

Fractal analysis of surface micro-topography for a rolled anisotropic thick sheet of aluminium alloy AA2024-T351

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Abstract. Fractal geometry has gained attention in recent years and represents a problem of high interest for the characterization of surface topography. In this study it was analyzed the surface micro-topography for a rolled thick sheet anisotropic metallic material of type 2000 series aluminium alloy (AA2024-T351). In order to analyze and to characterize the corresponding anisotropic surfaces, profile of particular samples were recorded with a specialized apparatus Mitutoyo SJ-301 (Japan). The random nature of the roughness height is described through statistical analysis. The irregularity of the surface profile has been measured using a lot of conventional surface roughness parameters such as: arithmetic average, mean square root, maximum height of the profile, etc. Fractal analysis provides a useful way to characterize the observed spatial complexity of surface micro-topography. For this study it was used the structural function method to calculate two specific fractal parameters: D (fractal dimension) and L (topothesy). The fractal dimension of all samples it's been be calculated by plotting curves on log-log axes.

1. Introduction

The roughness is one of the main characteristics of surface quality and it can be appreciated by determining the micrometric profile that results at the end of finishing operations. Given the complexity of profile surface and the difficulty of measuring it is useful to define the main geometry elements that are useful to describe the actual surface profile exactly how they are defined in the international standards. The corresponding roughness parameters are defined by [1] and represented in figures 1-3: the real surface - the surface that delimiting the body by separating it from the environment, the geometric surface - ideal theoretical surface neglecting all the errors, the real profile - the resulting profile by intersecting the actual surface with a conventional plane defined in relation to the geometric surface, the local prominence - the side profile comprised between two adjacent minimum profiles, the local gap - the side profile comprised between two adjacent maximum profiles, the reference line - the line that is chosen in a conventional manner to obtain a quantitative assessment of the real profile, the base length, l - the segment length measured on the reference line practically used to evaluate the real profile, the deviation profile y - the distance measured from a profile point to the reference line, the profile prominence p - the side profile oriented towards the exterior of the body and comprised between two consecutive points of the profile intersection with the reference line, the profile gap v - the side profile oriented towards the interior of the body and comprised between two consecutive points of the profile intersection with the reference line, the external profile line - the line which is parallel to a reference one and passes through the highest profile point, the internal profile



line parallel to the reference one and which passes through the lowest profile point, the irregularities profile step s_m - reference length of the segment between two successive projections profiles.

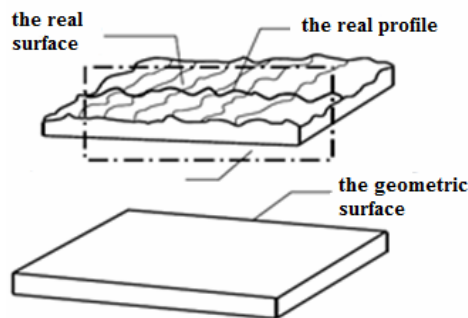


Figure 1. The real and the geometric surfaces [1].

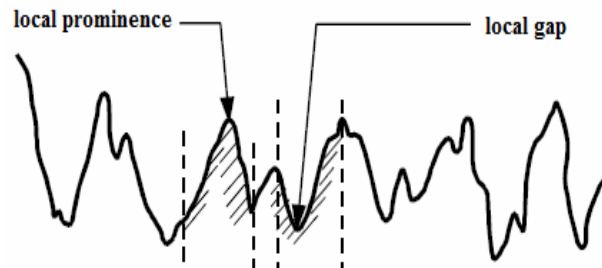


Figure 2. Local prominence and local gap [1].

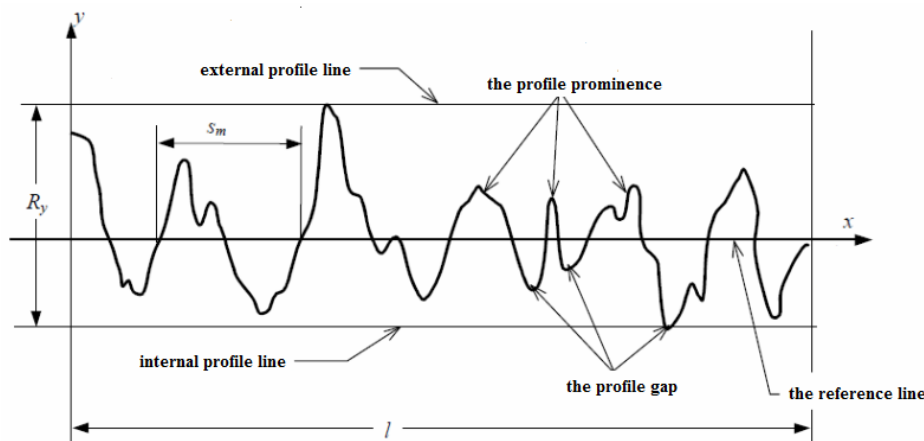


Figure 3. The main elements defining surface microtopography [1].

2. Experimental study

2.1. The material properties

For this study it's been used an aluminum alloy of 2000 series such as AA2024-T351 containing particularly copper and magnesium. This alloy it is one of the most popular high-strength aluminum alloys having good heat resistance and a low resistance to corrosion [2]. The major industrial applications concern generally the aeronautics structural components, aircraft accessories, parts for the transportation industry and molds manufacturing for plastics processing tools.

Table 1. Composition and properties of aluminum alloy AA2024- T351.

	Al	Si	Fe	Cu	Mn	Cr	Cr	Zn	Ti
Wt. %	---	0.5%	0.5%	3.8%-4.9%	0.3%-0.9%	0.1%	0.25%	0.25%	0.15%

2.2. The samples dimensions

Starting from a rolling plate having a 10 mm of thickness a lot of parallelepiped thick samples have been designed and cut out using a high speed machining process by the Mechanical Common Center (CCM) of INSA Rennes in order to analyses the material anisotropy [2] and to observe properties of the initial rolled sheet surface. On a normal size specimen surface there are several hundreds of different sized asperities which have different sizes and are randomly arranged. Measuring the size and location of these asperities is impossible; therefore, accurate measurements are made on samples (using representative areas) to allow extrapolating results to the whole surface area.

In order to analyze the quality of the anisotropic surfaces there were made measurements for the three main directions: LD - longitudinal direction (generally assumed to be identical with the rolling direction), TD - transversal direction and DD - diagonal direction.

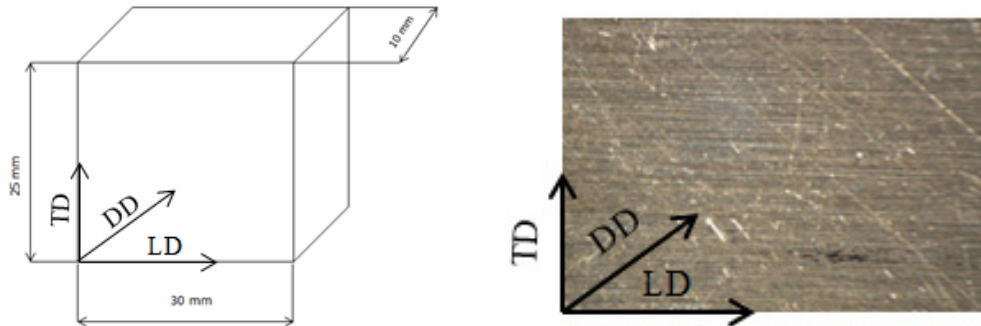


Figure 4. Samples dimensions and corresponding measuring directions.

2.3. The operating conditions

The roughness parameter is measured in micrometers (μm) and the used measurement methods are [1]: a) contact measurement by using special measuring devices such as profilometers and profilograme, b) non-contact measurement methods such as pneumatic, optical, electrical or electronic scanning of the surface shape. In order to analyses the quality of the anisotropic surfaces, the profile of each sample were recorded with a specialized apparatus Mitutoyo SJ-301 (Japan) from The Department of Machine Elements and Tribology, University Politehnica of Bucharest.

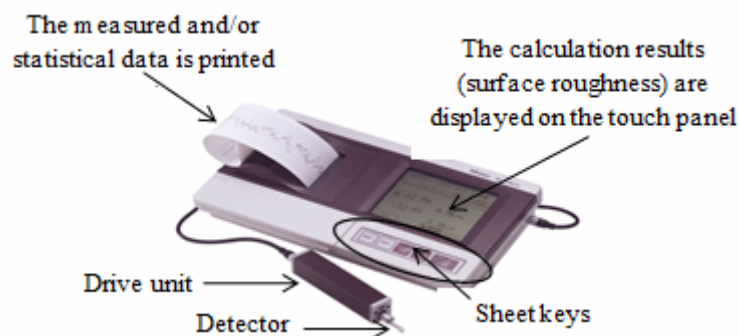


Figure 5. Profilometer SJ-301.

Mitutoyo profilometer SJ-301 has a measuring capacity of 5 mm. To measure the length of each specimen, there were taken more measurements for the three directions mentioned above: LD - longitudinal direction; TD - transversal direction and DD - diagonal direction. For the longitudinal direction were made six measurements, for the transversal direction five measurements and eight measurements for median direction.

3. Experimental results and statistical analysis

3.1. Amplitude parameters [3]

- R_a - arithmetic average of the profile, $R_a = \frac{1}{N} \sum_{i=1}^N |y_i|$
- R_q - mean square root, $R_q = \sqrt{\frac{1}{N} \sum_{i=1}^N (y_i)^2}$

- R_y - maximum height of the profile, $R_y = R_p + R_v$ where, R_p - maximum height of prominent profile i.e. $R_p = (\max(y))$ and R_v - maximum depth of the profile i.e. $R_v = (\min(y))$
- R_{sk} - asymmetry profile factor, $R_{sk} = \frac{1}{N \cdot R_q^3} \sum_{i=1}^N (y_i)^3$
- R_{ku} - flattening the profile factor, $R_{ku} = \frac{1}{N \cdot R_q^4} \sum_{i=1}^N (y_i)^4$

The following tables provide all the statistical data determined from the first measured sample of the three main directions (LD, TD and DD).

Table 2. Surface microgeometry parameters for LD direction (longitudinal direction).

Number and direction	R_a [μm]	R_q [μm]	R_y [μm]	R_{sk} [μm]	R_{ku} [μm]
LD_4.1-a	0.208	0.265	-0.16	-0.293	2.837
LD_4.1-b	0.187	0.241	-0.15	-0.649	3.472
LD_4.1-c	0.245	0.347	0.24	1.032	5.049
LD_4.1-d	0.366	0.481	1.35	0.797	5.63
LD_4.1-e	0.228	0.297	-0.63	-0.012	4.10
LD_4.1-f	0.226	0.283	0.003	-0.058	2.638

Table 3. Surface microgeometry parameters for TD direction (transversal direction).

Number and direction	R_a [μm]	R_q [μm]	R_y [μm]	R_{sk} [μm]	R_{ku} [μm]
TD_4.1-a	0.503	0.635	-0.16	0.03	2.834
TD_4.1-b	0.512	0.632	-0.36	0.026	2.708
TD_4.1-c	0.644	0.787	0.35	-0.106	2.709
TD_4.1-d	0.572	0.692	0.37	0.233	2.486
TD_4.1-e	0.549	0.712	-0.31	0.083	3.412

Table 4. Surface microgeometry parameters for DD direction (diagonal direction).

Number and direction	R_a [μm]	R_q [μm]	R_y [μm]	R_{sk} [μm]	R_{ku} [μm]
DD_4.1-a	0.526	0.654	-0.41	-0.407	2.922
DD_4.1-b	0.547	0.724	-0.76	0.232	4.23
DD_4.1-c	0.415	0.502	0.25	0.371	2.467
DD_4.1-d	0.614	0.776	0.87	0.741	3.038
DD_4.1-e	0.608	0.814	0.58	0.717	3.719
DD_4.1-f	0.497	0.627	0.09	0.489	3.473
DD_4.1-g	0.583	0.732	-0.04	0.402	3.037
DD_4.1-h	0.532	0.619	-0.06	-0.223	2.211

3.2. Functional parameters

To define the behaviour of the real contact area having a perfectly flat and tougher surface it is useful to plot a bearing area curve (Abbott-Firestone curve). For isotropic friction surfaces the bearing area curve is the same in all directions. For anisotropic surfaces (the most common case) it is desirable the knowledge of bearing area curve along a minimum of two perpendicular directions (in this case for three directions: LD - longitudinal direction; TD - transversal direction and DD - diagonal direction) [4]. The Abbott-Firestone curve is depending on [5]:

$$z = \text{sort}(y) + |\min(y)|; \quad z_a = \frac{z}{\max(z)} \quad (1)$$

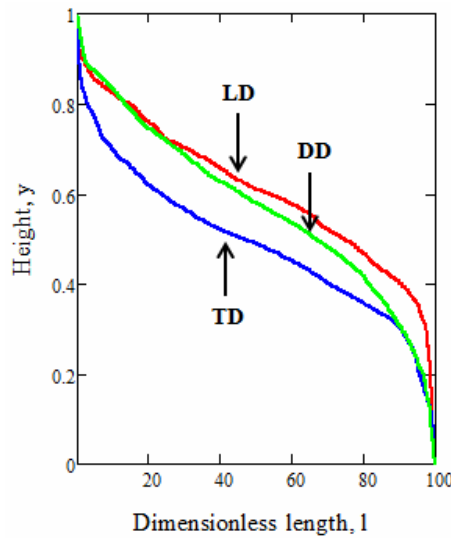


Figure 6. Normalized Abbott – Firestone bearing area curve for LD, TD and DD directions.

4. Fractal analysis

Several methods have been developed to characterize the dimension of a fractal set, such as the compass dimension, box dimension, mass dimension and area-perimeter dimension [6, 7]. The fractal dimension, D_f and the toposity, L_f are the most important aspects of fractal and they can be calculated using the following equations [8]:

$$D_f = \frac{4 - D_s}{2}, \quad 0 < D_s < 2 \quad (2)$$

$$\log L_f = \frac{C_2}{2D_f - 2} \quad (3)$$

The intersection of the structure function curve with the y axis, C_2 can be calculated as following [8]:

$$C_2 = \frac{y_{fs1} - D_s \cdot x_{fs1}}{1} \quad (4)$$

The slope of this function D_s , plotted on double logarithmic coordinates is calculated by using three points of structure function.

$$D_s = \text{slope}(x_{fs}, y_{fs}) \quad (5)$$

where:

$$x_{fs} = \begin{cases} x_{fs1} \\ x_{fs2} \\ x_{fs3} \end{cases}; \quad y_{fs} = \begin{cases} y_{fs1} \\ y_{fs2} \\ y_{fs3} \end{cases}$$

The roughness structure function, S can be written as [5]:

$$S(N, k) = \frac{1}{N - k} \sum_{i=1}^{N-k} (y_{i+k} - y_i)^2 \quad (6)$$

where k is the increment of x ordinate, $k = 1 \dots N - 1$; N is the length of the y vector, $N = 2000$

The straight line that approximates the structure function S and it can be determined as following:

$$y_f(x_f) = D_s \cdot x_f + y_{fs1} - D_s \cdot x_{fs1} \quad (7)$$

The table 5 provides the most important statistical data, such as arithmetic average of the profile, mean square root, fractal dimension and topothesy, determined from the mean of the measured sample for each direction longitudinal direction, transversal direction and diagonal direction.

Table 5. Surface microgeometry parameters for LD, TD and DD (longitudinal direction, transversal direction and diagonal direction).

Number and direction	R_a [μm]	R_q [μm]	D_f	L_f
LD	0.097	0.120	1.216	2.586×10^{-10}
TD	0.237	0.303	1.113	2.489×10^{-14}
DD	0.196	0.244	1.120	5.853×10^{-15}

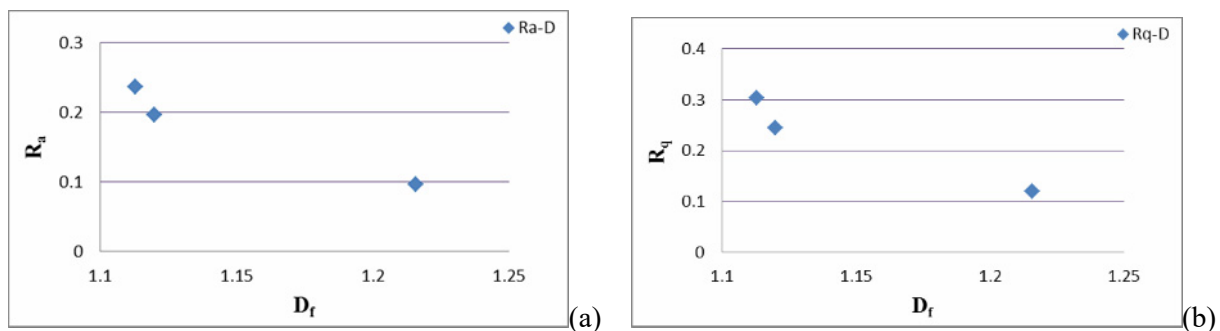


Figure 7. The effect of fractal dimension and statistical data determined from the mediation of the measured sample for each direction LD, TD and DD. (a) the effect of fractal dimensions D_f and statistical data R_a ; (b) the effect of fractal dimensions D_f and statistical data R_q .

The figure 8 describes the log-log curves (structure function - correlation length) and the straight line ($y_f(x_f)$) which approximates the structure function for the all measurement of the all three directions (LD, TD and DD).

The absolute scale-independent of parameters D_f and L_f is sometimes impossible and also unnecessary [9]. The Weierstrass-Mandelbrot (W-M) function was first used as an example of a real function which is continuous everywhere but differentiable nowhere. Later, its graph became a common example of a fractal curve. It can be used to simulate a fractal surface profile and is described as [8, 9]:

$$Z(x) = G^{(D_f-1)} \sum_{n=n_1}^{\infty} \frac{\cos(2\pi\gamma^n x)}{\gamma^{(2-D_f)n}} \quad 1 < D_f < 2, \eta > 1 \quad (8)$$

Here γ is the frequency factor; n - is an integer; n_1 - is the minimum of n ; G is the characteristic length scale of surface that determines the position of spectrum along power axis and which is invariant with respect to all frequencies of roughness [5]. It can easily be shown that the characteristic length scale can be evaluated as a function of topothesy L_f and of fractal dimension D_f , $G = L / C_4$ where C_4 can be computed from the formula [8]:

$$C_4 = \left\{ \frac{\Gamma(2D_f - 3) \sin\left[\pi(2D_f - 3)/2\right]}{2 - D_f} \right\}^{1/(2D_f-1)} \quad (9)$$

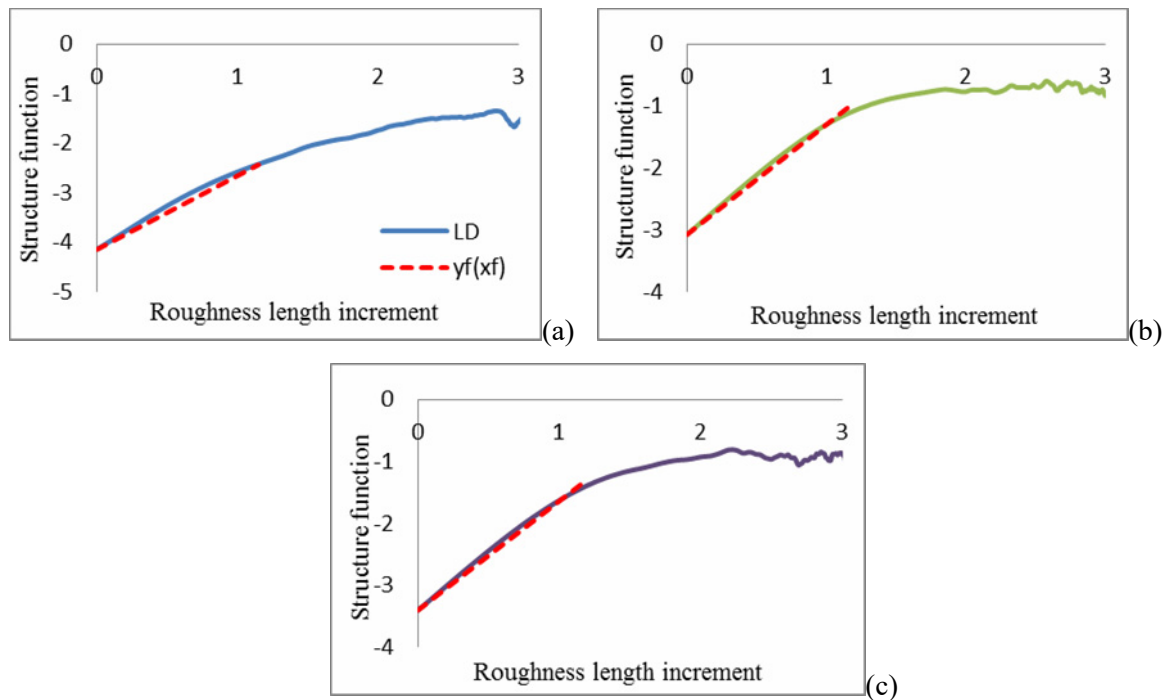


Figure 8. Log-Log curves (structure function- correlation length) and the straight line ($y_f(x_f)$); (a) longitudinal direction (LD); (b) transversal direction (TD) (c) diagonal direction (DD).

In figures 9 (a), 9 (b) and 9 (c) is shown the Weierstrass – Mandelbrot function (WM model) for all three directions: longitudinal direction (LD), transversal direction (TD) and diagonal direction (DD).

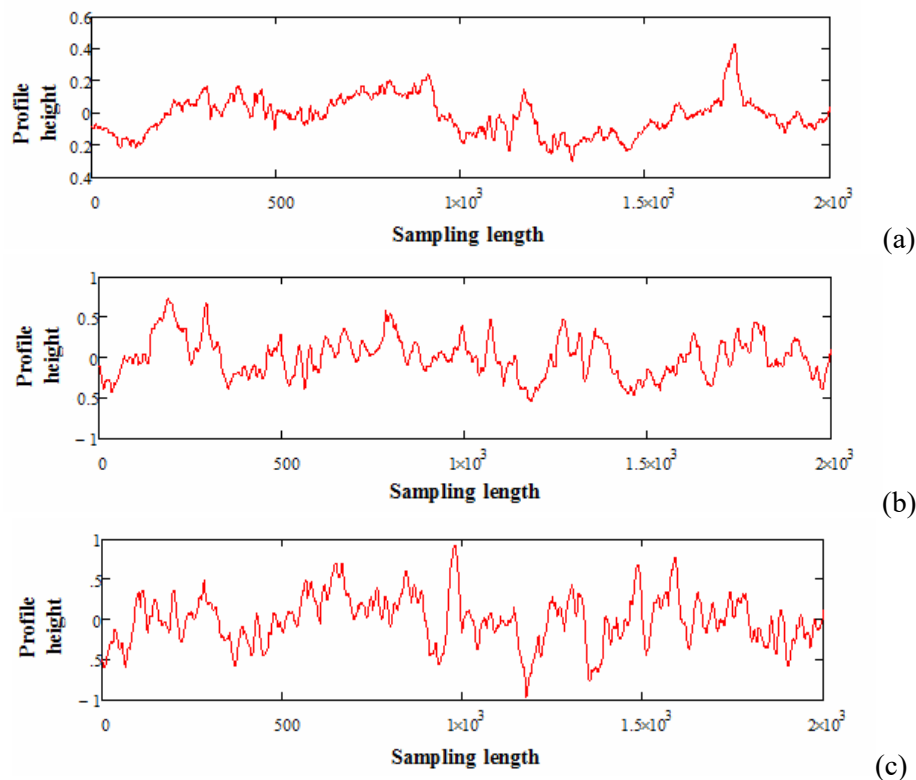


Figure 9. The Weierstrass – Mandelbrot function (WM); (a) longitudinal direction (LD), (b) transversal direction (TD), (c) diagonal direction (DD).

5. Conclusions

In this paper was studied the surface micro-topography for an anisotropic aluminium alloy AA2024-T351. In order to analyze the quality and morphology parameters of the anisotropic surfaces, the profile of each sample were recorded with a specialized apparatus Mitutoyo SJ-301 (Japan). To measure the length of each specimen, there were taken more measurements for the three main directions. For the longitudinal direction were made six measurements, for transversal direction five measurements and eight measurements for median direction. The random nature of the roughness height is described through statistical analysis, and the most important aspects of a fractal analysis are the obtained fractal dimension, D_f and the topothesy parameter L_f . It can be observed from the log-log curves (structure function - correlation length) and the straight line ($y_f(x_f)$) that the samples has a fractal behavior up to $1.1 \mu\text{m}$. From the graphics that shows the effect of fractal dimension and using statistical data results obtained from the mean of the measured sample along the three anisotropic directions (LD, TD and DD), it can be noticed that arithmetic average of the profile R_a and the mean square root R_q are smaller for the longitudinal direction and they have similar values for transversal and diagonal directions.

6. References

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