IOP Conf. Series: Materials Science and Engineering **147** (2016) 012001 doi:10.1088/1757-899X/147/1/012001

The bearing capacity of the assemblies through compression with intermediate biconical elements

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Abstract. The assemblies through elastic compression are made through forced contact between the mating surfaces of the pieces to be assembled. These assemblies function due to friction force developed between surfaces under pressure contact. If the surfaces in contact present elastic deformations, the assembly is removable, if not, the assembly is non-removable. This research presents the calculus algorithm of the capable torque of an assembly with biconical intermediate elements. To verify the accuracy of the theoretical results this research contains the appliance used to obtain the experimental value of the bearing capacity of the assemblies through compression with intermediate biconical elements.

1. General considerations

Presented as a better version of the assemblies through conical compression, the assembly with intermediate biconical elements which is analysed in this research is shown in figure 1 and it is used for the assembly of shaft-hub type parts [1].

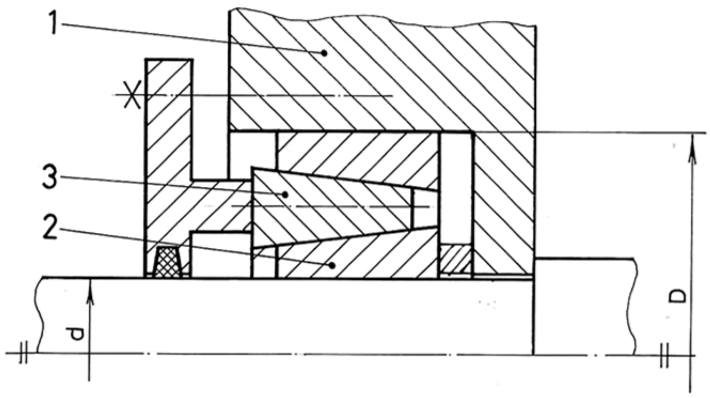

Figure 1. Componing elements of the assembly with intermediate biconical elements: 1 – outer ring; 2 – inner ring; 3 – intermediate ring.

The mating surfaces of the pieces to be assembled have a cylindrical form. Both the exterior and interior rings are unilateral sectioned (them being elastical rings) to adjust without big pre-tensioning on the shaft and hub [1]. The interior and outer surfaces have a roughness of $R_a \leq 0.6$ µm.

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The shaft-hub must have a roughness of $R_a \geq 3.2$ μm in the binding area. The sectioning milling width is chosen so that the ring could compensate a diametrical loose of (2…3) mm. The conical surfaces of the elements are being mounted with mineral oil.
As main advantages are mentioned:
- less severe tolerances and a lower quality than assemblies through pressure have.
- they provide a better shaft-hub centered structure.
- the maintenance of a precise axial and angular position.
- the possibility of repeated mounting and dismounting without damaging the contact surfaces.

2. The transmission capacity and the force system of the assemblies through compression with intermediate biconical elements.

The transmission of the torque is based on the friction formed on the contact surfaces between the rings and the parts to be assembled [2, 3]. The connection between the radial compressive force Q_1 and the axial pressure force F_{a1} is established by repeatedly applying the equilibrium condition on the direction of the axial force for each ring: (outer, intermediate, inner) based on the force system presented in figure 2.

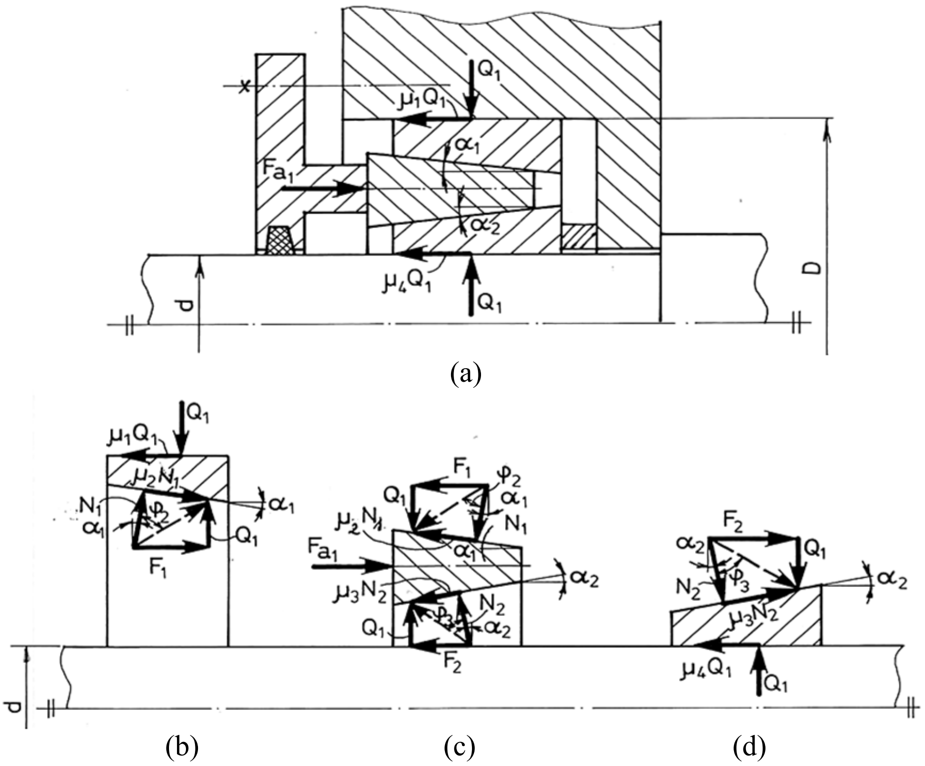

Figure 2. The forces with actuating on rings:
(a) assembling; (b) outer ring; (c) intermediate ring; (d) inner ring.

All four contact surfaces have different friction factors. For the intermediate ring we have the following equations:

$$F_{a1} - F_1 - F_2 = 0 \tag{1}$$

where:

$$F_1 = Q_1 \cdot \mathrm{tg}(\alpha_1 + \varphi_2) \tag{2}$$

and

$$F_2 = Q_2 \cdot \mathrm{tg}(\alpha_2 + \varphi_3) \tag{3}$$

as a result:

$$F_{a1} = Q_1 \cdot [\mathrm{tg} \cdot (\alpha_1 + \varphi_2) + \mathrm{tg}(\alpha_2 + \varphi_3)] \tag{4}$$

The compression force has the following formula:

$$Q_1 = \frac{1}{\mathrm{tg}(\alpha_1 + \varphi_2) + \mathrm{tg}(\alpha_2 + \varphi_3)} \cdot F_{a1} \tag{5}$$

From the equation of the forces projected after the assembly axis we obtain:

$$F_{a1} - \mu_1 \cdot Q_1 - F_{a2} - \mu_4 \cdot Q_1 = 0 \tag{6}$$

resulting:

$$F_{a2} = F_{a1} - (\mu_1 + \mu_4) \cdot Q_1$$

$$F_{a2} = F_{a1} - (\mu_1 + \mu_4) \cdot \frac{1}{\mathrm{tg}(\alpha_1 + \varphi_2) + \mathrm{tg}(\alpha_2 + \varphi_3)} \cdot F_{a1} \tag{7}$$

$$F_{a2} = \left[1 - \frac{\mathrm{tg}\varphi_1 + \mathrm{tg}\varphi_4}{\mathrm{tg}(\alpha_1 + \varphi_2) + \mathrm{tg}(\alpha_2 + \varphi_3)}\right] \cdot F_{a1}$$

$$F_{a2} = K \cdot F_{a1} \tag{8}$$

where K is the reduction factor of the axial force

$$K = 1 - \frac{\mathrm{tg}\varphi_1 + \mathrm{tg}\varphi_4}{\mathrm{tg}(\alpha_1 + \varphi_2) + \mathrm{tg}(\alpha_2 + \varphi_3)} \tag{9}$$

in which:

$$\mu_1 = \mathrm{tg}\varphi_1 \tag{10}$$

is the friction factor between the outer ring and the reaming of the part to be assembled.

$$\mu_2 = \mathrm{tg}\varphi_2 \tag{11}$$

is the friction factor between the outer ring and the intermediate ring.

$$\mu_3 = \mathrm{tg}\varphi_3 \tag{12}$$

is the friction factor between the intermediate ring and the inner ring.

$$\mu_4 = \mathrm{tg}\varphi_4 \tag{13}$$

is the friction factor between the inner ring and the axle.

The case of equal friction factors. If equal friction factors are admitted:

$$\mu_1 = \mu_2 = \mu_3 = \mu_4 = \mathrm{tg}\varphi \tag{14}$$

and

$$\alpha_1 = \alpha_2 = \alpha \tag{14'}$$

equation number (9) becomes:

$$K = 1 - \frac{\mathrm{tg}\varphi + \mathrm{tg}\varphi}{\mathrm{tg}(\alpha + \varphi) + \mathrm{tg}(\alpha + \varphi)}$$

$$K = 1 - \frac{\mathrm{tg}\varphi}{\mathrm{tg}(\alpha + \varphi)} = 1 - \frac{\mathrm{tg}\varphi(1 - \mathrm{tg}\alpha \cdot \mathrm{tg}\varphi)}{\mathrm{tg}\alpha + \mathrm{tg}\varphi}$$

$$\mathrm{K} = (1 + \mathrm{tg}^2\varphi) \cdot \frac{\mathrm{tg}\alpha}{\mathrm{tg}\alpha + \mathrm{tg}\varphi} = \frac{1}{\cos^2\varphi} \cdot \frac{\mathrm{tg}\alpha}{\mathrm{tg}\alpha + \mathrm{tg}\varphi} \tag{15}$$

The values of the reducing factor obtained in equation number (15) for different values of the α angle and friction factors are presented in table 1.

The variation diagram of *K* factor for $\mu = 0.15$ is presented in figure 3. The variation diagram of *K* factor for different friction factor values presented in table 1 is displayed in figure 4.

Table 1. Reduction factor values (K).

Friction factor values	Reduction factor of the axial force							
	Angle α (°)							
	6	8	10	12	14	16	18	20
μ_1=μ_2=μ_3=μ_4=0.04	0.72548	0.77968	0,81639	0.84296	0.86312	0.87898	0.89180	0.90242
μ_1=μ_2=μ_3=μ_4=0.06	0.63887	0.70333	0.74879	0.78266	0.80892	0.82993	0.84715	0.86157
μ_1=μ_2=μ_3=μ_4=0.08	0.57143	0.64133	0.69229	0.73119	0.76192	0.78686	0.80756	0.82505
μ_1=μ_2=μ_3=μ_4=0.10	0.51755	0.59011	0.64448	0.68685	0.72086	0.74884	0.77230	0.79291
μ_1=μ_2=μ_3=μ_4=0.12	0.47362	0.54718	0.60360	0.64835	0.68479	0.71512	0.74049	0.76288
μ_1=μ_2=μ_3=μ_4=0.14	0.43720	0.51078	0.56833	0.61470	0.65295	0.68510	0.71256	0.73636
μ_1=μ_2=μ_3=μ_4=0.15	0.42126	0.49460	0.55248	0.59945	0.63840	0.67131	0.69954	0.72408

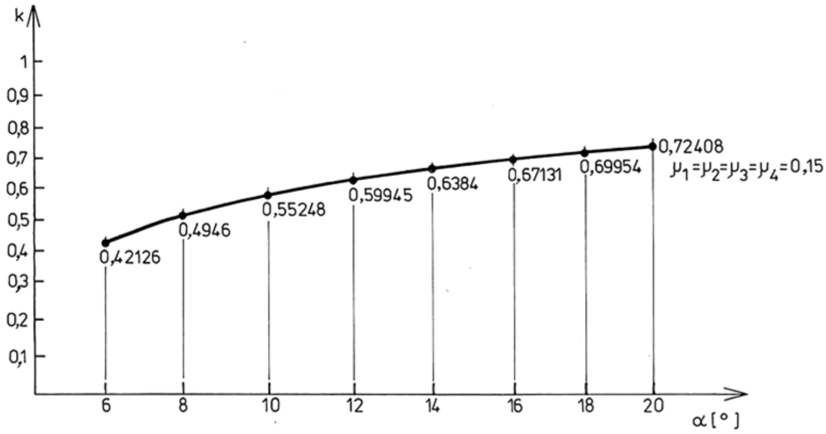

Figure 3. The variation diagram of *K* factor for $\mu = 0.15$.

The case of different values for each friction factor. According to equation (9), the following cases are going to be analysed:

- the case of big differences between the values of the friction factors (table 2):

$$\mu_1 = \mu_2 = \mu_3 = 0.15 \text{ and } \mu_4 = 0.04;$$
$$\varphi_1 = \varphi_2 = \varphi_3 = \text{arctg}0.15 = 8.53076°;$$
$$\varphi_4 = \text{arctg}0.04 = 2.29061°;$$

- the case of small differences between the high values of the friction factors:

$$\mu_1 = \mu_2 = \mu_3 = 0.15 \text{ and } \mu_4 = 0.12;$$
$$\varphi_1 = \varphi_2 = \varphi_3 = \text{arctg}0.15 = 8.53076°;$$
$$\varphi_4 = \text{arctg}0.12 = 6.84277°;$$

- the case of small differences between the lower values of the friction factors:

$$\mu_1 = \mu_2 = \mu_3 = 0.06 \text{ and } \mu_4 = 0.03;$$
$$\varphi_1 = \varphi_2 = \varphi_3 = \text{arctg}0.06 = 3.43363°;$$
$$\varphi_4 = \text{arctg}0.03 = 1.71835°;$$

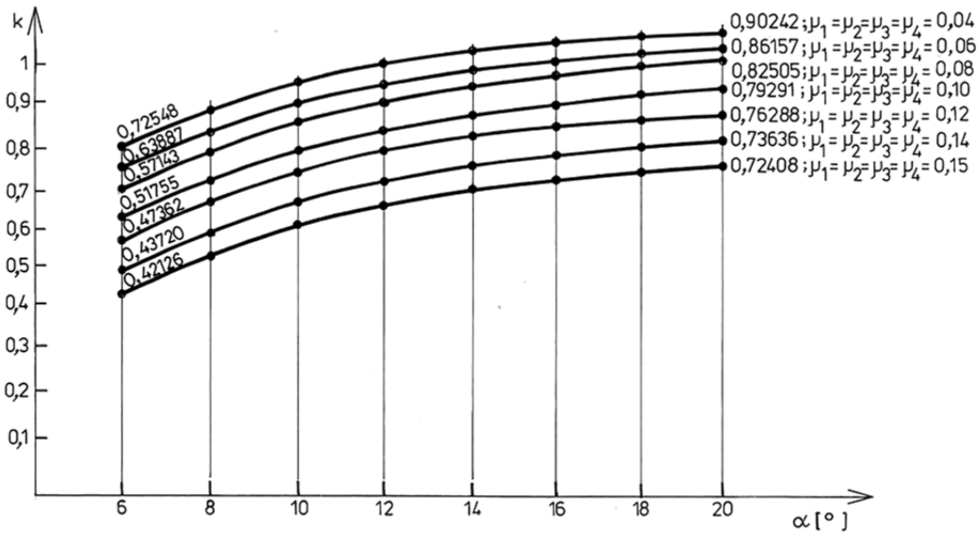

Figure 4. The variation diagram of *K* factor for different friction factor values.

Table 2. Values of *K*, the reduction factor of the axial force.

Friction factors values	Values of *K*, the reduction factor of the axial force							
	Angle α (°)							
	6	8	10	12	14	16	18	20
$\mu_1=\mu_2=\mu_3=0.15$ $\mu_4=0.04$	0.63347	0.67991	0.71657	0.74632	0.77099	0.79183	0.80971	0.82525
$\mu_1=\mu_2=\mu_3=0.15$ $\mu_4=0.12$	0.47914	0,54514	0.59724	0.63951	0.67457	0.70418	0.72959	0.75167
$\mu_1=\mu_2=\mu_3=0.06$ $\mu_4=0.03$	0.72916	0.77749	0.81159	0.83700	0.85669	0.87245	0.88537	0.89618

In figure 5 it is presented the variation diagram of *K* factor discussed about earlier.

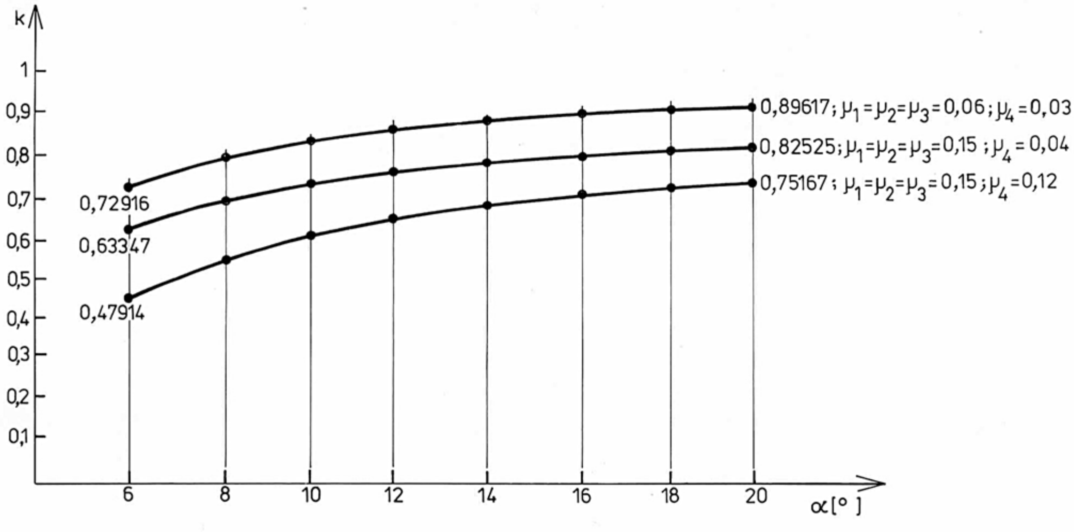

Figure 5. Variation diagram of *K* factor.

For $\mu_1 = tg\varphi_1 = 0.15$; $\mu_4 = tg\varphi_4 = 0.06$ and $\alpha_1 = 12°$; $\alpha_2 = 8°$, according to equation number (9) the result obtained for *K* factor is: *K* = 0.71430. Other variations of results are presented in table 3 and table 4 with graphical representations in figure 6 and figure 7.

Table 3. Values of *K*

Friction factors values	Values of *K*, the reduction factor of the axial force							
	Angle α (°)							
	6	8	10	12	14	16	18	20
$\mu_1=\mu_2=0.15$ $\mu_3=\mu_4=0.04$	0.53074	0.60280	0.65645	0.69803	0.73128	0.75854	0.78135	0.80075
$\mu_1=\mu_2=0.15$ $\mu_3=\mu_4=0.12$	0.44576	0.51940	0.57674	0.62277	0.66061	0.69234	0.71939	0.74278
$\mu_1=\mu_2=0.06$ $\mu_3=\mu_4=0.03$	0.70167	0.75903	0.79832	0.82698	0.84885	0.86613	0.88015	0.89179

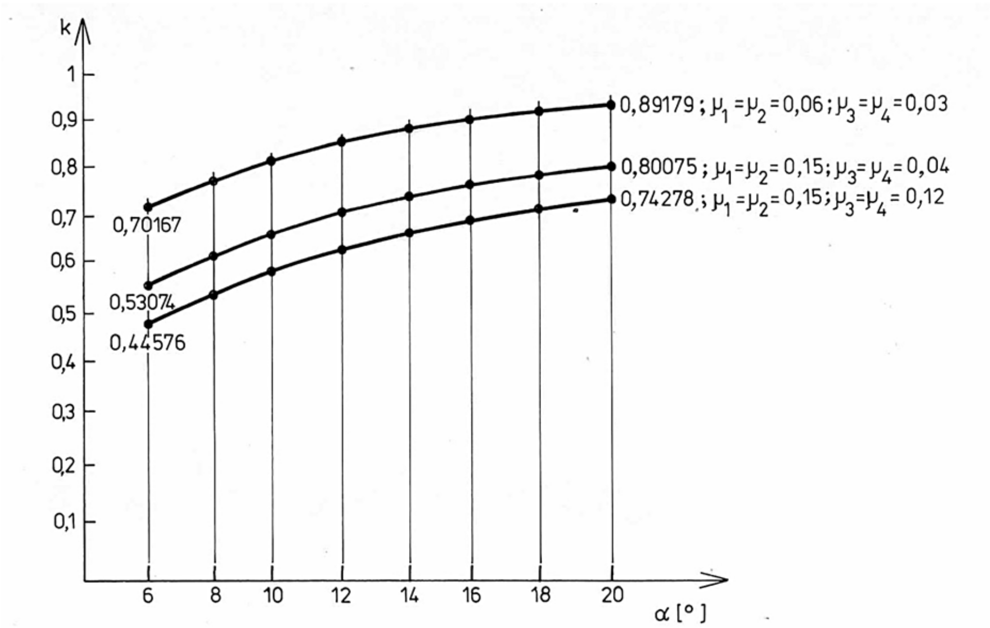

Figure 6. Variation diagram of K factor detailed in table 3.

Table 4. Values of K, the reduction factor of the axial force.

Friction factors values	Values of K, the reduction factor of the axial force							
	Angle α (°)							
	6	8	10	12	14	16	18	20
$\mu_1=\mu_4=0.15$ $\mu_3=\mu_2=0.04$	0.02944	0.17382	0.31148	0.41111	0.48672	0.54618	0.59428	0.63408
$\mu_1=\mu_4=0.15$ $\mu_3=\mu_2=0.12$	0.34201	0.43398	0.50450	0.56045	0.60599	0.64390	0.67600	0.70359
$\mu_1=\mu_4=0.06$ $\mu_3=\mu_2=0.03$	0.55729	0.64965	0.71073	0.75420	0.78680	0.81220	0.83259	0.84936

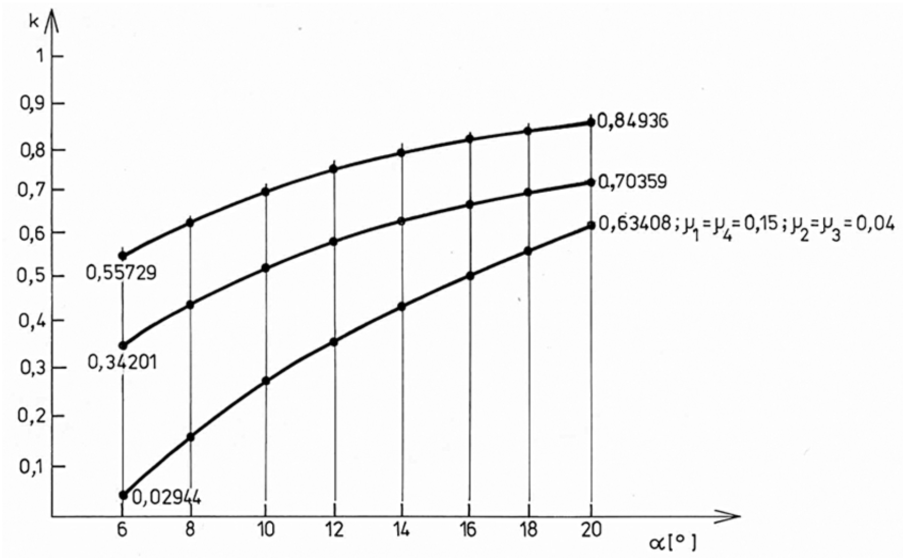

Figure 7. Variation diagram of K factor detailed in table 4.

The functional condition of transmitting the torque through friction:

$$M_{f1} \geq M_{tc} = \beta \cdot M_t \tag{16}$$

where β – slippage safety factor,
it is obtained:

$$M_{tc} = M_{f1} = \mu_4 \cdot Q_1 \cdot \frac{d}{2} \tag{16'}$$

$$\mu_4 \cdot \frac{1}{\mathrm{tg}(\alpha_1 + \varphi_2) + \mathrm{tg}(\alpha_2 + \varphi_3)} \cdot F_{a1} \cdot \frac{d}{2} = \beta \cdot M_t \tag{16''}$$

or

$$\mu_4 \cdot \frac{\cos(\alpha_1 + \varphi_2) \cdot \cos(\alpha_2 + \varphi_3)}{\sin(\alpha_1 + \alpha_2 + \varphi_2 + \varphi_3)} \cdot F_{a1} \cdot \frac{d}{2} = \beta \cdot M_t . \tag{16'''}$$

3. Experimental calculus of the capable torque

3.1. Constituents of the setting

This setting is composed of: assembly through compression with biconical intermediate elements (figure 8), the distortion stand of the assembly and the measurements tools.

By screwing up the bolt nut (9) with a dynamometric wrench the axial force of compression is developing. The assembly is being mounted on the distortion stand with its scheme presented below in figure 9.

The torque applied to the bolt nut is measured with the dynamometric wrench.

3.2. Calculus of the axial force and the torsion momentum of the assembly.

Using the dynamometric wrench, by tightening the bolt nut (12) the initial axial force F_0 is being developed. By neglecting the friction force developed between the bearing rollers which depends on the value of the torque applied on the dynamometric wrench M_{tp} we obtain the initial compression force [4, 5]:

$$F_0 = \frac{2 \cdot M_{tp}}{d_2 \cdot \mathrm{tg}(\alpha_m + \varphi')} = K_1 \cdot M_{tp} = F_{a1} \tag{17}$$

where d_2 – midle diameter of the fillet; φ' – reported friction angle:

$$\varphi' = \mathrm{arctg}\frac{\mu}{\cos\left(\frac{\beta}{2}\right)} \tag{18}$$

The value of K_1 factor from equation (17) being obvious, the normal reaction from equation (5) becomes:

$$Q_1 = \frac{1}{\mathrm{tg}(\alpha_1+\varphi_1)+\mathrm{tg}(\alpha_2+\varphi_3)} \cdot F_{a1} = K_2 \cdot F_{a1} \tag{5'}$$

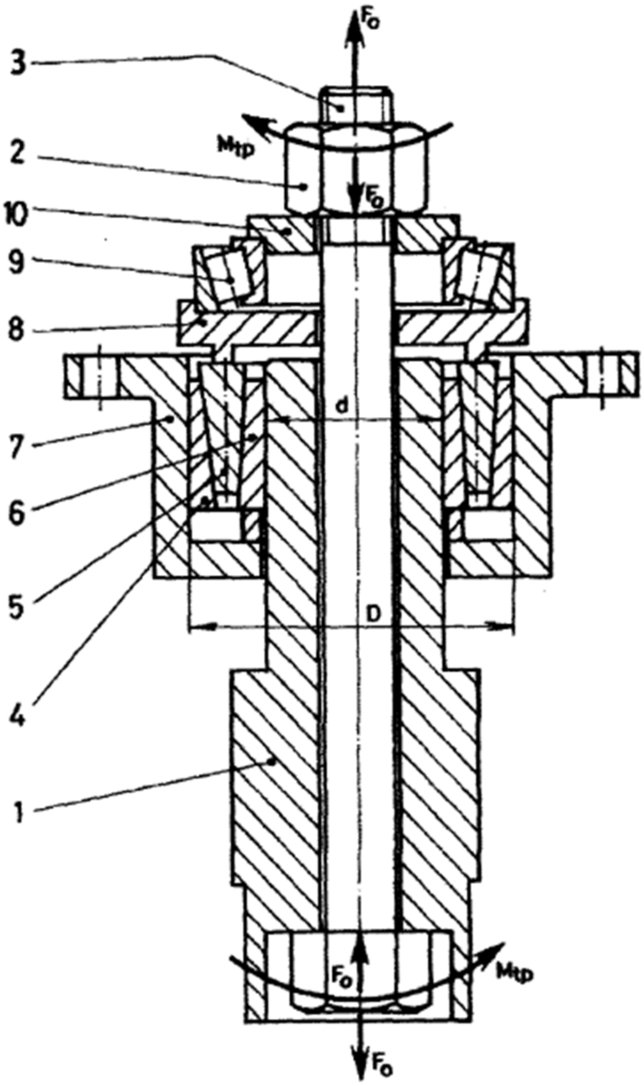

Figure 8. Principle scheme of assay pilot at torsion:
1 – cylinder; 2 – nut; 3 – screw; 4 – outer ring; 5 – intermediate ring;
6 – inner ring; 7 – protector; 8 – cap; 9 – roller; 10 – plaque.

The value of K_2 factor from being obvious in equation (5'), moving to (16') the capable torque of the assembly is:

$$M_{tc} = M_f = \mu_4 \cdot Q_1 \cdot \frac{d}{2} = K_3 \cdot Q_1 \tag{19}$$

where:

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$$K_3 = \mu_4 \cdot \frac{d}{2} \tag{19'}$$

3.3. Experimental calculus of the capable torsion momentum for assemblies with intermediate biconical elements.

Using the stand whose scheme is presented in figure 9, the torque that calls on the assembly is determined using a movement screw which operates a lever of length R.

Measuring the axial force developed in the movement screw we obtain:

$$M_{tm(exp)} = F_s \cdot R \cdot \eta_r \tag{20}$$

where: η_r – stand's mechanical performance.

Final results are presented in table 5.

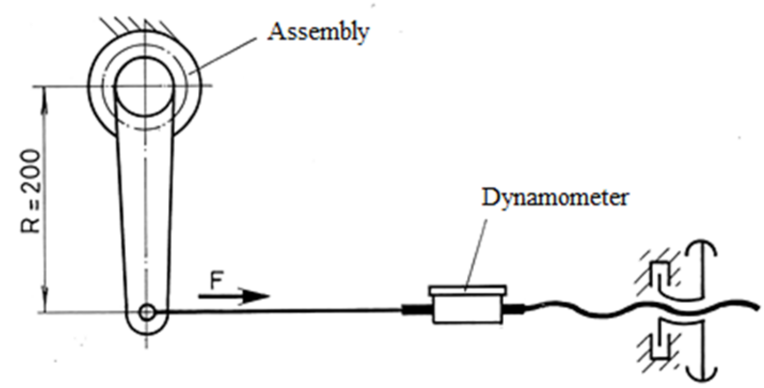

Figure 9. Principle scheme of assay pilot at torsion.

Table 5. Final results.

Current Number Parameter	1	2	3
M_{tp} (daN·cm)	400	600	800
μ_1		0.15	
μ_2		0.10	
μ_3		0.08	
μ_4		0.06	
φ_1 (°)		8.53076	
φ_2 (°)		5.71059	
φ_3 (°)		4.57392	
φ_4 (°)		3.43363	
α_1 (°)		12	
α_2 (°)		18	
α_m (°)		2.47	
φ' (°)		8.53	
K_1 (cm^{-1})		5.59	
F_0=F_{a1}(daN)	2236	3354	4472
K_2		1.36043	
Q_1 (daN)	3041.9	4562.8	6083.8
K_3 (cm)		0.12	
M_{tc} (daN·cm)	365.028	547.536	730.056
R (cm)		20	
F_s (daN)	20	29	39
M_{tmexp} (daN·cm)	380	551	741

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4. Conclusions

This research presents the calculus algorithm of the capable torque of the assemblies with intermediate biconical elements. The purpose of it is to analyse different variations of assemblies and it concludes that the most favourable situation it is presented in figure 4, where $\mu_1 = \mu_2 = \mu_3 = \mu_4 = 0.04$ and the K factor is $K = 0.90242$, all this data corresponding to an α angle $\alpha = 20°$.

In order to confirm the credibility of the theoretical results obtained during this research, it is presented in it the stand used for experimental calculus of the bearing capacity of assemblies through compression with intermediate biconical elements.

Any eventual inconsistencies between the capable torque of the assembly experimentally determined and the values of the friction factors can be explained using the reading errors of the torque displayed on the dynamometric wrench.

5. References

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