

# The singularities, forces transmission index and mechanism self-blocking

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**Abstract.** The paper represents a synthesis of the most representative results obtained by authors, along their researches in the field of mechanism singularities, forces transmission quality in mechanisms and self-blocking phenomenon. Our research is based on the idea that mechanism critical positions are a consequence of critical configurations in the component structural groups. At its turn, the critical configurations are noticed when the mathematical model of the kinematics and statics without friction, is affected by singularities. Thus, we have presented in the paper, the case of the principal structural groups, and we have established the critical configuration, the calculus formula of the configuration parameter and the existence conditions of the configuration (real solutions in the position problem).

## 1. Introduction

The singularities characterize those mechanism positions, when mechanism does not run properly, occurring the blocking phenomenon or kinematical indeterminations. These positions are related to the mathematical model depicting them. This model is an system of algebraic nonlinear equations. In singular positions, the system Jacobian equals zero and, as consequence, the determinant solving the kinematics and static without friction analysis also, equal zero. We named so defined singularities, as being of  $S_0$  type.

The singularity problem concerned researchers still at end of XIX century, when it founded the mechanism theory, because of their practical, applying character. In [7] a detailed analysis is being done, of the most important contributions in this field. It came to an important number of papers, characterized by a high theoretical degree and using an adequate mathematical apparatus [5,6,7,8,19,20]. It is also worth to mention that singularities imply defining of a index, in order to characterize the force transmission quality in mechanism. This index is depicting the ratio between useful forces and reactions in cinematic joints. The force transmission is the most unfavorable in singular positions and, their quality is increasing with distance from these positions. An other related to the two, problem is the mechanism self-blocking that occurs into a vicinity of the singular positions. It is noticed fact that approaching of these problems in technical literature is considering the system as a whole. This leads to a much increasing complexity of the approach with increasing of links number.

A different approach but with remarkable simplifying consequences consist of singularities analysing aided structural groups, composing the mechanisms. This approach were initiated in two papers worked out by Fr. Kovacs [1,2] and continued by M. Dranga [2,3]. We have brought, continuing this way, a series of contributions to solving these three problems [10-18]. In present paper we propose to synthesize these contributions in order to do more accessible the results contented in



these series of papers [10-18]. Of course we do not indicate here demonstrations and justifications that are being in the cited papers, but the results only.

## 2. The singularities

So we have shown, the singularities occur in those positions to which the determinant of the linear system depicting velocity and static analysis equals zero. The mathematical model materialized by this equation system depicts the mechanism structure, in the sense that it can be decomposed into subsystems, associated to the structural groups, which can be solved separately. If the determinant of such subsystem equals zero, we have a singularity to the level of that structural group. That transfers to the level of whole mechanism, because the global determinant equals zero too.

The configuration of a structural group, resulting from position analysis, is given by the extern joints position parameters. These, at their turn, depend on the mechanism position to which the group is attached. In order that position problem to have real solution, these parameters have to respect certain conditions. The singularities belong to the real domain but they lie at this domain boundary and have certain particularities regarding their configuration. Of course, when solutions lie in the complex domain, the group and the mechanism too, cannot be built. In paper [11] we studied these aspects and we have established the blocking configurations and also, the real solution conditions to the 022 structural groups. In table 1 we show synthetically these results for the 022 usual groups (with one prismatic joint at most). For the 033 and 042 groups, conditions to obtain real solutions are very difficult to express in analytical form, but blocking configurations are known [12] (table 2). In paper [16] we analysed the singularities analysis and we gave a new interpretation for the singularity types. This interpretation is based on the relation between extern joint positions and the defined above real solutions domain (table 1).

As we have being shown, it clearly results that singularity approaching for a complex mechanism, based on the structural groups, allows singularities identifying, locating and classifying without be necessary a complicated analysis of the position Jacobian. To show better this advantage, we will perform the possible singularities analysis for the mechanism in figure 1a, that comes from a Stephenson kinematical chain and has two structural groups RRT and RTR. In figures 1b and 1c, the mechanism is shown in those positions when the two structural groups have singular configurations. Identification of these positions is done without difficulties, applying to each group, the results from table 1.

## 3. The forces transmission

As we have being shown, the forces transmission quality in mechanisms is evaluated as report between useful forces and reactions in cinematic pairs, calculated neglecting frictions, in absence of a precise definition in this sense. An index depicting the forces transmission quality was being considered and is until now the pressure angle or it complement, the transmission angle. This opinion is correct, as we will show, in the case of simple mechanisms: four bar linkage and slider-crank. Attempts to extend the pressure angle to the complex mechanisms (with 6 or more links) do not succeeded [8].

The problems solution is given, as in the case of singularities, by an approach using structural groups. For each structural group, an index of movement transmission is defined, written down  $T$ . In [15] we proposed to use in defining the index  $T$ , the absolute value of determinant  $D$ , intervening into linear system of velocity and reaction without friction analysis. It can also adopt every monotone, reported to  $D$  function.

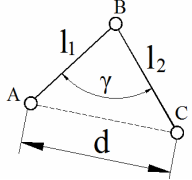
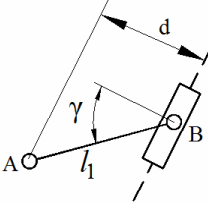
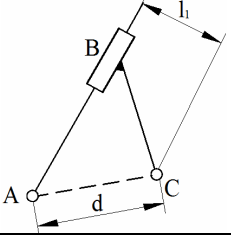
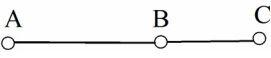

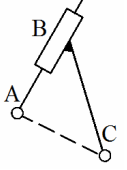
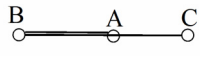
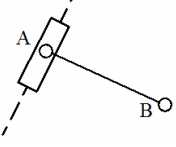
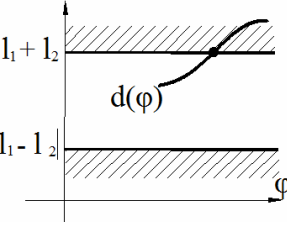
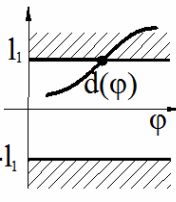
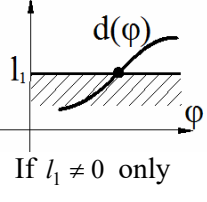
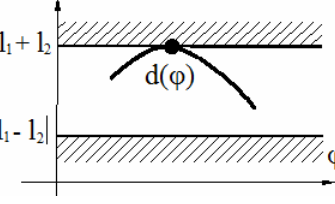
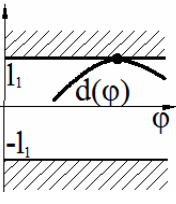
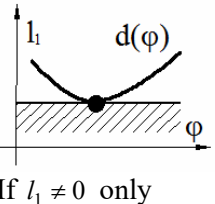
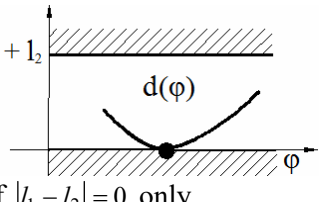
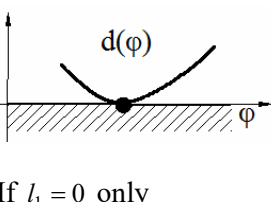
In the *RRR* structural group case,  $D$  is expressed as in [15]:

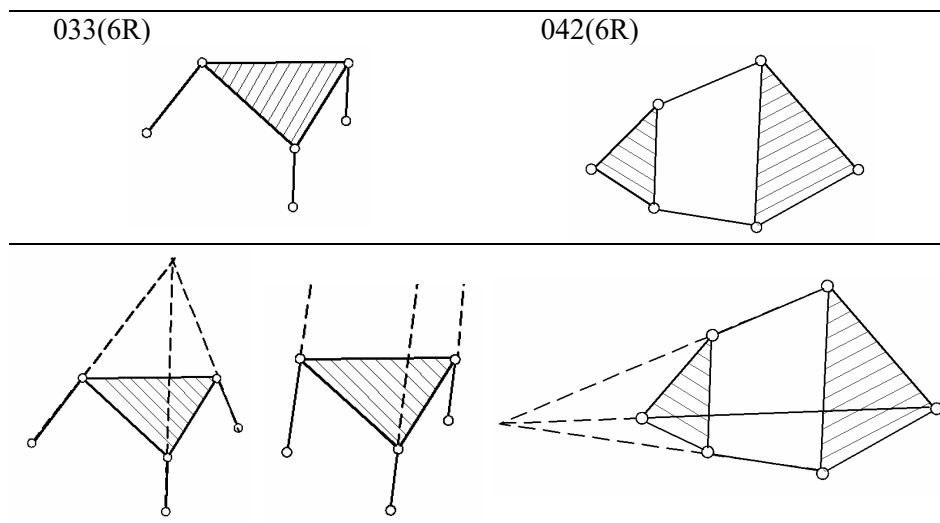
$$D = l_1 \cdot l_2 \cdot \sin \gamma, \quad (1)$$

and to *RRT* structural group:

$$D = l_1 \cdot \sin \gamma, \quad (2)$$

**Table 1.** Classification of the RRR, RRT and RTR dyads

	RRR	RRT	RTR
			
Conditions for real solutions	$l_1 + l_2 \leq d \leq  l_1 - l_2 $	$-l_1 \leq d \leq l_1$	$d \geq l_1$
Singular configurations ( $S_0$ )	 $d = l_1 + l_2, \gamma = 0$	 $d = l_1, \gamma = 0$	 $d = l_1$
	 $d =  l_1 - l_2 , \gamma = 0$	 $d = -l_1, \gamma = 0$	
Forces transmission index	$\gamma, \sin \gamma$	$\gamma, \sin \gamma$	$d$
Singularities classification	a) 		 If $l_1 \neq 0$ only
	b) 		 If $l_1 \neq 0$ only
	c)  If $ l_1 - l_2  = 0$ only		 If $l_1 = 0$ only

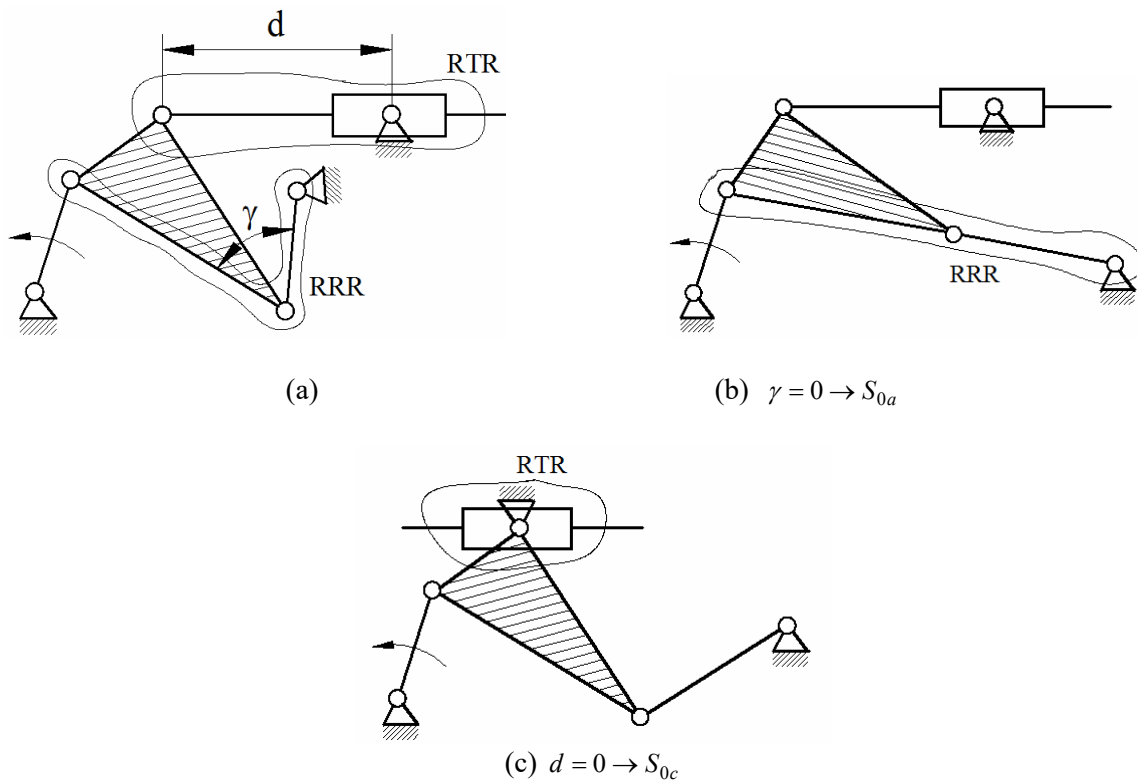
**Table 2.** 033(6R) and 042(6R) structural groups and its singularities

For the both structural groups,  $T$  can be adopted as:

$$T = D \text{ or } T = l_1 \cdot \sin \gamma \text{ or } T = \gamma, \quad (3)$$

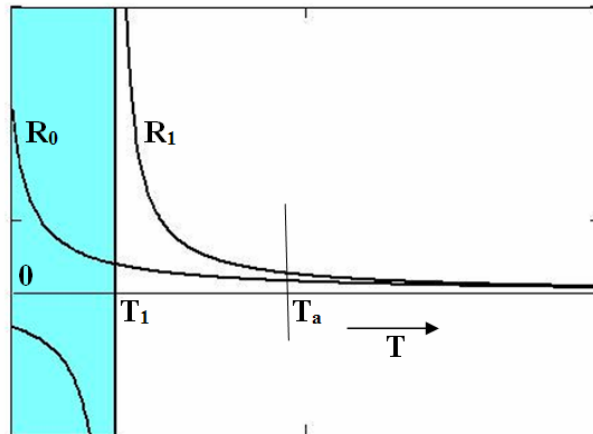
In the RTR structural group case [15] we have:

$$T = D = d, \quad (4)$$

**Figure 1.** Singularities analysis on an exemplifying mechanism.

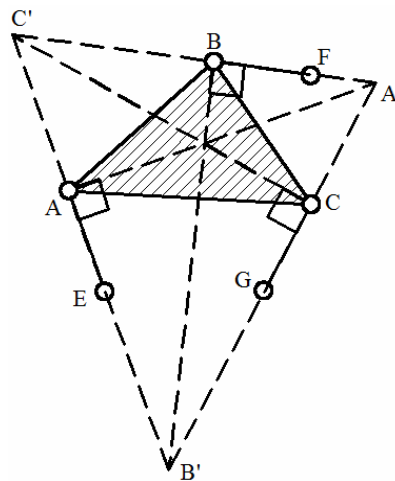
Parameters  $l_1$ ,  $l_2$ ,  $\gamma$  and  $d$  have the significations as in table 1.

In figure 2, the reaction without friction variation  $R_0$  is shown, reported to index  $T$ , that can be considered as group configuration parameter. It can be noticed that in singular configuration, at  $T = 0$ ,  $R_0$  tends to infinitum. At the same time with departing from this configuration  $R_0$  decreases. It result that forces transmission quality is as favorable as index  $T$  has the bigger value.



**Figure 2.** The reactions without friction ( $R_0$ ) and considering friction ( $R_1$ ).

In the 033(6R) case, the situation is different. Here, the singular configuration number is infinite, so that we cannot give as referential such a configuration but, we can use the fact that  $|D|$  has an unique maximum value. This value founds in the configuration when lines  $AE$ ,  $BF$  and  $CG$  are perpendicular on the  $AA'$ ,  $BB'$ ,  $CC'$ , where  $A'$  represents the intersection point of  $BF$  and  $CG$ ,  $B'$  - the intersection point of lines  $AE$  and  $CG$  and  $C'$  is the intersection point of the lines  $AE$  and  $BF$  [15,19,20], (figure 3).



**Figure 3.** Singular configurations of the 033(6R) group.

Calculating  $|D|_{\max}$ , for this configuration  $T$  is defined as:

$$T = \frac{|D|}{|D|_{\max}}. \quad (5)$$

When this structural group has the described configuration, the reactions have minimum value. Departing from this configuration leads to the decreasing of  $T$  in the same time with reactions increasing.

#### 4. Self-blocking

It is known, even we did not find a general and rigorous definition, that a mechanism blocks before it comes to a singular configuration of  $S_0$  type. If the friction is also taken into consideration, the reaction  $R_I$  advances as in figure 2, tending to infinitum in a point  $T_I$ , that corresponds to a configuration which we name, singularity of  $S_I$  type. This blocking configuration lies in a  $S_0$  vicinity. Into  $[S_0, S_I]$  interval, the friction forces, determined by calculus, are found with a physically incompatible sense. It is noticed that blocking phenomenon occurs in  $T_I$  point no matter of singularity type, according to classification from [16].

In [11] we indicated the calculus procedure for blocking configurations  $S_I$  determining to 022 groups. Based on these, we can show that blocking configuration depends on three groups of parameters: a) constructive cinematic parameters; b) machine part constructive parameters; c) parameters of tribology (friction coefficients). This problem were also been treated in [3,4] where indicates an approximated procedure, comparatively with our procedure, that is an exact one.

Even for  $T > T_I$ , running is possible, closely the blocking configuration, reactions have very high values, so that to come into this zone is not recommendable. Because of this, to designing, the index  $T$  is limited to an inferior value  $T_a$ , named admissible value, according to relation:

$$T \geq T_a > T_I \quad . \quad (6)$$

An interesting aspect, there is practically when sometimes  $S_I$  and  $S_0$  configurations can be passed through. This occurs when  $S_0$  singularity is of  $S_{0b}$  or  $S_{0c}$  type (table 1) and  $[0, T_I]$  interval is very small. As result from a qualitative designing, the value  $T_I$  comes really near to zero. We suppose the explanation of this contradicting theory phenomenon, being as follows. The dimensional errors, clearances in joints, links deformations modify structure and geometrical configuration of groups. These modifications, with small values, do not act in an aggressive way, when they are departed from the singularities, but they became of great importance, closely its, as result from [10]. Anyway, these situations have not practical importance as  $T_a$  index is more bigger than  $T_I$ .

#### 5. Conclusions

1) The singularity notion ( $S_0$ ) and that of forces transmission index ( $T$ ) are associated to each structural group, not to the mechanism as a whole. This fact, clearly shows that the way of singularity identifying, together with all afferent problems, is that appeals to structural groups. We consider that is the only possible way in the case of complex mechanism (containing more than one structural groups).

2) In our papers, dedicated to this problem [10-18] there are some contributions with priority character, that we show as follows:

- The singular configurations for the usual structural groups was established.
- It is indicated the calculus procedure of the 022 groups blocking configurations.
- It was also proposed as forces transmission index, the determinant  $D$ , associated to the linear system of the cinematic and static without friction analysis. This index is proved in the four bar linkage and slider-crank mechanism, because it is a monotone function respecting the transmission angle, used for this goal. Related to this, we have shown that the crank-shaper mechanism has a blocking position and a forces transmission index different from the transmission or pressure angle.
- The name of ( $S_I$ ) singularities for blocking configurations was proposed, because in this configurations the reactions with friction tend to infinitum.
- An original interpretation to the  $S_0$  singularities classifying is given, based on the conditions to obtain a real solution in position analysis.

- An attentive study of the 033(6R) group was performed, which has a particular character, showing that to this group, also can be adopted the determinant  $D$  as forces transmission index. We also have shown that to this group referential configuration is not a singular one, but it corresponds to the maximum value of the determinant  $D$ . In [17] we have established which the geometry of this configuration is and it calculus procedure.

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