

Displacements and stresses in bending of circular perforated plate

C Atanasiu¹ and St Sorohan¹

¹ Department of Strength of Materials, POLITEHNICA University of Bucharest, Splaiul Independenței 313, 060042, Bucharest, Romania

E-mail: atanasiucostica@yahoo.com

Abstract. The flat plates, perforated by a large number of holes are widely used in the engineering, especially in the component of the process equipment. Strength calculations and experimental methods used in the actual literature for study perforated plates, do not present the problem in all its complexity for stress distribution and displacements. Research and doctoral theses in last decades, with methods characteristic of the respective periods were engaged either perforated plates considered infinite and requested the median plane or rarely, plate loaded normal to the median plane, with a small number of holes. In this work the stress distribution and displacement is presented for a circular plate perforated by 96 holes arranged in a grid of squares, simply supported on the outline and loaded through a central concentrated force or by uniformly distributed load. It conducted a numerical analysis by finite element method (FEM) with a proper meshing of the plate and an experimental study by holographic interferometry. Holographic interferometry method permits to measure, with high accuracy, extremely small displacements and comparing the results with those obtained by FEM becomes sustainable. Supplementary, an analysis of a non-perforated plate with the same dimensions and stiffness, similar loaded, was performed, determining the coefficient of stress concentration for a particular arrangement of holes.

1. Introduction

Kirch [1] studied the distribution of stresses in an infinite plate perforated by a hole and subjected to uni-axial stretching and determined that the maximum stress on the perimeter of the hole is three times higher than nominal stress. Reissner and Godier reach the same conclusion [1] if the same plate is loaded in simple bending. Studies were later extended [2], and it was discover as the maximum stress decreases when the diameter of the hole increases. The problem was extended to the plate with two holes or a serie of unidirectional holes using the complex variable function [3]. The study of stress distribution in infinite plates punched through a network of circular holes arranged in a regular network symmetrical in relation to axes, loaded mono-axially or bi-axially is assimilated [3], [4] with a double issue periodical to which it applies the functions of complex variable and functions of elliptic type Weierstrasse for determining a characteristic element stress distribution. In papers [5], [6] is considered an equivalent perforated plate in terms of displacements, with a non-perforated plate, made from the same material, with the same shape and boundary conditions but having elastic characteristics changed from those of perforated plate. From equivalent displacements of the two plates are deducted the elastic characteristics of the actual perforated plate that are used to determine the equivalent stresses in perforated plate. These study methods considered the plates to be infinite and loaded in the median plane. In the design of circular plates when the stress distribution is no longer



double periodicals it was used many years the method by which the perforated plate was considered as a non-perforated plate with the stiffness modified by a less than one factor. This approximate method, used in the dimensioning of such plates, depended on the stiffness reduction coefficient. In Romania research in perforated plates were expanded in the doctoral thesis. Thus, Praisler in [7] studied the case of reinforced hollow in the median plane. The problem of circular plates perforated by circular holes and loaded out-of-plane is approached [8] numerically using FEM and experimentally by moiré interferometry, strain gauge and holographic. In the paper [9] the study of stress in rectangular perforated plates are made in the elastic-plastic field. Valuable contributions are made in the works [10] and [11]. The emergence of very powerful computers and the latest technology, enabling solving very large systems of algebraic equations contributed to widening of the FEM applications.

In this work is studied, using the finite element analysis (FEA) and experimentally by holographic interferometry, the stress distribution in a circular plate of Plexiglas (Young's modulus $E = 3400$ MPa and Poisson ratio $\nu = 0.3$), with a diameter of 300 mm and thickness 10 mm, perforated by 96 circular holes of diameter 12 mm, arranged in a grid of squares of 24 mm, as in figure 1. The plate is considered simply supported on its exterior margin, loaded through a central concentrated load or a load evenly distributed. It studied in parallel, a non-perforated plate of the same material, the same supports and the same load condition to make a comparisons between the behaviour of the two types of plates.

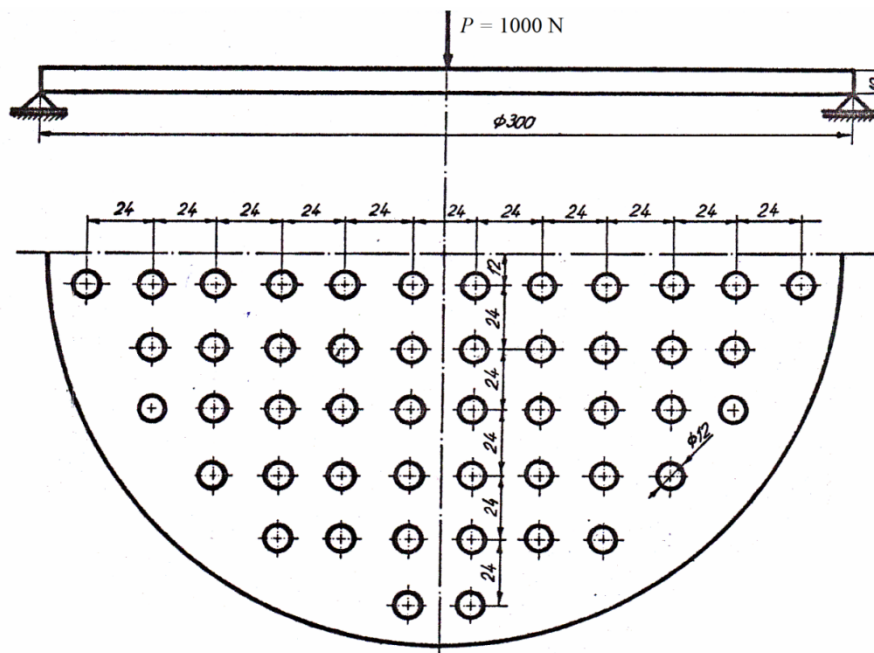


Figure 1. The geometry of the analyzed perforated plate.

2. Analysis of stress and displacement distribution using finite element method

Due to the symmetry only 1/8 of the plate was modelled and analyzed. For a direct comparison it was considered also a model 1/8 of the non-perforated plate. It was developed a 3D solid model using a finite element type with 20 nodes (SOLID95 in ANSYS). The model geometry of the perforated plate is given in figure 2 where the line next to the hole on the underside (line AB in subsequent text) will be used as reference representation of some quantities. These quantities will be presented in units of mm for displacements and MPa for stresses.

Initially it was achieved with only two rows of the finite elements on the plate thickness. In the figure 3 the mesh was refined in six rows of elements on the plate thickness resulting in 62 729 nodes and 12828 elements, all the analyzes were conducted for this second mesh. The vertical loads (concentrated load or load evenly distributed) sum the same value, $P = 1$ kN for both perforated and

non-perforated plates. The central concentrated load was considered uniformly distributed ($p = 12.7324$ MPa) on an area of small radius $r = 5$ mm, to avoid the occurrence of very high stresses (stress singularities) in the application zone of force. Timoshenko [1] shows that in calculating the displacements using this assumption there is a difference of up to 4% for displacements but large differences in what concerns stresses. In this area of concentrated load the stress distribution is tri-dimensional and in the rest of the plate the stress distribution is roughly a plane stress. For the perforated plate loaded with a uniformly distributed load on the effective surface the load is $p = 0,016714$ MPa and for equivalent non-perforated plate $p = 0.014147$ MPa. All the results (displacements and stresses) are presented in cylindrical coordinates. Some parts of resulting stresses are small in relation to the other and will not be shown in the paper. The mesh of non-perforated plate was kept as the mesh in perforated plate and supplementary mesh were inserted into the holes (see figure 4) for proper facilitate a direct comparison.

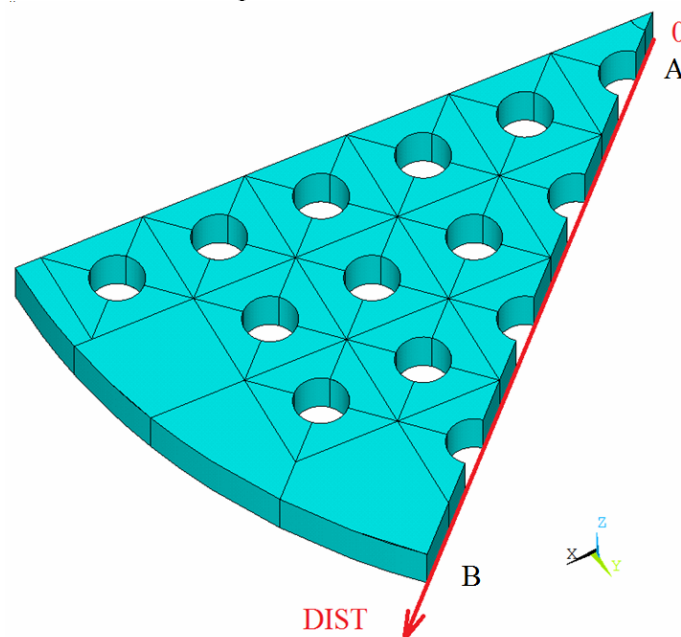


Figure 2. The geometry of the perforated plate considered for finite element analyses.

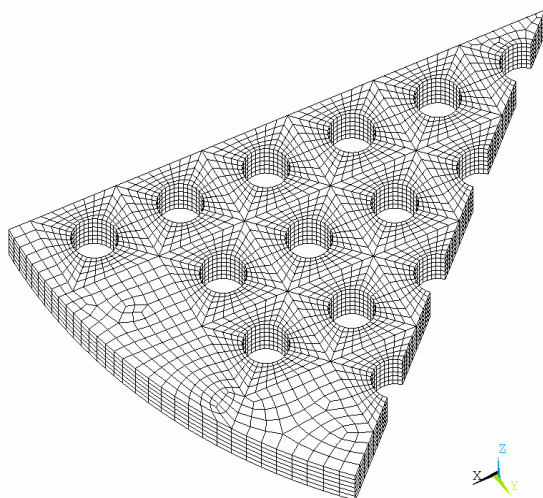


Figure 3. The finite element mesh of the perforated plate.

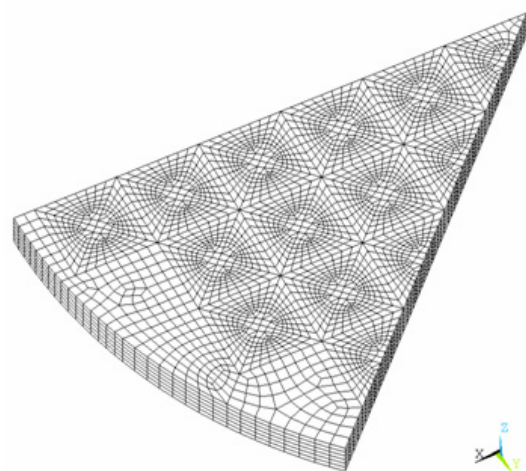


Figure 4. The finite element mesh of the reference plate.

The stress concentration factor k , is defined as the ratio between the maximum equivalent stress in the perforated plate and maximum equivalent stress in the reference plate [12], [13]. Displacement and stress distribution analysis was performed on the two set of plates for following situations:

- plate simply supported on the contour and loaded with a concentrated load, $P = 1$ kN;
- plate simply supported on the contour and uniformly loaded with an equivalent concentrated load of the same value, $P = 1$ kN ($p = 0,016714$ MPa for perforated plate and $p = 0,014147$ MPa for reference plate).

The deflection along line AB, are shown in figure 5 for perforated plate, loaded by the concentrated force and in figure 6 for reference plate in same load and support conditions. One can observe that the maximum deflection is $w = 5.73$ mm for perforated plate and $w = 3.47$ mm for non-perforated plate, i.e. a ratio of 1.65 between these maximum displacements.

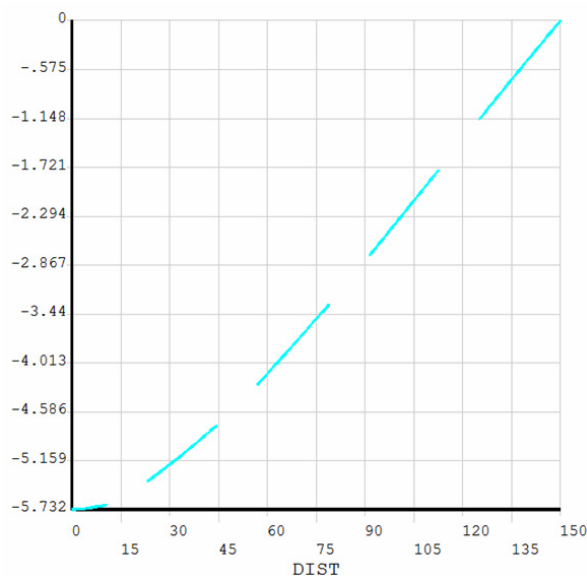


Figure 5. The deflection of the perforated plate loaded with concentrated load.

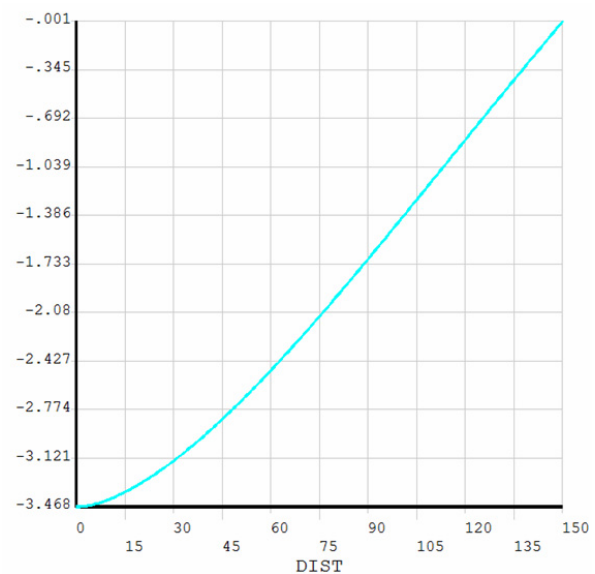


Figure 6. The deflection of the reference plate loaded with concentrated load.

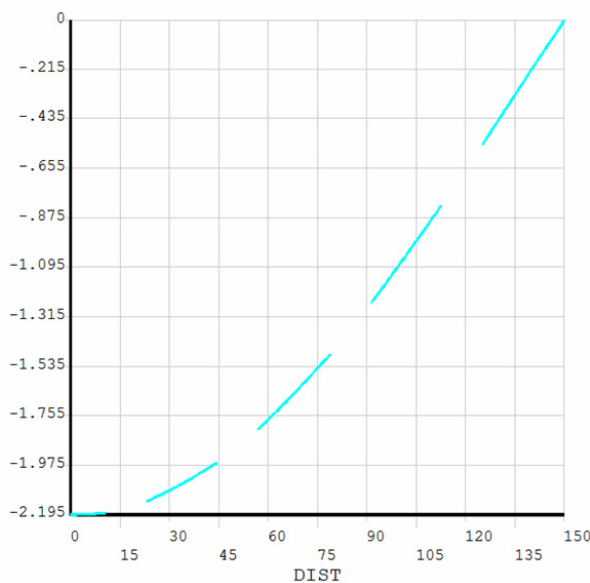


Figure 7. The deflection of the perforated plate loaded with uniformly distributed load.

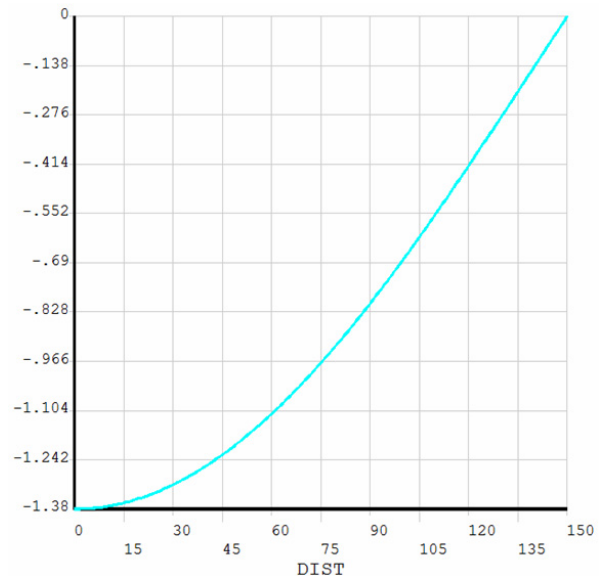


Figure 8. The deflection of the reference plate loaded with uniformly distributed load.

For perforated plate loaded with a uniformly distributed load, the variation of displacement along the same line AB, is given in figure 7. Figure 8 presents the same results for non-perforated or reference plate. It can be observed a decrease of maximum values compared to the previous load case to $w = 2.19$ mm and respectively $w = 1.38$ mm for the centre of plates, i.e. a ratio between these displacements is equal to 1.59.

From the obtained results one can observe that the maximum deflections for the concentrated load is 2.6 times higher than those produced by evenly distributed equivalent load, whether it is full or perforated plate. The maximum deflection of the perforated plate is 50% higher than those of non-perforated plate for both load cases. The displacements distribution obtained using FEA are very close to the calculated one using the plate theory [1] which shows that the meshes of the models were adequate to these analyses.

Stress distribution in perforated plate is more complex than that of displacement. In figure 9 and figure 10 are given variation of stresses along line AB for perforated and reference plates loaded with the concentrated central load and in figure 11 and figure 12 the same stress variation when the load was uniformly distributed with the same equivalent concentrated load.

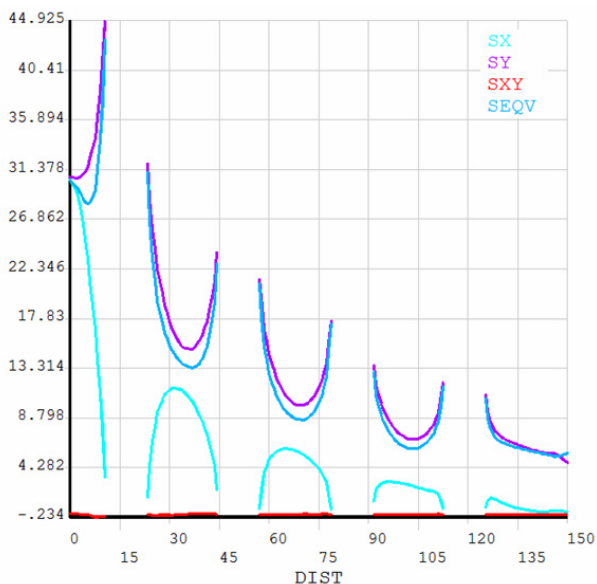


Figure 9. The stress distribution in the perforated plate loaded with concentrated load.

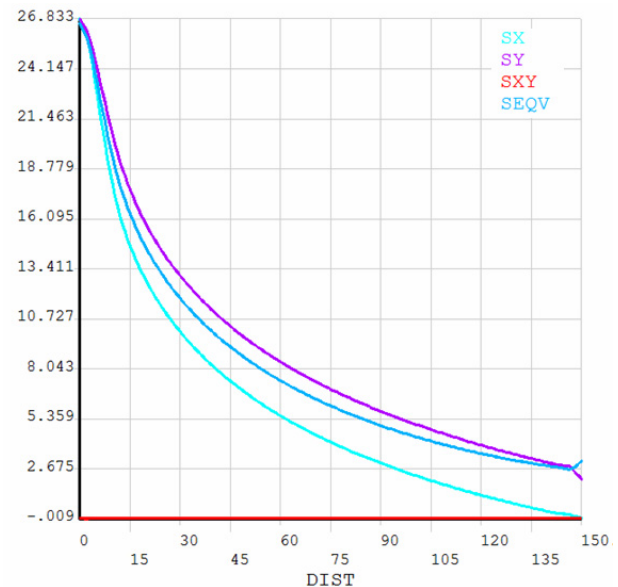


Figure 10. The stress distribution in the reference plate loaded with concentrated load.

One can see large stress jumps on the perimeter holes, especially in case of application of concentrated force. The most important being the hole nearest the centre plate where the concentrated load is applied. This distribution is similar also for the load uniformly distributed, but with much lower stresses (8.18 MPa compared to 44.9 MPa for the first holes on the perimeter). For non-perforated plates the maximum stress decrease from 26.8 MPa to 4 MPa. The stress concentration coefficients grow from $k = 1.67$ to $k = 2.04$ for the two load cases, the difference between the coefficients are due to a pronounced decrease of the maximum tensions in those two perforated plates. In the central area, around concentrated force the stress is tri-dimensional and the shear stresses is of the order of magnitude of the normal stresses and their effect can not be neglected in determining the equivalent stress using the von Mises's hypothesis. In the remaining part of the plate the stress state correspond to plane stress.

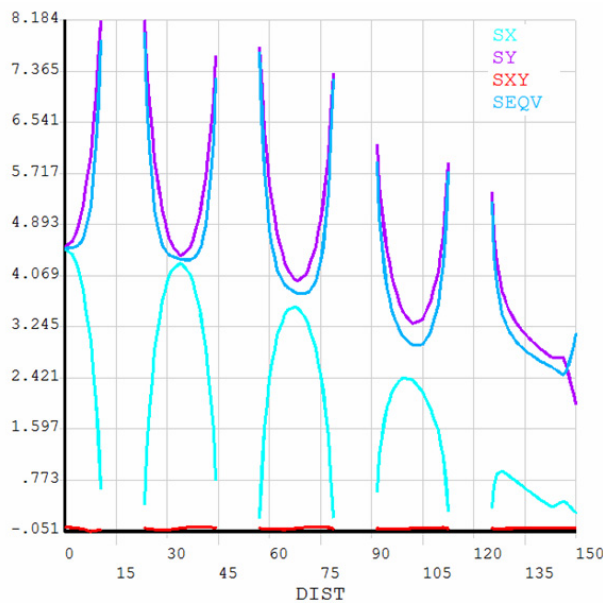


Figure 11. The stress distribution in the perforated plate loaded with uniformly distributed load.

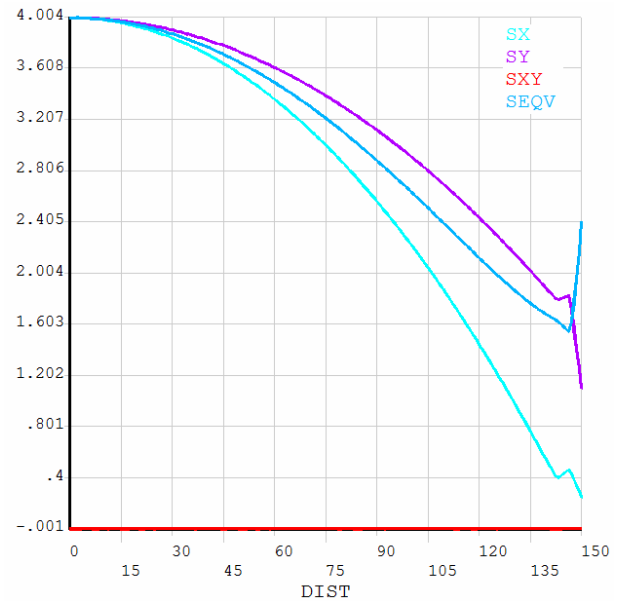


Figure 12. The stress distribution in the reference plate loaded with uniformly distributed load.

3. Experimental determinations

The experimental method chosen for study the displacement distribution in the perforated plate is holographic interferometry [14], [15] and [16]. This method consists in recording on the same photographic plate (in classical holography) or a camera (digital holography) of the hologram card of the unloaded and loaded specimen. By re-lightening the obtained hologram with the reference beam it is simultaneously rebuilding the undistorted picture and the deformed image. If the plaque surface is broadcast two images interfere and get an interference pattern depends on the geometry and optical system used by the three components of the displacement field for each point of the surface plate during his request. In digital holography [16] a single hologram recorded on a video camera can reconstruct the amplitude and phase simultaneously an object, chemical processes are avoided developing a holographic plate. The object is digitally reconstructed by an algorithm based on propagation simulation, available in electronic format image of the object, that which allows software image comparison of the same object in different situations. This method, applied to the study of displacements in perforated sheets shows advantages over conventional methods in that the contact plate is not required, the obtained interference fringes are visible all over the plate and displacements that can be measured are extremely small. For bending of thin plates, the displacements u and v are null fringes thus the image obtained is the locus of points of equal displacement w and have the equation [14]:

$$w = \frac{(2N+1)\lambda}{2(1+\cos\beta_0)}, \quad (1)$$

where N is the order of fringe; λ = wavelength of light used and β_0 the angle that makes the light source with the centre plate normal.

Experimental setup used to study the plaque is shown in figure 13. It includes an argon laser with a power of 2 W and wavelength of light emitted $\lambda = 4900 \text{ \AA}$, three flat mirror (O), a half mirror (SO) two goals microscope (OM) three diaphragms (D), holographic plate (H) and perforated plate to be studied. Parasitic vibrations that could disrupt the installation measurements are avoided by placing the setup on a holographic table. Study of displacements in the perforated plate was used by the classic double exposure. Through this process [14] two wave fronts object were recorded on the same

photographic card on perforated plate before and after deformation are rebuilt simultaneously. Interference of two wave fronts object through reconstruction by illumination hologram with the reference beam are obtained. They are curved fringes of equal points away plaque and that have the expression of equation (1).

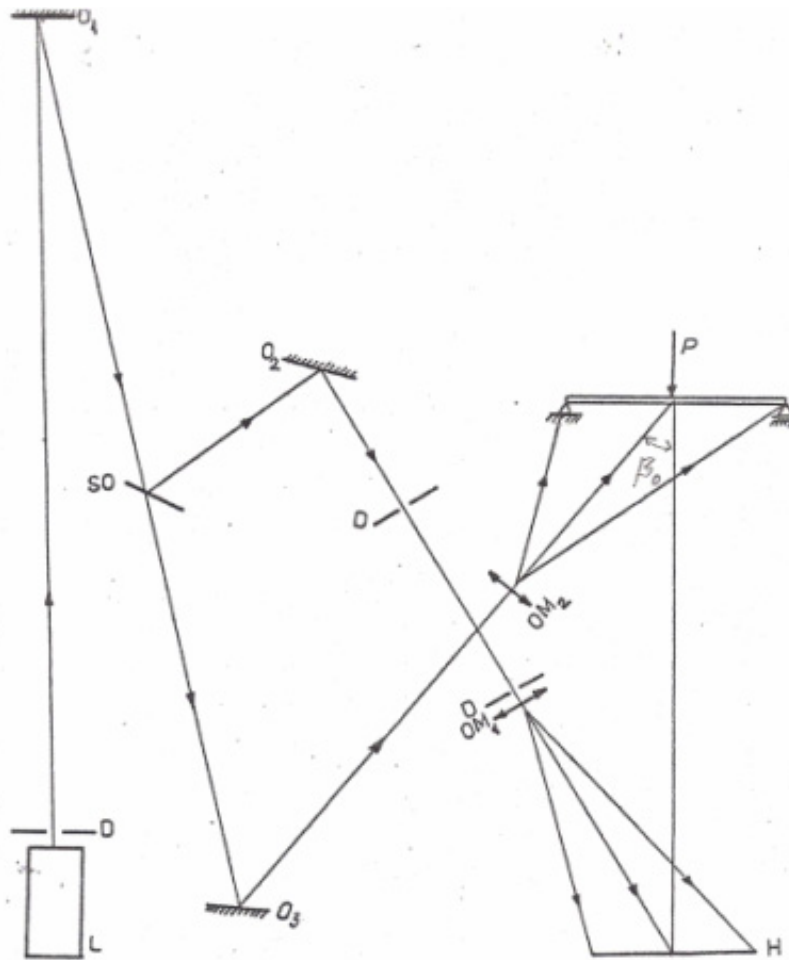


Figure 13. Holographic interferometry setup.

The experimental studied perforated plate was made of the same material and had the same geometry, dimensions, number of holes and holes arrangement as the one studied by FEM. Perforated plate surface was covered by an aluminium coating to be more diffuse. From experimental geometry setup resulting $\beta_0=45^\circ$ in the expression (1). For so prepared perforated plate and simply supported on the contour has been obtained an unsolicited hologram plate and a hologram plate requested by a central concentrated load $P = 2.5$ N applied by a special loading system.

In figure 14 it is presented the global image of the fringes representing the $w = \text{constant}$ and the deflection variation along the number of fringes N . Substituting in equation (1) is obtained, depending on the geometry used for holographic installation, maximum deflection of the perforated plate

$$w = (2N_1 + 1) 1435 \cdot 10^{-7} \text{ [mm]}, \quad (2)$$

where N_1 represents the number of fringes in the centre of perforated plate.

It can be seen that the maximum deflection in the centre of plate is $w = 0.0127715$ mm, for $N_1 = 44$ fringe. For deflection in different points the number N from figure 14 must be considered in (2).

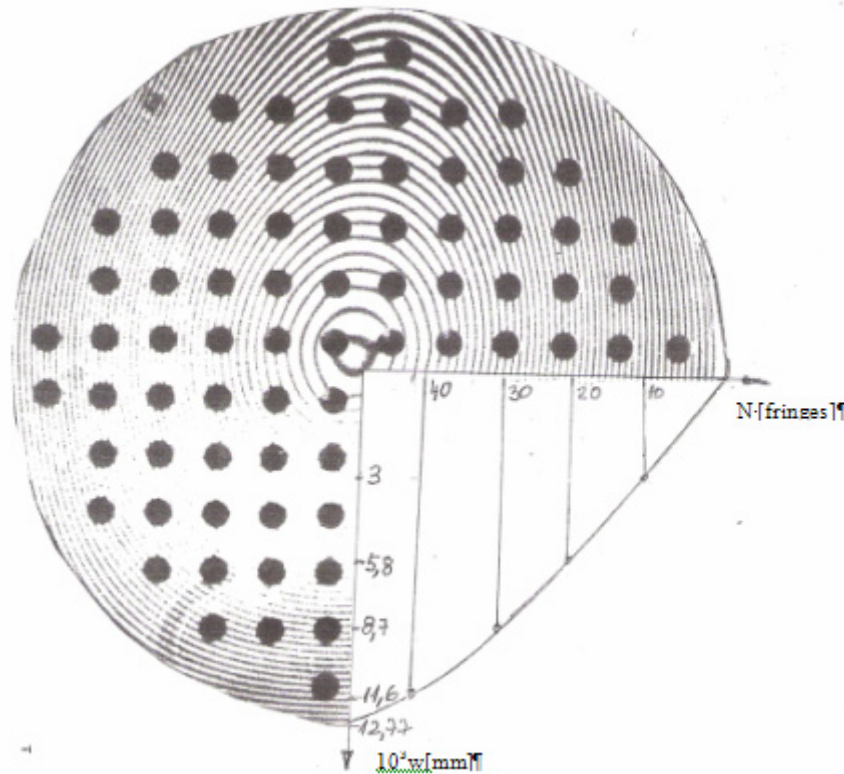


Figure 14. Fringes of $w=\text{const.}$ and deflection variation of the perforated plate.

4. Conclusions

For comparing the results obtained by FEA and by experimental holographic interferometry we must keep in mind that in the FEM it was considered a concentrated force $P = 1$ kN and in the experimental research only $P = 2.5$ N. The maximum deflection in the centre of the plate, calculated by FEA for a concentrated load $P = 2.5$ N, according to linear analysis, results $w = 0.01433$ mm, which is relatively close to the experimentally determined value $w = 0.01277$ mm. Considering the results in the paper, we can observe several general conclusions: the existence of holes in the plate do not disturb the global distribution of displacements; stress concentrations occur around holes in the perforated plate, higher when applying concentrated load compared to the case where the load was evenly distributed of the same equivalent concentrated load; the distribution of stress and displacement is expected to be influenced, as happens in case of perforated plates loaded in the median plane, by the diameter of the holes, the distance between them and the arrangement of holes (network in rectangle, square or triangular isosceles); at present, for the reason of constant development of automated computing, finite element method allows accurate geometry modelling of plaques and modes of load application, obtaining results very close to actual cases encountered in engineering practice; experimental verification using holographic interferometry, by the equipment made in recent years, can lead quickly to determine, with precision, the displacement and stress distribution in plates; using FEA and experimental verification by digital holographic interferometry can enable the establishment of methodologies for proper calculation of perforated plates with circular holes in bending or holes of a different form, with diverse support systems (simply supports, supports on contour in points, clamping etc.) and loaded static, dynamic, or on a field of temperature.

5. References

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Acknowledgments

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