

Concepts and fundamental equations in Thermodynamics with Finite Speed

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Abstract. This paper presents the basic concepts and fundamental equations of the Thermodynamics with Finite Speed (TFS) resulted by the systematically study of the thermal reciprocating machine in relation with the piston finite speed and thermal molecular speed measured in the considered thermodynamic system. These concepts are based on the idea that any propagation of the interaction in the thermodynamic systems of finite dimensions is achieved by finite speeds: (1) - *piston speed*, (2) - *average speed of the gas molecules* inside the cylinder. A specific approach (scheme of calculation) for non-equilibrium (irreversible) thermodynamic processes is developed within TFS in order to find *the fundamental equations* appropriate for *Optimizing Efficiency* or *COP* and *Power* of thermal reciprocating machines. *Analytical equations for all 5 irreversible thermodynamic processes in gases* (isometric, isothermal, isobaric, adiabatic, polytropic) are deduced by *integration of the combined First and Second Laws equation for processes with Finite Speed*. This paper is limited to *Irreversible Processes with Finite Speed*, without taking into account the Friction and Throttling effects. It also notes the *main moments* in the development of TFS that led to these *concepts and fundamental equations*.

1. Introduction

The Thermodynamics with Finite Speed (TFS) [1-12] is one of the Thermodynamic branches developed in the last 52 years by the Romanian School of Thermodynamics and can be considered a possible Romanian brand in historical development of Thermodynamics. TFS is focussed on a deeper understanding of the real physical process performed by a thermodynamic system, in thermal reciprocating machine. In order to optimize the machine's performances (Efficiency or COP and Power), their specific causes of internal and external irreversibility are considered.

“The Thermodynamics (<https://www.grc.nasa.gov/www/k-12/airplane/thermo.html>) is the branch of physical science that deals with the relations between heat and other forms of energy (such as mechanical, electrical, or chemical energy), and, by extension, of the relationships between all forms of energy”.

The *Statistical and Molecular Thermodynamics* defines first of all the concept of *Thermodynamic System*, like a collection of a high number of particles in movement and in interaction. A second fundamental concept is the *Thermodynamic State of the system*, which can be characterised by a *property of the system*, represented by its parameters measured in a defined moment (i.e. Pressure,



Temperature, Volume). The concept of the *Process in the system* is defined by the *Thermodynamic parameters changes from one state to another state*. Each System State can be characterized by a relation between its parameters named *Equation of State*. Any *Process* in the *System* is determined by heat and work *interaction* with the *environment* (by energy and mass transfer). The parameters can be *extensive parameters* (their values are *additive* in rapport with the *dimensions of the system* – i.e., *volume, mass, numbers of mols*), and *intensive parameters* (their values are independent of the system dimensions – i.e., *pressure, density, temperature*). A system is in a *stationary state* if their parameters are not changed in time. In Thermodynamics there are two types of systems: *closed system* (with *heat* and *work* interaction) and *opened system* (with *heat, work* and *mass* interaction with surroundings). The Kinetic-Molecular Theory and Statistical Theory can provide a conceptual definition of these parameters in relation with the statistical behaviour of the microscopic system components in stationary status and in evolutionary status. In a real system its proprieties can vary from point to point. We can usually analyse a general system by sub-dividing it (either conceptually or in practice) into a number of simple systems in each of them the properties being assumed as uniform. The state of a system in which properties have definite, unchanged values as long as external conditions are unchanged is called an Equilibrium State.

The *Classical Thermodynamics* consider only the processes that are performed by quasi-equilibrium state path of the system. In this assumption the system can follow the main path between two thermodynamic states and all the processes will be reversible. Reversible Thermodynamics its a axiomatic deductive science built by the classics (Carnot, Clausius, Kelvin, Boltzmann, Planck) based on Four Laws (Zero Law introducing *Temperature(T)*, First Law introducing the concept of *Energy(U)*, Second Law introducing the concept of *Entropy(S)*, and Third Law stating: “at zero Kelvin *Temperature, Entropy(S)* tends to zero”).

Introducing the fundamental concepts of Thermodynamics with Finite Speed has started from the need to provide a better understanding of the role of the interaction between the piston (which is moving with the speed w_p) and the molecules of the system (which are moving with the *average molecular speed* c). The study of this interaction (between the piston and the molecules) conducted to a new formula of the *Irreversible Work(W_{ir})* which depends on the ratio w/c .

$$\delta W_{ir} = P_{m,i} \cdot \left(1 \pm a \frac{w}{c} \right) \cdot dV = P_p \cdot dV \quad (1)$$

Two different equations of the *First Law of Thermodynamics* can be written to characterise the process evolution as "*coupled processes*": a "*classical equation*" that consider the whole system homogenous and isotropic (in this system the processes are reversible) and "*the finite speed equation*" related to the movement of the surface transferring the work to the mechanical system (in this system all processes are irreversible). The *second equation* is the result of *the contribution of the gradients* of the *thermodynamic parameters* of the system, which occurs if we consider the *need to meet* the requirements of the *Second Law of Thermodynamics*.

This fundamental concept was promoted and gradually developed by a dedicated team under the coordination of Prof. Stoian Petrescu in order to clarify the level of the irreversibility contribution inside the thermal machinery with piston, if the effect of speed, mechanical friction effect, fluid throttling, thermal transfer to the environment or internal combustion effect to the total amount of mechanical work transferred to the mechanical system are considered. These analyses are in progress, and the team can provide step by step new proofs of the concept validity and new results extensions are expected to be proved in biological and electrical processes future approaches.

This paper is limited to describe only the TFS achievements related to the concept of “*finite piston speed*” versus “*finite molecular speed*” contribution and the related equations obtained by the research team.

2. Early developments and fundamental concepts (1964-1974)

The development of TFS started in the years 1964-1965 with 5 fundamental papers published in IPB Bulletin by Stoicescu and Petrescu [1-5] which conducted to the First PhD Thesis in this domain written by Petrescu in 1969 [6], and several papers that followed [7-10], which were essential for Promotion and Extension of „*The new Theory on Irreversible Processes with Finite Speed*”, in the period 1965-1972, which eventually became what is called today *Thermodynamics with Finite Speed*. Several achievements of TFS regarding the optimization of thermal machines and electro-chemical devices were reported [11, 12].

In 1937 a German Physicist Paul [13] studied the first Process with Finite Speed in a piston cylinder arrangement, only for military application and obtained the first equation of the adiabatic irreversible Process with Finite Speed:

$$TV^{(k-1)} \left(1 \pm 2.6067 \frac{w}{c} + 3 \frac{w^2}{c^2} \pm 3.6743 \frac{w^3}{c^3} \right) = \text{Constant} . \quad (2)$$

Much later (1964) and independently of this achievement, inspired by the books of Make [14], Sommerfeld [15] and Einstein's Theory of the Relativity [16], Petrescu, working for his Ph.D Thesis [6], with Prof. Stoicescu as adviser [3-5] has written for the first time the new equation for the First Law of Thermodynamics for Processes with Finite Speed in closed systems (Piston Cylinder arrangement):

$$dU = \delta Q - P_{m,i} \cdot \left[1 \pm a \frac{w}{c} \right] \cdot dV \quad (3)$$

where $a = \sqrt{3k}$, with $k = C_p / C_v$, w is the piston speed and $c = \sqrt{3RT_{m,i}}$.

The second term in the right hand side of Eq. (3) is the *irreversible work* δW_{ir} :

$$\delta W_{ir} = P_{m,i} \cdot \left[1 \pm a \frac{w}{c} \right] \cdot dV . \quad (4)$$

Equation (3) is a more developed approach in comparison with Paul's one [13]. It seems that Paul was not interested in finding *a new equation of the First Law for irreversible processes*, like Eq. (3), but just to get an equation of *irreversible adiabatic process with Finite Speed*.

Equation (3) includes for the first time the causes for the process irreversibility, in a fundamental equation of Thermodynamics, namely the *Mathematical Expression of the First Law*, written in an explicit way, *especially for irreversible process* as function of *Finite Speed of the Piston* w , and *Finite Speed of the Molecules* c . The analogy with Einstein's Theory of Relativity [16,17] consist in the fact that in the piston cylinder arrangement the waves generated by the piston motion with *finite speed* w , depend on *the average molecular speed* c , in a similar way like the electromagnetic waves interactions depend on the *speed of light*. The final result stating that *the irreversible elementary Work* δW_{ir} *must depend of the ratio* w/c was found by the author using 4 different approaches, namely:

a) *The kinetic-molecular model* [1,6] leads to the following relationship:

$$\delta W_{ir} = P_{m,i} \cdot \left(1 \pm a \frac{w}{c} + b \frac{w^2}{c^2} \pm \dots \right) \cdot dV = P_p \cdot dV \quad (5)$$

b) *The advanced kinetic-molecular model* [6, 8], based on *Maxwell-Boltzmann distribution* [18] on *finite relaxation time in the system* leads to:

$$\delta W_{ir} = P_{m,i} \cdot \left(1 \pm 2.764 \frac{w}{c} + 3 \frac{w^2}{c^2} \pm 1.283 \frac{w^3}{c^3} + \dots \right) \cdot dV = P_p \cdot dV \quad (6)$$

c) The phenomenological analysis based on the propagation of pressure waves, generated by the finite speed of the piston and the speed of sound in the gas [1,6] leads to:

$$\delta W_{ir} = P_{m,i} \cdot \left(1 \pm a \frac{w}{c}\right) \cdot dV = P_p \cdot dV \quad (7)$$

d) The Second Law approach [19], based on Linear Irreversible Phenomenological Thermodynamics (LIPT) of Onsager [20] and Prigogine [21] and the concept of Entropy Generation, leads to:

$$\delta W_{irr} = P_{m,i} \cdot (1 \pm K_1 w) \cdot dV = P_p \cdot dV \quad (8)$$

In the four approaches developed above, the irreversible work is generated by the pressure on the piston surface (P_p), and one can note that this pressure is different from the main pressure in the gas (P_{mi}). This conclusion is another fundamental concept in all futures applications and TFS developments.

3. New Concepts introduced in Irreversible Finite Speed Thermodynamics (FST)

Figure 1 illustrates the simplest closed system in which a finite speed irreversible process may take place. In this case, the only interactions between the system and the surroundings are the thermal one, δQ_{irr} , and the finite speed mechanical one, δW_{irr} . They are provided as function of the piston speed in relation with the thermal molecular speed measured in the considered thermodynamic system.

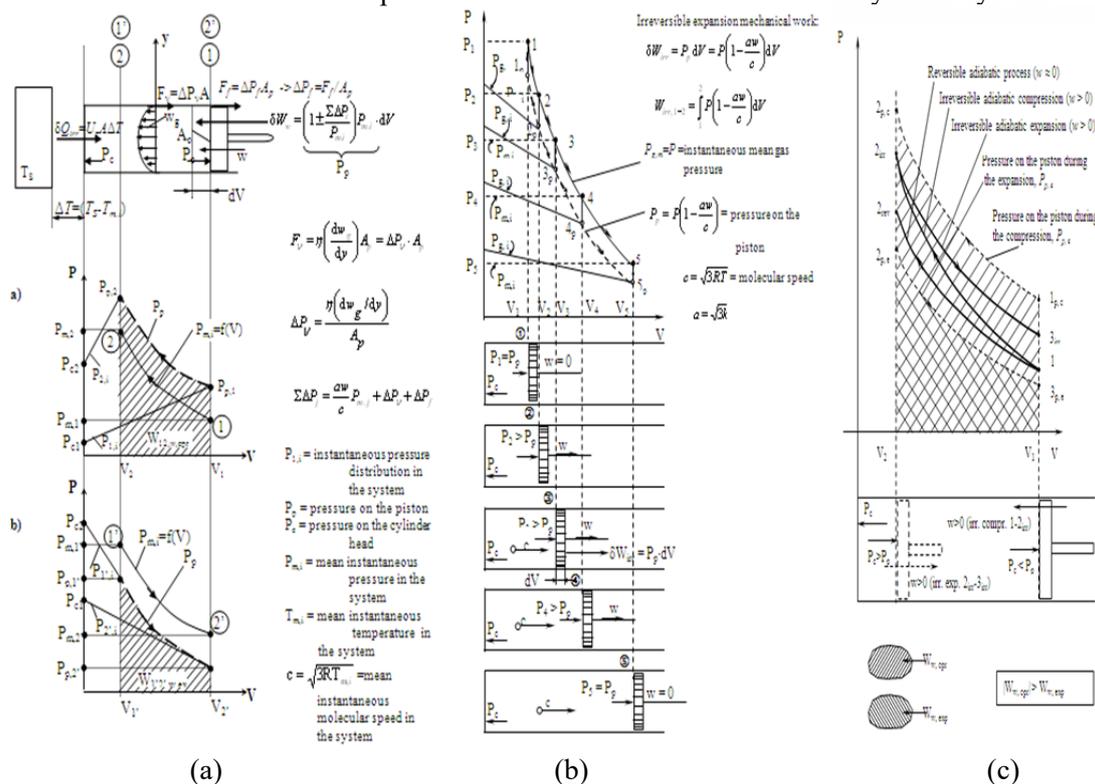


Figure 1. The pressure of the system gas $P_{1,i}$ (P_{gas} for compression) and $P_{2,i}$ (P_{gas} for expansion) and the pressure on the piston, P_p are different representations of the same diagram; the representations (a), (b) and (c) are similar. Source [22, 23]

In Irreversible Thermodynamics with Finite Speed, where conditions of non-equilibrium prevail, new concepts and conditions are needed to describe the state of the system and the processes the system undergoes. Thus, the analysis becomes much more complex by comparison to classical reversible processes. However, the models in Finite Speed Thermodynamics accurately predict the real system's behaviour.

Figure 2 illustrates some of these new concepts with regard to the pressure variation during the expansion process with Finite Speed.

3.1. Instantaneous pressure distribution in the system

The curve labelled $P_{1,i}$ (P_{gas} for compression) and $P_{2,i}$ (P_{gas} for expansion) describes in the *new type of P-V diagram*, necessary to be introduced in *The Finite Speed Irreversible Thermodynamics*, the instantaneous pressure of the gas in the cylinder (system), for a certain moment during the motion of the piston with the speed w . We do not know exactly how this distribution looks like, but in the *simplest first order mathematical approximation* we could consider it *as linear* as was represented in *figure 1A.a) and b)*, for simpler understanding of these *new concepts*.

3.2. Gas instantaneous mean pressure

The horizontal line labelled $P_{m,1}$ (for compression) in *figure 1A.a)* and labelled $P_{m,2}$ (for expansion), in the *figure 1A.b)* indicates the uniform distribution that would occur in the system if the piston is stopped and the *finite relaxation time* of the system is achieved.

This pressure ($P_{m,1}$ or $P_{m,2}$) will be used as a basis for the description of the processes in the system instead of the equilibrium pressure, P (see *figure 2*), from Equilibrium-Reversible Thermodynamics.

The equilibrium-reversible processes *is the limit of TFS* where the piston speed, w approaches *to zero*). The main difference between what occurs in the Finite Speed processes compared to the equilibrium-reversible processes is that TFS use of two pressures instead of only one in reversible processes in order to properly describe finite speed processes. This is because in Finite Speed Thermodynamics the pressure of the system gas P_{gas} and the pressure on the piston, P_p are different.

3.3. The pressure on the piston

The pressure on the piston (P_p) differs from the gas pressure (P_{gas}) at any point in the system, as shown by the pressure distribution curves of *figure 1*.

During a *compression process* the pressure on the piston, P_p , will be greater than the pressure at any other point in the system. The pressure of the gas in the system reaches a minimum at the cylinder head, P_c . However, in the case of an expansion process, the pressure on the piston, P_p , will be less than the pressure in any other point in the system, reaching a maximum at the cylinder head, P_c , as shown in *figures 1A, 1B and 1C*.

This implies that a Finite Speed Process cannot be described on P - V coordinates by a single curve, versus mean pressure in the gas (see $P_{m,1}$ or $P_{m,2}$ in *figure 1A*) as can be done for classic reversible processes, but requires two curves (P_p and $P_{m,i}$) - as shown on *figure 2*.

A better perspective on the important differences between *Classical Reversible Equilibrium Thermodynamics* (RET) and the new *Finite Speed Irreversible Thermodynamics* (FSIT) may be gained by referring to *figure 2*. The pressure during simple reversible compression and expansion processes is shown in *figure 2a* and *figure 2c*, for purposes of comparison with the more complex plot of pressure during irreversible compression and expansion as shown on *figure 2b* and *figure 2d*. It might also be noted that the irreversible processes were plotted using the simplification of the only two parameters, P_p and $P_{m,i}$.

Figure 2 provides a comparison between reversible processes and irreversible processes with finite speed during compression and expansion. The *reversible compression process* 1-2_r, in *figure 2a* overlaps the *reversible expansion process* 1-2_r, in *figure 2c*. Therefore the areas under these two curves, which represent the work of compression and the work of expansion respectively, are equal and the net work is zero. When the irreversibility due to finite speed is considered, two curves are

needed to describe each process, as shown by the curves 1_p-2_p and $1-2_{ir}$ in figures 2b and 2d. These curves during the compression process differ from those during the expansion process. Consequently, the system reaches a different state than the initial one after the compression process and also after the expansion process. Thus, point 2_{irr} in figure 2d differs from point 1 in figure 2b and, in particular, the area under the pressure curve during compression differs from the area under the pressure curve during expansion. Accordingly, the net mechanical work is different. It is reduced in proportion to the difference between these areas.

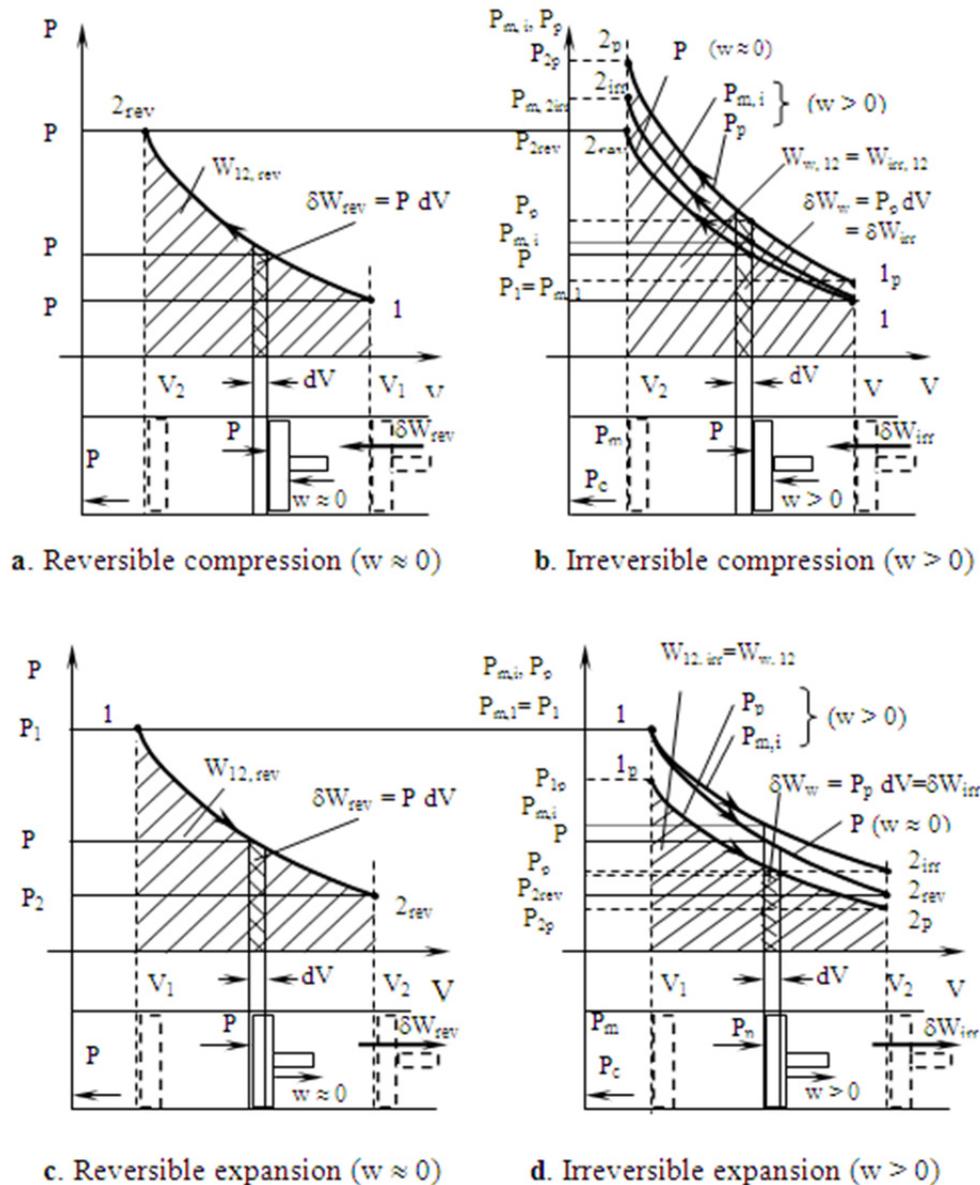


Figure 2. Comparison between P-V diagrams for reversible and irreversible processes. [22, 23]

The difference between the work of adiabatic compression and expansion is clearly shown in figure 2a as the difference between the area under the curve $1_{p,c} - 2_{p,c}$ during the compression process and the area under the curve $2_{p,e} - 3_{p,e}$ during the expansion process.

4. Thermodynamic processes with constant Finite Speed

The Irreversible Thermodynamics with Finite Speed is started with the S. Petrescu Ph.D Thesis [6], and the papers [3-5] where he worked with his adviser Prof. L. Stoicescu, to establish the equation for the 5 irreversible Processes with Constant Finite Speed: *isobaric, isometric, isothermal, adiabatic* and much later (1997), the *polytropic process with constant finite speed* in the Costea's Ph.D Thesis [24](advisers: S. Petrescu and M. Feidt). These equations are presented in *table 1*. For a condensed representation of the equations, parameters α_1 , α_2 and γ , are defined as functions of the coefficient a and the speeds ration w/c .

Table 1. How to compute the 5 irreversible processes with finite speed, in thermodynamic closed systems by taking into account the piston speed contribution and the work and heat transfer to the environment. Source [25] improved.

Process	Mass & Eq. of State	Eq. for the Process $P_2, V_2, T_2 = ?$	Energy & Enthalpy variation on the Reversible Process: $\Delta U, \Delta H = ?$	Work transferred in the Irreversible Process: $W_{12} = ?$	Heat transferred in the Irreversible Process: $Q_{12} = ?$
Isobaric $P = \text{const}$	$m = \frac{P \cdot V}{R \cdot T}$	$P_1 = P_2$; V_2 given; $\frac{T_1}{V_1} = \frac{T_2}{V_2}$	$\Delta U = m \cdot c_v \cdot \Delta T$ $\Delta H = m \cdot c_p \cdot \Delta T$	$W_{12,P} =$ $= p_1 \cdot \gamma \cdot (V_2 - V_1)$	$Q_{12,P} = \gamma \cdot \Delta H =$ $= m \cdot c_p \cdot \gamma \cdot \Delta T$
Isometric $V = \text{const}$	$m = \frac{P \cdot V}{R \cdot T}$	$V_1 = V_2$; V_2 given; $\frac{P_1}{T_1} = \frac{P_2}{T_2}$	$\Delta U = m \cdot c_v \cdot \Delta T$ $\Delta H = m \cdot c_p \cdot \Delta T$	$W_{12,V} = 0$	$Q_{12,V} = \Delta U =$ $= m \cdot c_v \cdot \Delta T$
Isothermal $T = \text{const}$	$m = \frac{P \cdot V}{R \cdot T}$	$T_1 = T_2$; V_2 given; $P_1 \cdot V_1 = P_2 \cdot V_2$	$\Delta U = 0$ $\Delta H = 0$	$W_{12,T} = \beta \cdot P_1 \cdot V_1 \cdot \ln \frac{V_2}{V_1}$	$Q_{12,T} = W_{12,T} =$ $= \beta \cdot P_1 \cdot V_1 \cdot \ln \frac{V_2}{V_1}$
Adiabatic $\delta Q = 0$ $dS = \frac{\delta Q}{T}$ $S = \text{const}$	$m = \frac{P \cdot V}{R \cdot T}$	$P_1 \cdot V_1^k = P_2 \cdot V_2^k \cdot \frac{\alpha_2}{\alpha_1}$ $T_1 \cdot V_1^{k-1} = T_2 \cdot V_2^{k-1} \cdot \frac{\alpha_2}{\alpha_1}$ $\frac{T_1}{P_1^{\frac{1}{k}}} = \frac{T_2}{P_2^{\frac{1}{k}}} \left(\frac{\alpha_2}{\alpha_1} \right)^{\frac{1}{k}}$	$\Delta U = m \cdot c_v \cdot \Delta T$ $\Delta H = m \cdot c_p \cdot \Delta T$	$W_{12,ad} = \frac{P_1 V_1 - P_2 V_2}{k-1} =$ $= -\Delta U = m c_v \cdot (T_1 - T_2)$	$Q_{12,ad} = 0$
Polytropic $N = 1 +$ $+B' \cdot (k-1)$	$m = \frac{P \cdot V}{R \cdot T}$	$P_1 \cdot V_1^N = P_2 \cdot V_2^N$ $T_1 \cdot V_1^{N-1} = T_2 \cdot V_2^{N-1}$ $\frac{T_1}{P_1^{\frac{N-1}{N}}} = \frac{T_2}{P_2^{\frac{N-1}{N}}}$	$\Delta U = m \cdot c_v \cdot \Delta T$ $\Delta H = m \cdot c_p \cdot \Delta T$	$W_{12,pol} = \frac{P_1 V_1 - P_2 V_2}{N-1}$	$Q_{12,pol} = \Delta U + W_{12,pol} =$ $= m c_v \Delta T + \frac{P_1 V_1 - P_2 V_2}{N-1}$
Ideal/ Perfect gas, closed system			H = U + PV		
$k = 1.66$ (mono atomic gas) $k = 1.4$ (diatomic gas) $k = 1.3$ (triatomic gas)	$c_v = \frac{R}{k-1}$ $c_p = \frac{kR}{k-1}$	$c_{v,av}, c_{p,av}$, from tables	$m = \text{const}$	$R = \frac{R_u}{M}$ $R_u = 8315 \text{ J / (kmol} \cdot \text{K)}$	
Irreversible adiabatic process corrections	$\alpha_2 = \left(1 \pm \frac{aw}{c_2}\right)^2$; $c_2 = \sqrt{3RT_2}$; $a = \sqrt{3k}$	$\beta = \left(1 \pm \frac{aw}{c_T}\right)$ ← Irreversible isothermal process correction			
	$\alpha_1 = \left(1 \pm \frac{aw}{c_1}\right)^2$; $c_1 = \sqrt{3RT_1}$; $a = \sqrt{3k}$	$\gamma = \left(1 \pm \frac{aw}{c_{12,av}}\right)$; ← Irreversible isobaric correction ; $c_{12,av} = \frac{c_1 + c_2}{2}$			
	$(-B) = \left[\pm \left(1 - \frac{T_0}{T}\right) \cdot U_g \cdot \frac{A \cdot V}{w \cdot A_p \cdot m \cdot R} - \left(1 \pm \frac{aw}{c_{12}}\right) \right]$	$\gamma_Q = \left(1 \pm \frac{aw}{c_{12,av}} \cdot \frac{W_{12,rev}}{\Delta H}\right)$ ← Irreversible isobaric correction for heat			

The equation for *Polytropic Irreversible Process with Finite Speed* was proposed starting from the Fundamental Equation of Thermodynamics with Finite Speed:

$$dU = \delta Q_{irrev} - p_{med} \left(1 \pm \frac{aw}{c} \pm \frac{aw^2}{c^2} \pm \dots \right) \cdot dV = mc_v \cdot dT \quad (9)$$

and using the *global heat transfer coefficient* (U_g) for the heat transfer at a given temperature difference, between internal mean temperature ($T_{med,i}$) and the ambient temperature (T_0), we obtained the expression:

$$\delta Q_{ir} = -U_g A (T_{med,i} - T_0) d\tau = -U_g A (T_{med,i} - T_0) \frac{|dV|}{(A_p w)} \quad (10)$$

we get a differential equation of the polytropic irreversible process, with two causes for irreversibility: finite speed and heat transfer at a finite difference of temperatures:

$$\left(\frac{dT}{T} \right) = \left[\pm \left(1 - \frac{T_0}{T} \right) \cdot U_g \cdot \frac{A \cdot V}{w \cdot A_p m \cdot R} - \left(1 \pm a \cdot \frac{w_p}{c_{12,av}} \right) \right] (k-1) \cdot \left(\frac{dV}{V} \right) \quad (11)$$

where A , A_p , are the *average cylinder area* and the *piston area* respectively, and U_g , is function of the piston speed w_p , the medium instantaneous temperature ($T_{med,i}$) with an average molecular speed ($c_{12,av}$) and pressure ($p_{med,i}$) in the gas. The previous equation can be expressed as:

$$\frac{dT_{med,i}}{T_{med,i}} = -B' (k-1) \cdot \frac{dV}{V} \quad ; \text{ with the notation:} \quad (12)$$

$$(-B') = -B_{ad} = \left[\pm \left(1 - \frac{T_0}{T} \right) \cdot U_g \cdot \frac{A \cdot V}{w \cdot A_p m \cdot R} - \left(1 \pm a \cdot \frac{w_p}{c_{12,av}} \right) \right]. \quad (13)$$

Considering an average value of the B' during the process, we can integrate the above differential equation of the polytropic irreversible process, exactly like the differential equation of the reversible process in Reversible Thermodynamics:

$$P_1 V_1^{B' \cdot (k-1)} = P_2 V_2^{B' \cdot (k-1)} \quad (14)$$

$$T_1 V_1^{1+B' \cdot (k-1)} = T_2 V_2^{1+B' \cdot (k-1)} \quad (15)$$

From eqs. (14) and (15) results:

$$\frac{p_1}{p_2} = \left(\frac{V_2}{V_1} \right)^{B' \cdot (k-1)} \Rightarrow \frac{V_2}{V_1} = \left(\frac{p_1}{p_2} \right)^{\frac{1}{B' \cdot (k-1)}} \quad (16)$$

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1} \right)^{1+B' \cdot (k-1)} \Rightarrow \frac{V_2}{V_1} = \left(\frac{T_1}{T_2} \right)^{\frac{1}{1+B' \cdot (k-1)}} \quad (17)$$

By combining eqs. (16) and (17), one gets:

$$\left(\frac{T_1}{T_2} \right)^{\frac{1}{1+B' \cdot (k-1)}} = \left(\frac{p_1}{p_2} \right)^{\frac{1}{B' \cdot (k-1)}} \quad (18)$$

Then, eq. (18) will be raised to the power $1 + B'(k - 1)$:

$$\left[\left(\frac{T_1}{T_2} \right)^{\frac{1}{1+B'(k-1)}} \right]^{1+B'(k-1)} = \left[\left(\frac{p_1}{p_2} \right)^{\frac{1}{B'(k-1)}} \right]^{1+B'(k-1)} \quad (19)$$

$$\Rightarrow \frac{T_1}{T_2} = \left(\frac{p_1}{p_2} \right)^{\frac{1+B'(k-1)}{B'(k-1)}} \quad (20)$$

If we denote $B'(k-1)+1=N$ we get finally to the question of irreversible polytropic process, *with finite speed and heat transfer at finite gradient*, in a very similar format as for the reversible process equations:

$$\begin{aligned} P_1 V_1^{N-1} &= P_2 V_2^{N-1} \\ T_1 V_1^N &= T_2 V_2^N \end{aligned} \quad (21)$$

$$\left(\frac{T_1}{T_2} \right) = \left(\frac{p_1}{p_2} \right)^{\frac{N}{(N-1)}}.$$

We can give to N a similar name like in Reversible Thermodynamics: *finite speed polytropic exponent*.

A very interesting particular case is the one in which the heat transfer tend to zero, and we obtain an adiabatic irreversible process with finite speed.

Thus, if we impose the condition $\alpha_g = 0$, we get:

$$-B' = -B_{ad} = \left[-\left(1 \pm aw / c_{12} \right) \right]. \quad (22)$$

In this case N becomes equal to k_{irr} =*adiabatic exponent for finite speed process*:

$$k_{irr} = B_{ad} \cdot (k - 1) + 1 = \left(1 \pm \frac{aw}{c_{12}} \right) (k - 1) + 1. \quad (23)$$

If such a process takes place with a speed tending to zero, B_{ad} becomes equal to 1 , and we regain the adiabatic exponent k from *Classical Reversible Thermodynamics*. This is a proof that the equations of TFS tend to the limit to the equations of Reversible Thermodynamics, which is a proof that our equations are very well build.

With the equations of polytropic irreversible processes we can study any real cycle in which such processes could appear.

$$T_1 V_1^{(k-1)} \cdot \alpha_1 = T_2 V_2^{(k-1)} \cdot \alpha_2. \quad (24)$$

5. Thermodynamic processes with variable Finite Speed

An extension of the paper [6] are study the effect of the variable speed in the fundamental equations of the thermodynamic Process with Finite Speed. In this case the authors fund the need to use a correction factor which amplifies the term (aw/c) with 1.24 in order to take into account that in real machines, the piston has a quasi-sinusoidal motion. This correction is not critical in the practical applications taking into consideration the linear contribution of the term (aw/c) and the technological limits for the speed w in thermal machines under 40 m/s.

6. Thermodynamic cycles with Finite Speed

The first approach of a Thermodynamic Cycle with Finite Speed [1] was relative to an irreversible Otto cycle with finite speed and determined by using the equations from *table 1*, the irreversible Efficiency of such cycle as:

$$\eta_{O_{to.w}} = \left(1 - \frac{1}{\varepsilon^{k-1}}\right) \times (\text{Correction which takes into account the piston speed}). \quad (25)$$

For all irreversible cycles studied after that moment (1965) we have tried to get a similar analytical expression, namely:

$\eta_{irrev} = (\text{Efficiency expression from Classical Reversible Thermodynamics}) \times (\text{Correction accounting for the effect of finite speed generating irreversibilities in the cycle})$

Here we present the results obtained in 1965 in the paper of L. Stoicescu and S. Petrescu [1]. Formulas and figures are original photocopies from paper [1].

The explicit analytical formula for the Irreversible Otto Cycle with Finite Speed, corresponding to Eq. (25) from [1] is:

$$\eta_{tir} = \left(1 - \frac{1}{\varepsilon^{k-1}}\right) \left[1 - \frac{2a \cdot w}{c_1} \cdot \frac{1}{\varepsilon^{\frac{k-1}{2}} \left(\varepsilon^{\frac{k-1}{2}} + 1\right) (\lambda^{1/2} - 1)}\right] = \eta_{tr} \cdot \eta_{irrev} \quad (26)$$

where:

$$\eta_{irrev} = \left[1 - \frac{2a \cdot w}{c_1} \cdot \frac{1}{\varepsilon^{\frac{k-1}{2}} \left(\varepsilon^{\frac{k-1}{2}} + 1\right) (\lambda^{1/2} - 1)}\right]. \quad (27)$$

The original P-V diagram (from paper [1]) for the two compared cycles is presented in *figure 3*.

Based on expression (27), in *figure4* (original figure from paper [1]) we represented *the decrease of irreversible Otto Cycle Efficiency with the increase of piston Finite Speed (w_{med})*.

The expression of the *Maximum Possible Speed of the piston, for the Irreversible Otto Cycle*, also determined in paper [1] was:

$$w_{max} = \frac{c_1}{2a} \cdot \varepsilon^{\frac{k-1}{2}} \left(\varepsilon^{\frac{k-1}{2}} + 1\right) (\lambda^{1/2} - 1) \quad (28)$$

where: $k = \text{adiabatic exponent of the gas in cylinder}$; $\lambda = \text{pressure increase ratio, during isometric process 2-3}$; $c_1 = \text{average molecular speed in the initial state 1}$; $a = (3 \cdot k)^{1/2}$; $\varepsilon = \text{compression ratio} = V_1/V_2$.

In all of the papers after 1965, where we studied the *Efficiency of a Real Irreversible Cycle* in the framework of Thermodynamics with Finite Speed, *we have used the same pattern of the analytical expression of Efficiency* (for engines cycles) *or COP* (for Refrigeration and Heat pump cycles) *as in eq. (26)*, as a product between the *ideal (reversible cycle) efficiency, η_{tr}* , and a parenthesis depending on the speed w and other constructive and functional parameters that can also be met in correlation with the ratio w/c .

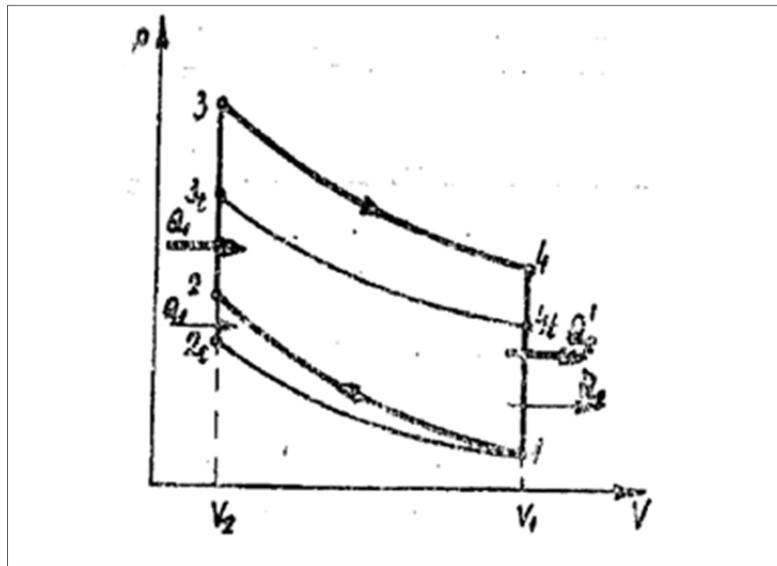


Figure 3. Otto Reversible Cycle 1-2t-3t-4t-1 (in the case $w = 0$) and Otto Irreversible Cycle with Finite Speed $w > 0$ (original figure 1 from the paper [1] 1965).

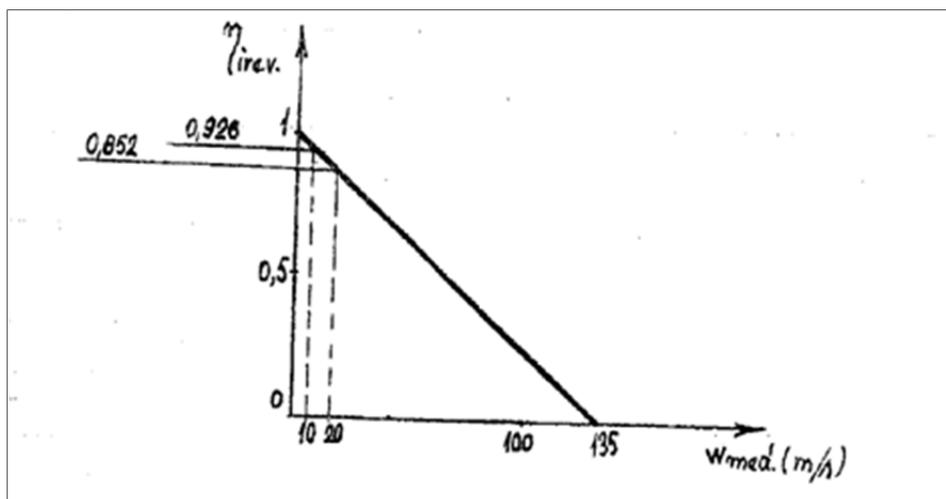


Figure 4. Decrease of irreversible Otto Cycle Efficiency with the increase of Finite Speed (w_{med}) of the piston (original figure 3 from the paper [1] 1965).

The extension of the conceptual approach in these specific areas is envisaged to be performed soon by the authors of this paper.

7. Conclusions

This paper presents and systematizes the concepts and fundamental equations introduced by the Thermodynamics with Finite Speed (TFS) in order to describe both the state and especially the evolution of a thermodynamic system in the conditions occurring in the cylinders of the actual thermal pistons machines, where the piston moving with finite speed. The achievements of this paper are related to an improved theoretical approach of the last 52 years of TFS development, in order to *systematizes basic concepts* that lead to the *new fundamental equations in closed thermodynamic systems for irreversible processes with finite speed* taking into account *the molecular speed and the piston speed* contribution and the *work and heat transfer* to the environment. The basic concepts in the

Thermodynamic Irreversible System with Finite Speed are: The *Concept of a System in Interaction* by statistical thermal movement – the *Thermodynamic Irreversible System*, is a collection of a high number of particles in movement and in interaction; The *Concept of the Process* - Any Process in the System is determined by its interaction with the environment, with heat and work interaction in closed systems and with heat, work and mass interaction with surroundings in opened system. The statistical movement and interaction cannot be stopped by any physical or chemical process in the system both in stationary and transitory state between two stationary states; The *Concept of the Irreversible Work*, as result of interaction between the molecules of the thermodynamic system (that move with a mean thermal speed c) with the piston surface (that move with a finite speed w_p versus a reference point of the system considered motionless), as an external mechanical system component by which the internal thermodynamic system transfer the work; The *irreversibility* is generated by the gradients of pressure in the realistic thermodynamic system by the piston movement in the long of the cylinder versus the top of the cylinder. The difference between the statistical mean pressure in the gas used to compute the work in the frame of Classical Reversible Equilibrium Thermodynamics (RET), and the gas pressure of the piston surface is positive in compression process and negative in expansion process. In this respect, the difference of these two pressures gives in every cases an opposite work, as irreversible work that is proportional with the ratio “ w/c ” of the piston speed and the mean thermal speed, and a coefficient “ a ” in relation with the gas characteristics (i.e. mono atomic, diatomic, triatomic). The heat transfer with the environment can be also take into consideration if we study the polytropic process. Based on these new concepts, an entirely analytical computation of Efficiency or COP and Power of any Thermal Machines operating upon irreversible cycle can be developed. Also, the main achievement of the analytical computation in the frame of *Irreversible Thermodynamics with Finite Speed* consists in finding the corresponding expressions for reversible processes for very low piston speeds ($w \ll c$), i.e. $W_{ir} \rightarrow W_{rev}$. This is also a validation of the method.

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