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Kinematics and tribological problems of linear guidance systems in four contact points

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Abstract. A procedure has been developed to determine both the value of the ball's angular velocity and the angular position of this velocity, according to the normal loads in a linear system with four contact points. The program is based on the variational analysis of the power losses in ball-races contacts. Based on this the two kinematics parameters of the ball (angular velocity and angular position) were determined, in a linear system type KUE 35 as function of the C/P ratio.

1. Introduction

The contact between the balls and races of the carriage and of the guide in a gothic-arch grooves linear system is realised in four contact ellipses. If initially all the contacts are preloaded with the same normal forces, in operation during various loads, the four contact ellipses are not equal loaded. Consequently, the balls motion is complex and both rolling and pivoting motion can appear in all four contact ellipses. If the carrying capacity and rigidity are higher that the two contact points, important friction losses in a gothic-arch grooves linear system can be observed, especially due to the pivoting friction.

2. Theoretical model for determining the kinematics parameters of the balls

In figure 1 is presented the position of the ball and the angular velocity of it in a linear system type KUE with 4 contact points. The ball exhibits 4 contact ellipses, 2 with the rail-corresponding with the points C1 and C2, and 2 with the carriage-corresponding with the points G1 and G2. Under the action of the force applied to the assembly Fy the value of the angle α is usually $\pi/4$. If the carriage has a linear velocity *V*, in the direction X, then the ball will rotate with an angular velocity ωb in the direction indicated in Figure 1, situated at an angle β to the vertical axis of the assembly. There are 3 cases of the relationship between the angle α and β regarding the contact loads Q1 and Q2. If the loads Q1 and Q2 are equal then the angle β is zero, but if Q1 is different than zero and Q2 is equal to zero, then β and α are equal between themselves and the angular velocity is perpendicular to the contact line comprised by C1-G1. The final dependency is if Q2 is greater than Q1, the angle β is positive but smaller than the angle α.

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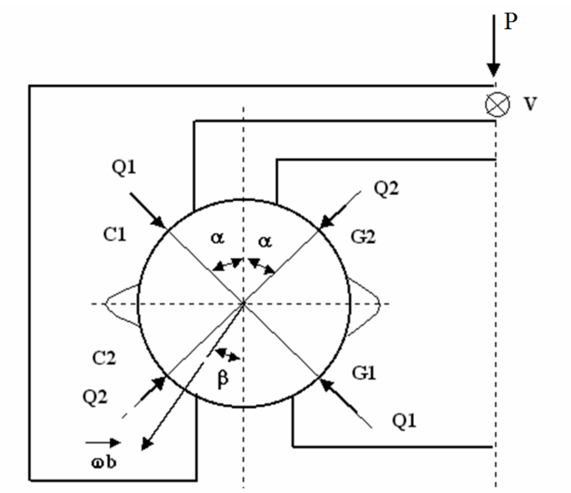

Figure 1. The position of the ball and the angular velocity of the ball in a lineal system type KUE with 4 contact points.

If the value of the angle β is known, it can be deducted if on the contact surfaces C1-C2 and respectively G1-G2 the balls produce a spin motion or just a rolling one. If β is equal to zero the spin motion is exhibited on the contact points C1-C2, but if the value of angle β is equal to that of the angle α, then the spin motion only appears in the contact point C2.

The problem consists in establishing an analytical model to calculate the value of the angle β and of the angular velocity ωb, based on the values of the load Q1 and Q2.

The proposed model is based on minimal friction power losses on the contact surfaces C1 and C2. The following hypothesis is made:

- the friction coefficient on the contact ellipses is constant;
- only the power losses due to the friction caused by the sliding velocity on the contact points C1 and C2 have been taken into consideration.

For the contact ellipse presented in figure 2, the power loss for a slice of the ellipse is calculated based on the following formula:

$$dP = |vs \cdot dFs| \tag{1}$$

where dFs is the friction force and vs is the sliding speed.

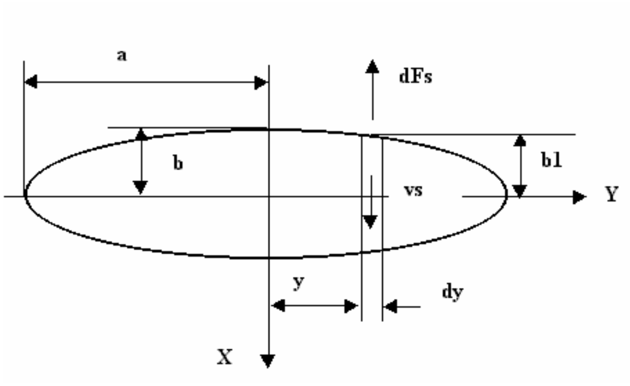

Figure 2. The sliding speed and the friction force on the contact ellipse.

The friction force is defined by the following relation:

$$dFs = \int_{-b1}^{b1} (\tau \cdot dx) \cdot dy \tag{2}$$

The tangential tension present in the contact ellipse is given by the relation:

$$\tau = \mu_m \cdot p_H \cdot \left(1 - \frac{x^2}{b^2} - \frac{y^2}{a^2}\right)^{\frac{1}{2}} \tag{3}$$

The contact pressure can be expressed in a different manner:

$$p_H \cdot \left(1 - \frac{x^2}{b^2} - \frac{y^2}{a^2}\right)^{\frac{1}{2}} = p_H \cdot \left(1 - \frac{x^2}{b1^2}\right)^{\frac{1}{2}} \cdot \left(1 - \frac{y^2}{a^2}\right)^{\frac{1}{2}} \tag{4}$$

were *b1* is the contact ellipse semi-axes and p_H is the maximum Hertzian contact pressure, a and b are the semi-major and semi-minor axes of the ellipse. b1 can be determined by:

$$b1 = p_H \cdot \left(1 - \frac{y^2}{a^2}\right)^{\frac{1}{2}} \tag{5}$$

The semi-major and semi-minor axes of the contact ellipse are computed based on the following formula:

$$a \approx 1.1552 \cdot R_X \cdot k^{0.4676} \cdot \left(\frac{Q}{E \cdot R_X{}^2}\right)^{\frac{1}{3}} \tag{6}$$

$$b \approx 1.1502 \cdot R_X \cdot k^{-0.1876} \cdot \left(\frac{Q}{E \cdot R_X{}^2}\right)^{\frac{1}{3}} \tag{7}$$

where E is the Young's modulus for the materials in contact, R_X is the equivalent radius in the rolling direction determined by $R_X = 0.5 \cdot d_w$ and k is the radius ratio based on:

$$k = \frac{R_{Y,Z}}{R_X} \tag{8}$$

$R_{Y,Z}$ – the transverse radius for a ball linear race contact can be determined by the relations:

for ball-carriage contact: $$\frac{1}{R_{Y,Z,C}} = \frac{2}{d_w} - \frac{1}{d_w \cdot f_c} \tag{9}$$

for ball-guide contact: $$\frac{1}{R_{Y,Z,G}} = \frac{2}{d_w} - \frac{1}{d_w \cdot f_g} \tag{10}$$

where d_w is the ball diameter, f_c is the carriage conformity and f_g the guide conformity.

Taken into consideration all the equation above, the friction force equation can be rewritten in the following form:

$$dFs = \pm \frac{3}{4} \cdot \mu \ \cdot Q \cdot \left(1 - \frac{y^2}{a^2}\right) \cdot \frac{1}{a} \cdot dy \tag{11}$$

where the (+) or (-) will be based on the sliding speed direction.

The sliding speed can also be expressed as a function of a distance y to the centre of the ellipse, in regards to:

- the contact ellipse C1:

$$vsc1(y)=v-\omega b\cdot\left[\left(Rd_c^2-y^2\right)^{\frac{1}{2}}-\left(Rd_c^2-a1^2\right)^{\frac{1}{2}}\right]\cdot\cos(\alpha-\beta)-\omega b\cdot\left(Rb^2-a1^2\right)^{\frac{1}{2}}\cdot\cos(\alpha-\beta)+\omega b\cdot y\cdot\sin(\alpha-\beta) \tag{12}$$

- the contact ellipse C2:

$$vsc2(y)=v-\omega b\cdot\left[\left(Rd_c^2-y^2\right)^{\frac{1}{2}}-\left(Rd_c^2-a2^2\right)^{\frac{1}{2}}\right]\cdot\sin(\alpha-\beta)-\omega b\cdot\left(Rb^2-a2^2\right)^{\frac{1}{2}}\cdot\sin(\alpha-\beta)+\omega b\cdot y\cdot\cos(\alpha-\beta) \tag{13}$$

where: a1 and a2 are the semi-major axes of the contact ellipse C1 and C2 which are dependent on the normal loads Q1 and Q2, respectively; v is the tangential speed in a ball-carriage and ball-guide contact, based on the fact that V is the carriage speed in a ball linear system ($v=0.5\cdot V$); Rd_c is the deformed contact ball-race radius for the ball-carriage contact and Rd_G is the deformed contact ball-race radius for the ball-guide contact, that are determined by following relation:

$$Rd_{C(G)}=\frac{2\cdot d_w\cdot f_{c(g)}}{2\cdot f_{c(g)}+1} \tag{14}$$

The power losses due to friction can be expressed integrating relation (1) for:

- the contact ellipse C1:

$$PS1=\int_{-a1}^{a2}|vsc1|\cdot dFs1=\frac{3}{4}\frac{\mu\cdot Q1}{a1}\cdot\int_{-a1}^{a1}|vsc1|\cdot\left(1-\frac{y^2}{a1^2}\right)dy \tag{15}$$

- the contact ellipse C2:

$$PS2=\int_{-a2}^{a2}|vsc2|\cdot dFs2=\frac{3}{4}\frac{\mu\cdot Q2}{a2}\cdot\int_{-a2}^{a2}|vsc2|\cdot\left(1-\frac{y^2}{a2^2}\right)dy \tag{16}$$

The moment of spin of the ball is calculated with the following formula:

- for the contact ellipse C1:

$$Mp1=\frac{3}{8}\cdot\mu\cdot Q1\cdot a1c(Q1) \tag{17}$$

- for the contact ellipse C2:

$$Mp2=\frac{3}{8}\cdot\mu\cdot Q2\cdot a2c(Q2) \tag{18}$$

Projecting the angular velocity vector ωb on the C1-G1 or C2-G2 axis, based on the value of the angular position of the ball β, the resulting angular velocity vector can be calculated:

- for the C1-G1 axis projection:

$$\omega p1=\omega b\cdot\cos(\frac{\pi}{4}+\beta) \tag{19}$$

- for the C2-G2 axis projection:

$$\omega p2=\omega b\cdot\cos(\frac{\pi}{4}-\beta) \tag{20}$$

The power loss due to the spin moment of the ball is:

- for the C1-G1 axis:

$$Pp1 = Mp1 \cdot \omega b \tag{21}$$

- for the C2-G2 axis:

$$Pp2 = Mp2 \cdot \omega b \tag{22}$$

The total power loss for both the contact ellipse C1 and C2, including the spin moments is:

$$PS = PS1 + PS2 + Pp1 + Pp2 \tag{23}$$

For a specific geometry and known values for the carriage speed, the normal loads Q1 and Q2 and the friction coefficient μ, the total energy losses can be considered dependent on the two parameters β and ωb:

$$PS = PS(\beta, \omega b) \tag{24}$$

It can assume that for any values of the Q1 and Q2 loads, the value of the parameters β and ωb can lead to a minimum value of the total power loss, furthermore the function $PS = PS(\beta, \omega b)$ will be minimum when the parameters β and ωb have the adequate values.

3. Numerical results

The graphical representation of the 3D and 2D variations of power based on the two parameters ß and ωb for the different values of the C/P ratios are presented in the following figures.

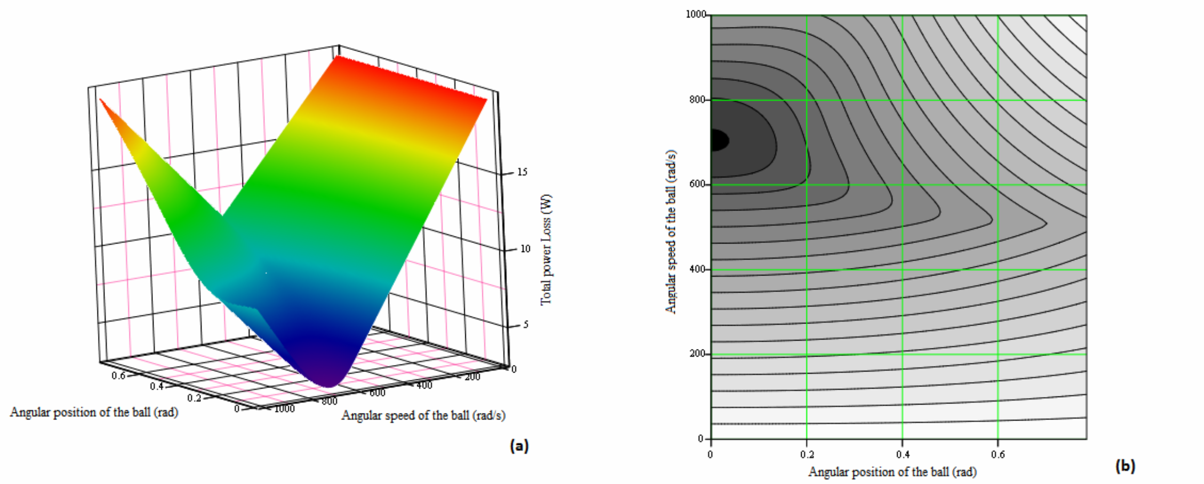

Figure 3. C/P=∞ (a) 3D representation of total power loss variation based on the two parameters ß and ωb, (b) 2D projection of the power loss variation in regards to ωb.

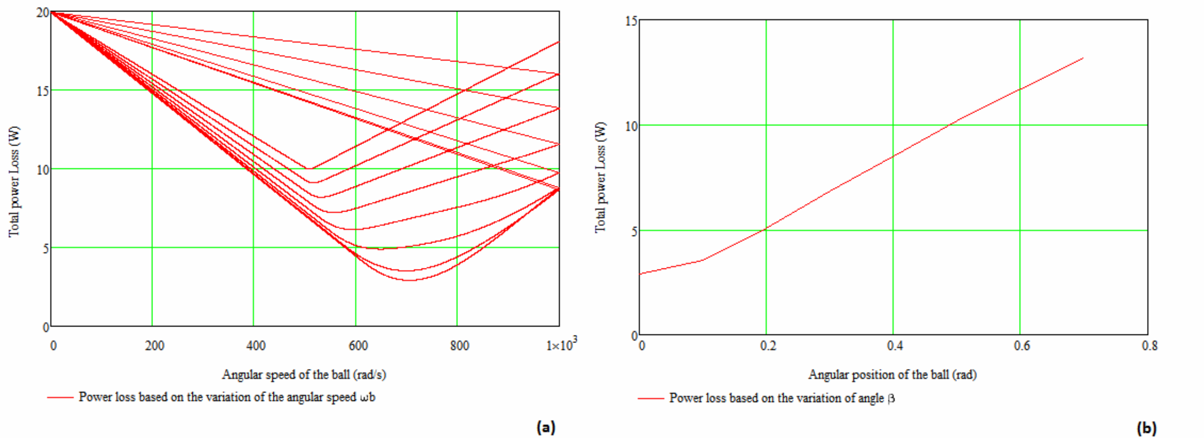

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Figure 4. C/P=∞ (a) Power loss variation in relation to ωb, (b) Power loss variation in relation to ß.

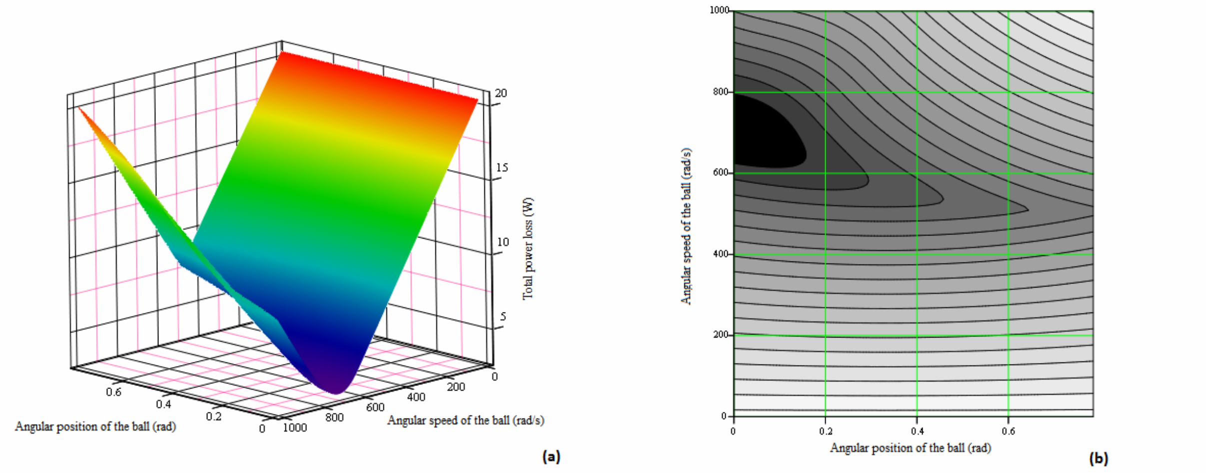

Figure 5. C/P=20 (a) 3D representation of total power loss variation based on the two parameters ß and ωb, (b) 2D projection of the power loss variation in regards to ωb.

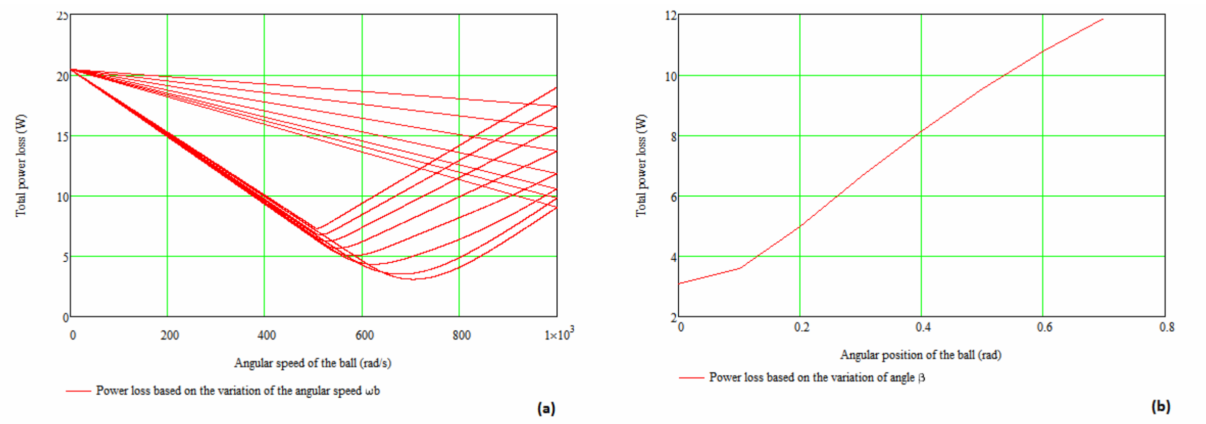

Figure 6. C/P=20 (a) Power loss variation in relation to ωb, (b) Power loss variation in relation to ß

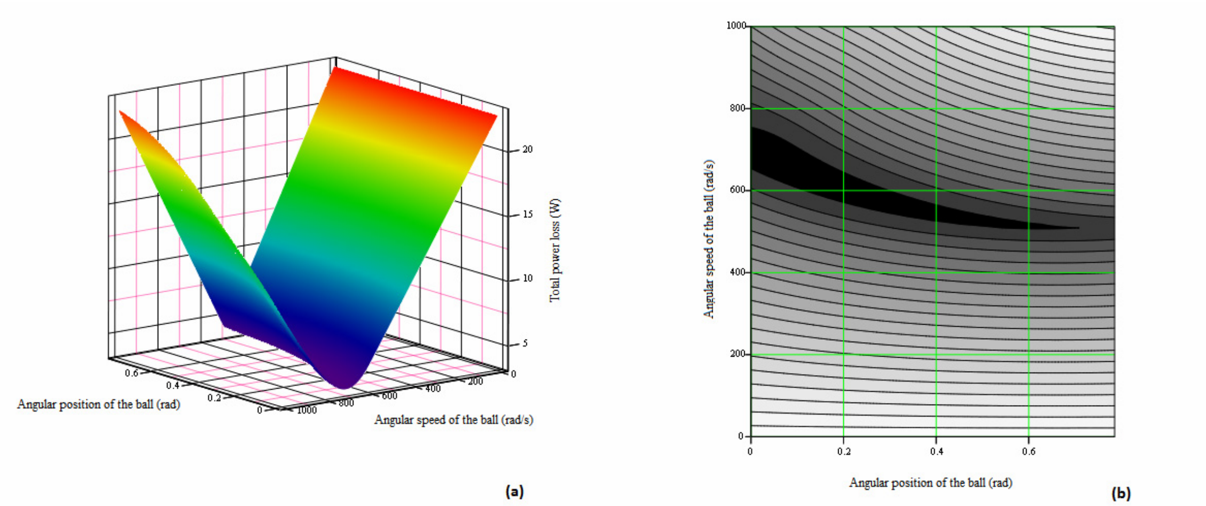

Figure 7. C/P=7.75 (a) 3D representation of total power loss variation based on the two parameters ß and ωb, (b) 2D projection of the power loss variation in regards to ωb

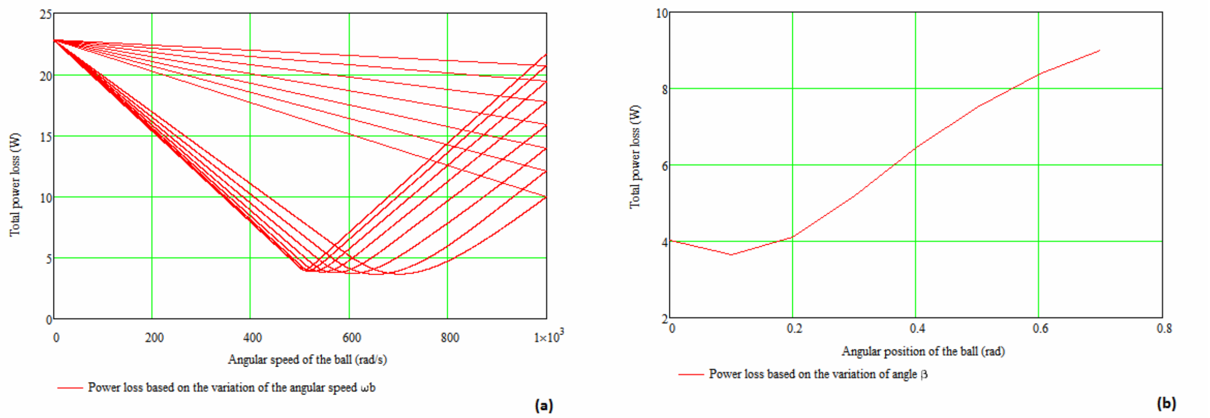

Figure 8. C/P=7.75 (a) Power loss variation in relation to ωb, (b) Power loss variation in relation to ß

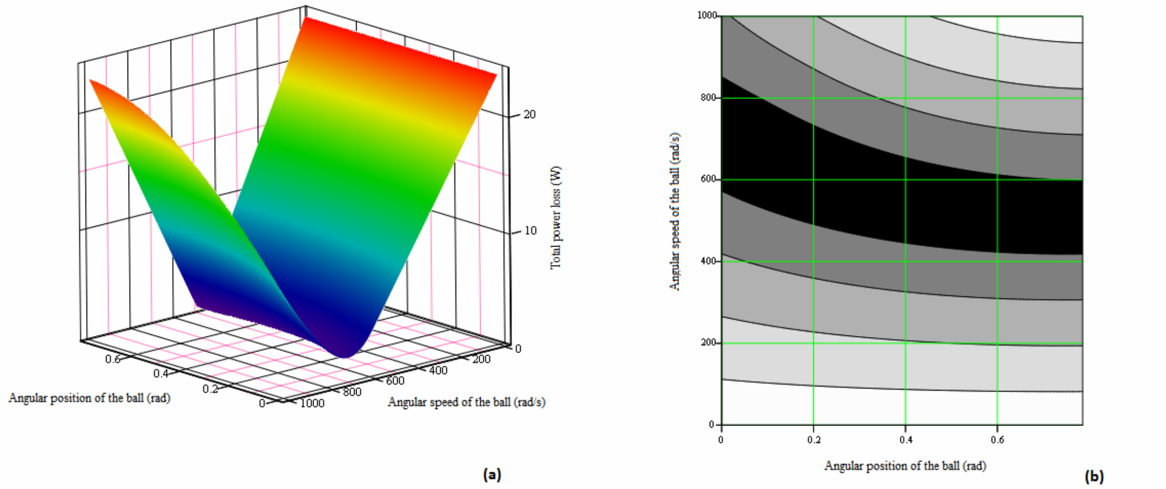

Figure 9. C/P=6 (a) 3D representation of total power loss variation based on the two parameters ß and ωb, (b) 2D projection of the power loss variation in regards to ωb

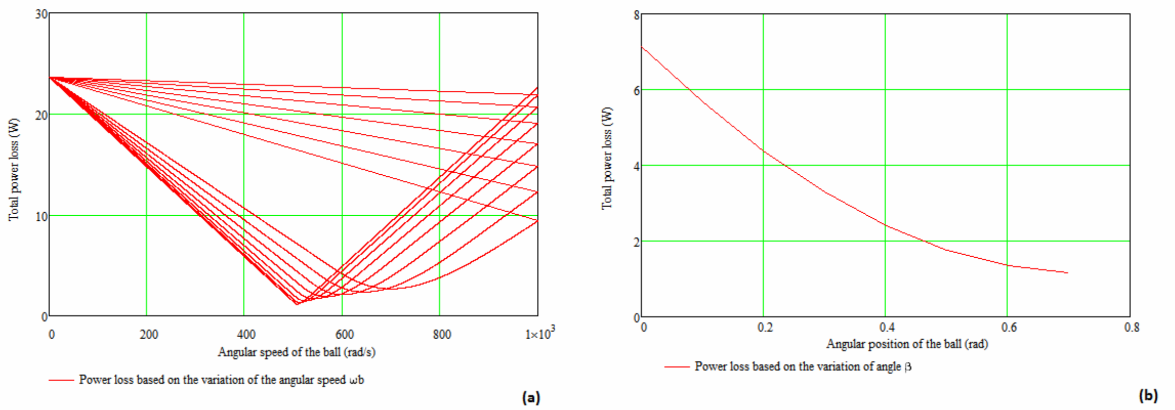

Figure 10. C/P=6 (a) Power loss variation in relation to ωb, (b) Power loss variation in relation to ß

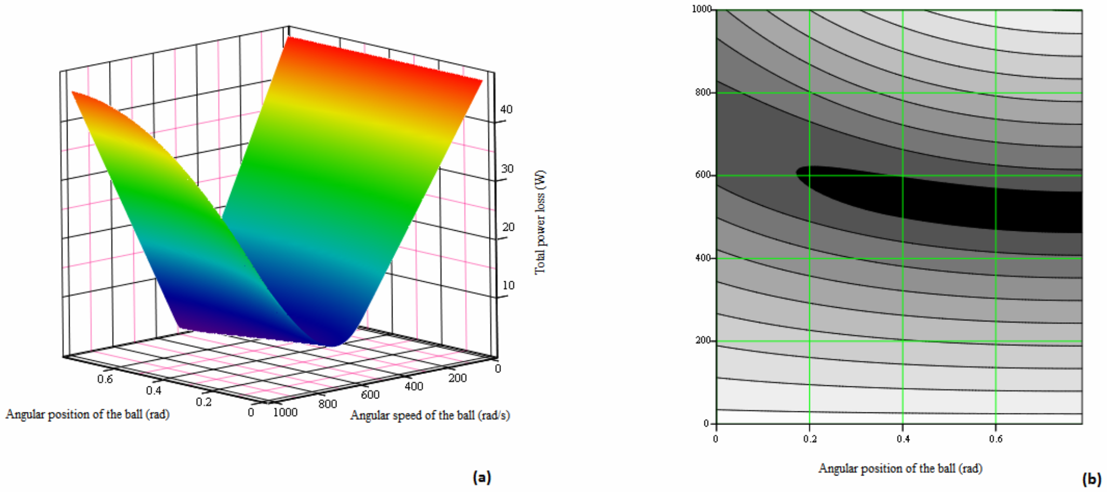

Figure 11. C/P=minimum (a) 3D representation of total power loss variation based on the two parameters ß and ωb, (b) 2D projection of the power loss variation in regards to ωb

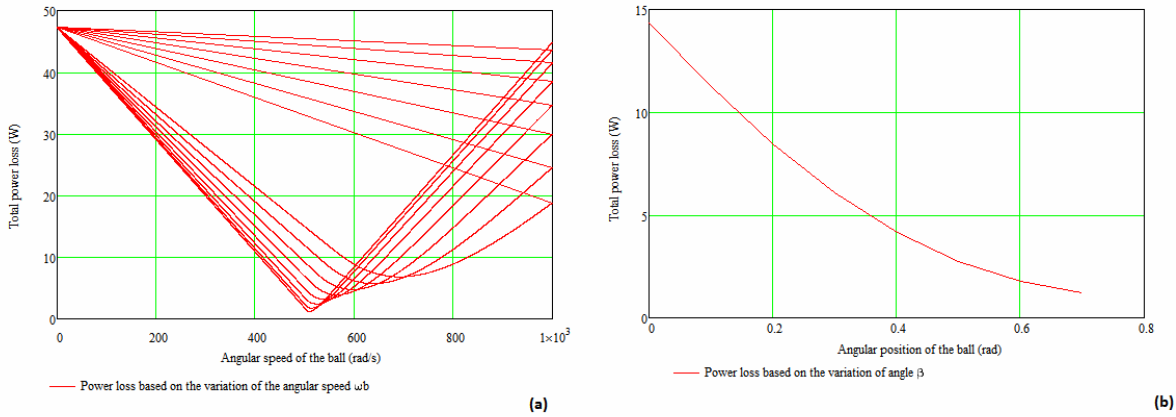

Figure 12. C/P=6 (a) Power loss variation in relation to ωb, (b) Power loss variation in relation to ß

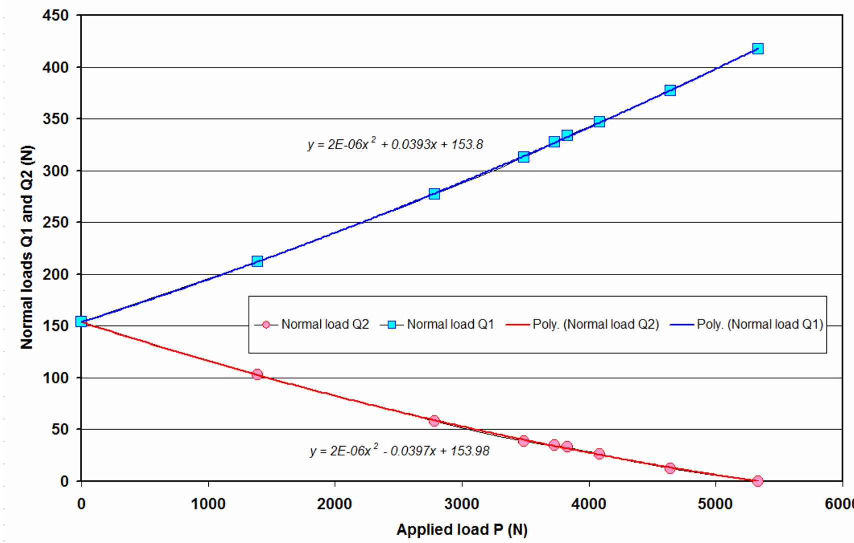

Figure 13. The loading forces Q1 and Q2 in relation to the power loss and a polynomial approximation of these values

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4. Conclusions

The motion of a ball in a linear system with four contact points is complex. The angular velocity varies in a large domain, depending on the normal loads on the contacts. Also, the position of the ball's angular velocity vector changes from zero degrees to 45 degrees, in regards to the normal loads on the contact points. The complex motion of the ball is the cause of the large sliding on the contact ellipses. So, if the two contacts between ball and carriage (or between ball and guide) are loaded, the pivoting motion appears on both contacts and the friction losses are high. If only one contact is loaded (for low values of the ratio C/P), the sliding between ball and race is small, thus two pure rolling points appear and the friction losses are significantly reduced. For the linear system KUE 35, it was analytically established that the important change in the ball motion is between C/P = 8 and C/P = 6.

5. References

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