The 033(6R) structural group existence condition in real field

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Abstract. This paper proposes an approach to identify and study the critical configurations of the 6R (033) structural group, based on an new idea, similar to the case of dyad groups. Starting up from the idea that the critical configurations of a structural group correspond to the singularities of the equation system, modeling the velocity problem and the critical positions are obtained when the lines through the binary link joints are concurrent in a point or are parallel. The velocity equation system for the case of 0/3/3 group with revolute joints, in conditions that the (6x6) system determinant equals zero.

1. Introduction

The control to obtain real solutions in the position problem, is essential for the mechanism designing process. The problem takes to singularities existence and to fully turning possibility of the driving elements. A global approach for a mechanism containing more than four elements, is complicated and references in technical literature are very restricted and do not solve complete the problem. The method, we exposed in a past series of papers [1-8], consists in approaching this problem, aided structural groups. As it is well known, structural groups have the property of being structural and kinematic determined. This property offers many opportunities, among which, that referring to the before mentioned problem. Of course, if every structural group of a mechanism has real solutions, the mechanism as a whole, has real solutions too. In a previous paper we studied from this point of view, the structural group 022 [1]. Even the problem solution of this group is ordinary it has very interesting consequences regarding singularities interpretation and classification. In present paper we deal with 033 structural group, covering so, an important part of usual mechanisms.

2. Formulation of the problem and discussions

The 033 (6R) structural group depends on the three variable, independent parameters. Logically, these parameters are the d_{12}, d_{13}, d_{23} distances (figure 1), because they are imposed when the group is attached to a mechanism in order to form a more complex mechanism. We propose to establish the condition which these parameters must respect, in order to obtain a real solution in configuration analysing (we name configuration - the group pose, reported to the central link). The mathematical model to determine the configuration is an algebraic, nonlinear equation system. Unfortunately mathematics can offer solution of this problem, in numerical form and for the concrete situations only. We however know an important aspect: the group singularities intervene when the lines *AE*, *BF* and *CG* (figure 2) are concurrent into a point or are parallel [6-10]. But, the singularities being located at the real domain border [1], this geometrical condition allows to determine a surface in d_{12}, d_{13}, d_{23} coordinates, limiting the real and the complex domains.
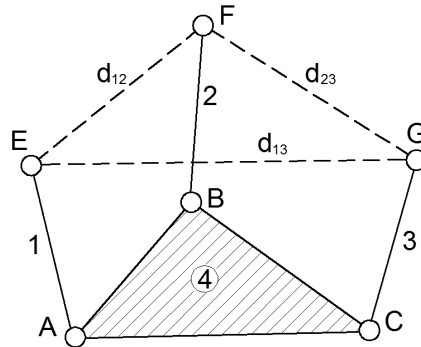
Figure 1. 033(6R) Structural group and configuration parameters d_{12}, d_{13}, d_{23}.
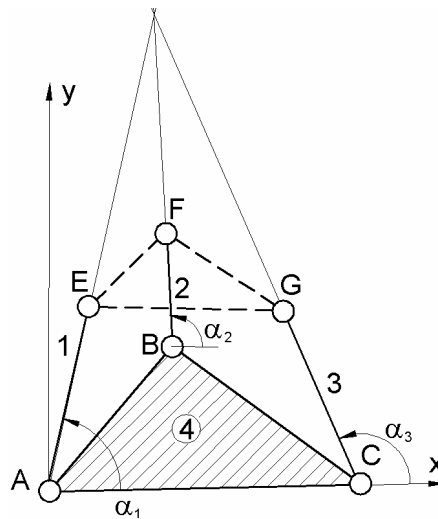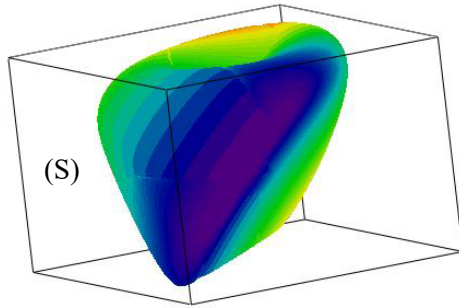

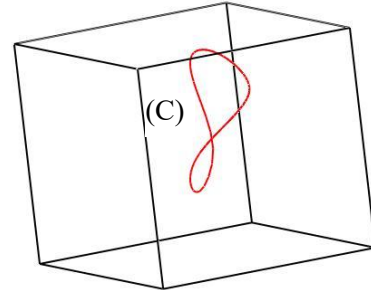**Figure 2.** A singular configuration of the 033(6R) structural group.

Let consider in the beginning as independent configuration parameters, the angles α_1, α_2, α_3. The points *E*, *F* and *G* positions are expressed in the *Axy* coordinate system (figure 2) as functions of α_1, α_2, α_3 and then the lengths d_{12}, d_{13}, d_{23} are calculated. If we impose the condition, the lines *AE*, *BF* and *CG* to be concurrent, one of the three parameters, for example α_3, can be eliminated. In this manner, the functions $d_{12}(\alpha_1, \alpha_2)$, $d_{13}(\alpha_1, \alpha_2)$, $d_{23}(\alpha_1, \alpha_2)$. Unfortunately, the problem cannot be solved in analytical way. Numerically proceeding, we impose variations in the domain $[0, 2\pi]$ to the α_1, α_2 parameters and represent in a three orthogonal system, the lengths d_{12}, d_{13}, d_{23}. We obtain a surface $S(d_{12}, d_{13}, d_{23})$ as in figure 3. The points with coordinates d_{12}, d_{13}, d_{23}, located inside the surface (*S*), lead to real solutions for the group configuration and the points outside this surface - to complex ones. The points placed on the surface lead to real solutions, but they represent critical (singular) positions.

Considering now the group connected into a mechanism, the $d_{12} = EF$, $d_{13} = EG$, $d_{23} = FG$ parameters are given and they depend on the given mechanism, to which the group is attached (figure 4). The parameters d_{12}, d_{13}, d_{23} can be expressed as function of one single independent variable, that can be an angle φ (figure 4) or even the time. The functions $d_{12}(\varphi)$, $d_{13}(\varphi)$, $d_{23}(\varphi)$, determine in the
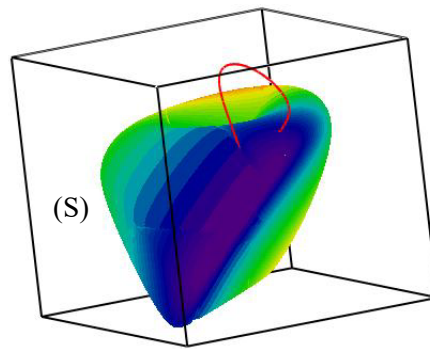
d_{12} , d_{13} , d_{23} system of coordinates a curve (*C*) generally a spatial one. We report the surface (*S*) and curve (*C*), to the same system of coordinates (figure 3).

(a) The spatial surface $S(d_{12}, d_{13}, d_{23})$

(b) The spatial curve $C(d_{12}, d_{13}, d_{23})$

(c) The spatial surface $S(d_{12}, d_{13}, d_{23})$ and curve $C(d_{12}, d_{13}, d_{23})$ in their relative position

Figure 3. The spatial surface (*S*) and curve (*C*).
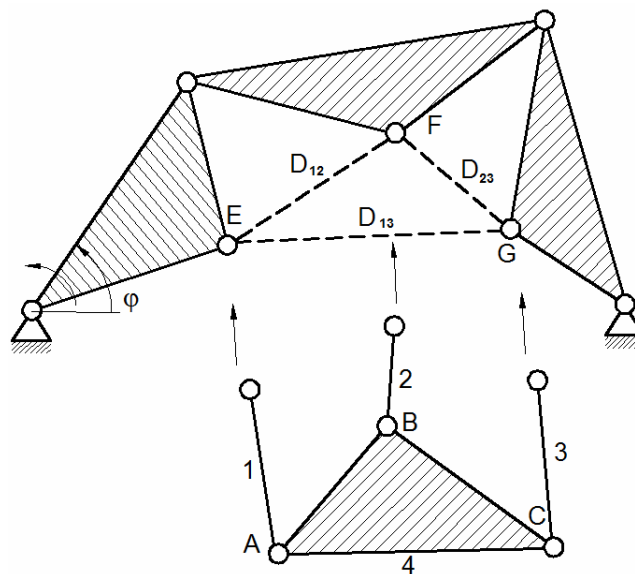

Figure 4. The 033(6R) structural group and the procedure to integrate it into a complex mechanism

The pose of curve (C) reported to the surface (S) leads to the following cases with interpretations regarding the mechanism running.

1) The curve (C) is placed entirely inside the space, bordered by surface (S), without common points between them. In this case, the mechanism can running in all domain of the independent parameter φ.

2) (C) is entirely located outside the surface (S), without common points between them. In this case, the mechanism cannot be assembled.

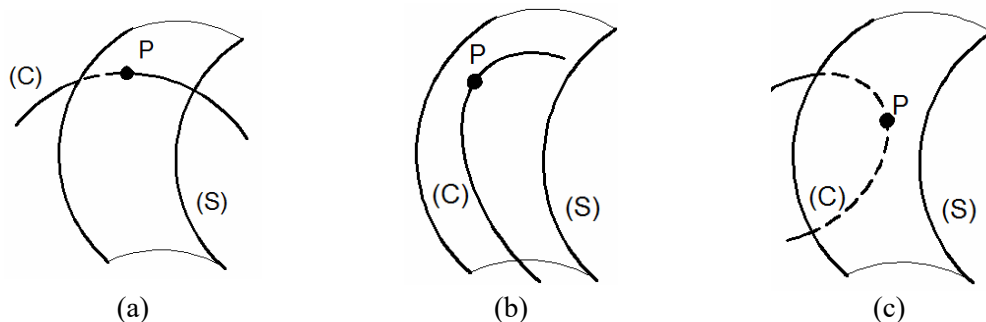3) There are common (intersection) points between surface (S) and curve (C). This case requires three subcases:

(a)   (b)   (c)

Figure 5. The possible positioning between (S) and (C).

3a) The curve (C) intersects the surface (S) in a single point P (figure 5a). In this case, the mechanism is running properly for the arc placed inside the surface (S), but it blocks in the point P. In this point there is a singularity, that we call of the *Soa* type.

3b) The curve (C) touches the surface (S) in a point P, (C) being placed inside (S) (figure 5b). In this case, passing through point P, a velocity indetermination occurs - the movement sense of some links can be uncontrolled changed. This situation generates in P singularities of *Sob* type. If curve (C) touches the surface (S) in P, but being placed outside (S), the mechanism has only one fixed position, in point P (figure 5c).

3c) The surface (S) and curve (C) pass through the point *P(0,0,0)*. In this case, a indetermination occurs, in the position problem, this is a singularity of type *Soc*.

3. Conclusions

1) The existence condition for a real solution in the position (configuration) problem, solves by a graphical, numerical procedure. It results a surface, separating the real domain from the complex one.

2) This condition, expressed by mentioned above surface leads to a new interpretation of the singularities, intervening to this structural group.

3) Starting from this interpretation of singularities, a development needs, referring to the mechanism particularities establishing, to which these singularities occurs.

4. References

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