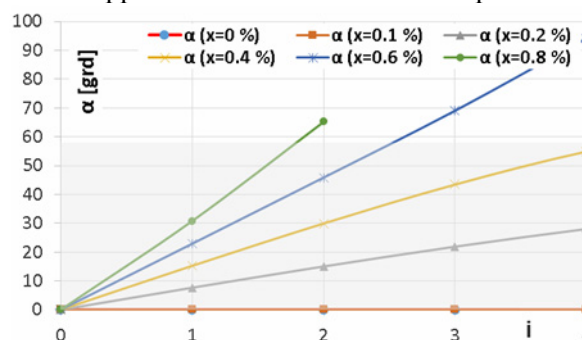


Figure 5. The influence of the specific elongation of the chain over the contact angle.

5. Results and discussions

One of the most decisive factors regarding the replacement of a chain transmission is the specific elongation of the chain, which needs to be in a certain interval (usually 0.2 % of the nominal pitch). This difference influences the positioning of the contact point between the bush and sprocket, thus, the contact

angle. As the contact angle is bigger, the dynamics of the chain transmission is more influenced considering the losses through friction. This contact point influences the behaviour of the normal and transversal forces appeared during operation of the transmission, as its dynamics, too.

In this work, there are analysed the contact angle between the sprocket and the bush (α_i), the deviation of positioning the bush on the filleted radius at sprocket dedendum (λ_i) and the influence of the specific elongation (x) over the contact angle, for a standardized profile of the sprocket teeth. The tooth profile is chosen according to ISO 606:2004, for which there are considered the nominal pitch $p = 9.525$ mm, the diameter of the bush $d_B = 5.08$ mm and the number of teeth $z = 16$. Based on the calculus presented before, the principal parameters are obtained, needed for the geometric modelling of the contact angle: the pitch radius of the sprocket $R_A = 24.55$ mm, the roller gap radius $r_A = 2.68$ mm and the radius of the bush $r_B = 2.54$ mm.

From figure 3a, it can be seen that, for the contact between the bush and chain ($i = 0, \dots, 4$), starting from the initial assembled position (see figure 2), the contact angle (α) has a nearly linear increase, as also the deviation angle (λ) caused by the specific elongation of the chain, which is modified proportionally to the contact angle. As the contact angle increases, the deviation angle also increases, but in reduced proportion; for $i=1$, $\alpha = 7.648651^\circ$, $\lambda = 0.044986^\circ$ and for $i=4$, $\alpha = 28.03914^\circ$, $\lambda = 0.158779^\circ$.

According to figure 4, the behaviour of the sprocket rotation (θ) is similar to the behaviour of the contact angle (α), they have the particularity that, with rotation of one pitch angle (τ), the value of the pitch angle corresponds to the value of the second contact if the sprocket is considered in the initial position (ex: $\alpha_i (\theta = \tau) = \alpha_{i+1} (\theta = 0)$, respectively for $\alpha_{(i=3)} = \alpha_3 (\theta = \tau) = 28.03914^\circ = \alpha_{(i=4)} = \alpha_4 (\theta = 0) = 28.03914^\circ$).

Based on the discussion presented above, there can be stated that the influence of the specific chain elongation is representative in the correct functional behaviour of the chain transmission. From figure 5, there can be stated that, with the increase of the specific elongation of the chain (x), the contact angle also increases, thus, the wear will be increased. For the considered example, for a specific elongation bigger than 0.4 %, the bush moves out from the contact with the filleted radius, the chain meshes with the sprocket's peaks, which assumes the appearance of some supplementary vibrations, thus, an accelerated wear or a miss meshing of the chain.

The deviation angle (λ) is positioning the bush relative to the sprocket and the contact angle (α) determines the direction of the normal force in bush-sprocket contact. Together, these two parameters are the main inputs for the establishment of the dynamics as well as the losses through friction of the chain transmission with bushes and short links.

6. Conclusion

During the functioning of a chain transmission, chain elongation develops continuously, as result of wear, from a minimum specific elongation of a maximum 0.2% to a maximum of 2%, the first contact point between bush and sprocket and the contact angle in that point depending on it.

In this work, a generalized algorithm is proposed for the deviation angle (determining the contact point) and the contact angle determination, depending on the specific chain elongation. This is part of the base of the friction model for bush chain transmission since bush – sprocket friction depends on the length of the contact line bush – sprocket and also on the normal force, which depends on the contact angle.

Based on the determination of these influences, there can be done optimisations of these types of transmissions by introducing some criteria, like minimum friction or minimum transversal vibrations.

7. References

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