

Wheel-rail dynamic forces induced by random vertical track irregularities

M A Spiroiu¹

¹Department of Rolling Stock, University POLITEHNICA of Bucharest, Bucharest, Romania

E-mail: marius_spiroiu@yahoo.com

Abstract. The present paper investigates the wheel-rail dynamic forces produced by railway vehicles in motion, which are an important issue especially for the high-speed rail transport from the point of view of traffic safety, ride quality and undesirable effects on vehicles, on track and on the land in the vicinity of railways. The research is carried out on a model which includes track system, vehicle unsprung mass, vehicle primary suspension and the bogie sprung mass. The wheel-rail dynamic overloads are evaluated assuming random vertical irregularities of the track. The estimation of wheel-rail dynamic forces is made for a range of vehicle speeds up to 300 km/h and the influence of track and vehicle various parameters is investigated.

1. Introduction

One of the main trends in rail passenger transport is to increase the traveling speed of trains. Running at high speed makes however that some phenomena of relatively low impact in the domain of conventional speeds to have a high intensity in high speeds domain. Among these, one of the most important is the wheel-rail dynamic forces produced by railway vehicles in motion. These forces are caused by wheel and rail defects, the latter ones being represented by vertical irregularities of the track. The arising forces are a practically dynamic overload which adds to static and quasi-static loads and have a negative effect primarily on vehicle and track but also on the land in the vicinity of the railway.

Given the importance of the issue, the wheel-rail dynamic forces topic has been studied by many authors. For example, in [1] is made a thorough state of the art in the field of the effect of vehicle characteristics on ground- and track borne-vibrations from railways. There are presented models for track, soil and vehicle dynamics as well as for analytical representations of track random irregularities under the form of unevenness rail spectra.

In [2] is presented a model for estimating wheel-rail dynamic forces, consisting of the vehicle unsprung mass (wheelset) and the track, considered as an elastic damped system. The dynamic overloads are determined assuming stochastic rail unevenness. A model related to the previous one is proposed in [3] but, in addition, the primary suspension of the vehicle is included. The influence of various parameters - of the vehicle and of the track - on wheel-rail dynamic loads is investigated. A relatively similar model is adopted in [4] including also a Hertzian contact spring, model based on which the dynamic wheel-rail contact forces are calculated for a sample population of measured rail welds.

A detailed model, including vehicle sprung and unsprung masses, the primary and secondary suspension, as well as a complex track model is presented in [5]. However, in this paper only



deterministic excitations are used to evaluate displacements of the wheel and the rail at the contact point and wheel-rail contact force.

2. Model for Vehicle-Track Dynamic Interaction

2.1. Mechanical Model of Vehicle-Track System

In terms of the vehicle-track ensemble as an oscillating system, the track can be considered as a system having its own elasticity and damping, which are depending on the characteristics of its superstructure and infrastructure elements. Taking into account that track stiffness is much higher than the vehicle suspension ones, the natural frequencies of the vehicle are much lower than those of the unsprung masses (in the order of Hertz and tens of Hertz, respectively). Under these conditions, the two systems can be considered decoupled in frequency, so the wheel-rail dynamic interaction can be studied, basically, considering a system consisting only of the track elements and the unsprung mass of the vehicle.

However, the present paper proposes a more complete mechanical model, which includes, besides track system and vehicle unsprung mass, the primary suspension and the bogie sprung mass. The track elastic and damping characteristics and the primary suspension connecting the wheelsets and the bogie frame have been modeled as a parallel combination of a linear spring and a damping element.

Therefore, the mechanical model (see figure 1) consist of a wheelset of mass m_w , the track reduced mass m_t , half of bogie sprung mass m_b , track stiffness k_t and damping constant c_t , wheelset primary suspension stiffness k_1 and damping constant c_1 ; z_t , z_w and z_b are the vertical displacements of track, wheelset and bogie, respectively, η is the track vertical unevenness (deflection) under the effect of static load and Δq denotes the wheel-rail dynamic force.

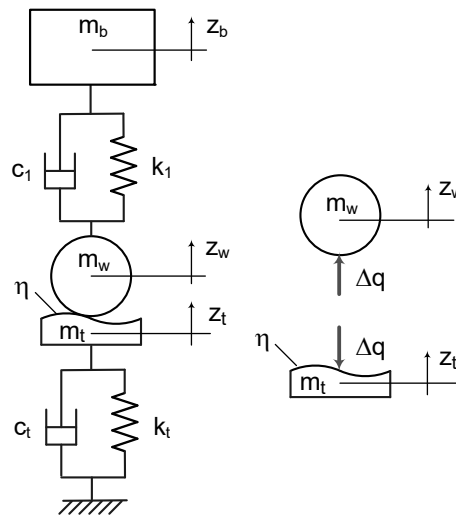


Figure 1. Mechanical model of vehicle-track system.

2.2. Transfer Function of Wheel-Rail Dynamic Forces

Equations of motion for the mechanical model in figure 1 are:

$$\begin{aligned} m_b \ddot{z}_b + c_1(\dot{z}_b - \dot{z}_w) + k_1(z_b - z_w) &= 0 \\ m_w \ddot{z}_w + c_1(\dot{z}_w - \dot{z}_b) + k_1(z_w - z_b) &= \Delta q \\ m_t \ddot{z}_t + c_t \dot{z}_t + k_t z_t &= -\Delta q \end{aligned} \quad (1)$$

and the displacements equation is

$$z_w = z_t + \eta. \quad (2)$$

Using the complex notations

$$\bar{\eta} = \eta_0 \exp(i\omega t); \quad \bar{z}_t = z_{t0} \exp(i\omega t + \varphi_1); \quad \bar{z}_w = z_{w0} \exp(i\omega t + \varphi_2); \quad \bar{z}_b = z_{b0} \exp(i\omega t + \varphi_3)$$

and substituting in equations (1) and (2), after some calculations is obtained the complex transfer function

$$\bar{H}_{\eta\Delta q}(\omega) = \frac{\Delta \bar{q}}{\bar{\eta}} = \omega^2 \frac{(k_t - m_t \omega^2 + i\omega c_t)[m_w m_b \omega^2 - (k_1 + i\omega c_1)(m_w + m_b)]}{(k_t - m_t \omega^2 + i\omega c_t)(k_1 - m_b \omega^2 + i\omega c_1) + \omega^2 [m_w m_b \omega^2 - (k_1 + i\omega c_1)(m_w + m_b)]} \quad (3)$$

which gives the relationship between the wheel-rail dynamic forces and the track vertical irregularities.

Defining the natural circular frequencies ω_1 , ω_t and the track and vehicle damping ratios ζ_t , respectively ζ

$$\omega_1^2 = \frac{k_1}{m_b}; \quad \omega_t^2 = \frac{k_t}{m_t}; \quad \zeta = \frac{c_1}{2m_b \omega_1}; \quad \zeta_t = \frac{c_t}{2m_t \omega_t}$$

and denoting

$$\lambda = \frac{\omega}{\omega_1}; \quad \lambda_t = \frac{\omega}{\omega_t}$$

the transfer function given by equation (3) can be written under the form:

$$\bar{H}_{\eta\Delta q}(\omega) = m_w \omega^2 \frac{\left[(1 - \lambda_t^2) + 2i\lambda_t \zeta_t \right] \left[\lambda^2 - \left(1 + \frac{m_b}{m_w} \right) (1 + 2i\lambda \zeta) \right]}{\left[(1 - \lambda^2)(1 - \lambda_t^2) - 4\lambda \lambda_t \zeta \zeta_t + \lambda_t^2 \left(\frac{m_w(\lambda^2 - 1) - m_b}{m_t} \right) \right] + 2i \left[\lambda_t \zeta_t (1 - \lambda^2) + \lambda \zeta \left(1 - \lambda_t^2 \frac{m_t + m_w + m_b}{m_t} \right) \right]} \quad (4)$$

2.3. Power Spectral Density of Wheel-Rail Dynamic Forces

In this paper wheel-rail dynamic forces are evaluated considering random vertical irregularities of the track, i.e. the rail unevenness are defined by random function of space. To describe the distribution of these irregularities is used the power spectral density (PSD) as a function of the spatial circular frequency Ω . A widely used relation for the distribution of track vertical unevenness is the power spectral density given in [1, 6]

$$S_v(\Omega) = A \frac{\Omega_c^2}{(\Omega_r^2 + \Omega^2)(\Omega_c^2 + \Omega^2)} \quad (5)$$

where $\Omega_c = 0.8246$ rad/m, $\Omega_r = 0.0206$ rad/m and A is a roughness constant, its value depending on the level of track irregularities, thus being a measure of the quality of the track.

The power spectral density in the frequency domain for a vehicle traveling at a speed v is given by:

$$G_\eta(\omega) = \frac{1}{v} S_v\left(\frac{\omega}{v}\right), \quad (6)$$

or, using equation (5)

$$G_{\eta}(\omega) = \frac{A}{v} \frac{\Omega_c^2}{\left(\Omega_r^2 + \frac{\omega^2}{v^2}\right) \left(\Omega_c^2 + \frac{\omega^2}{v^2}\right)} = \frac{A \Omega_c^2 v^3}{(\Omega_r^2 v^2 + \omega^2)(\Omega_c^2 v^2 + \omega^2)} \quad (7)$$

which represents the wheelset excitation temporal spectrum.

The PSD of wheel-rail dynamic forces is given by

$$G_{\Delta q}(\omega) = G_{\eta}(\omega) H_{\eta \Delta q}^2(\omega), \quad (8)$$

where $H_{\eta \Delta q}(\omega)$ is the modulus of the transfer function in (4):

$$H_{\eta \Delta q}^2(\omega) = \frac{m_w^2 \omega^4 \left\{ \left[\lambda^2 - \left(1 + \frac{m_b}{m_w}\right) \right]^2 + 4 \lambda^2 \zeta^2 \left(1 + \frac{m_b}{m_w}\right)^2 \right\} \left[(1 - \lambda_t^2)^2 + 4 \lambda_t^2 \zeta_t^2 \right]}{\left[(1 - \lambda^2)(1 - \lambda_t^2) - 4 \lambda \lambda_t \zeta \zeta_t + \lambda_t^2 \left(\frac{m_w(\lambda^2 - 1) - m_b}{m_t} \right) \right]^2 + 4 \left[\lambda_t \zeta_t (1 - \lambda^2) + \lambda \zeta \left(1 - \lambda_t^2 \frac{m_t + m_w + m_b}{m_t} \right) \right]^2} \quad (9)$$

The wheel-rail dynamic forces mean square value can be calculated by integrating the corresponding PSD given in equation (8) [2]:

$$\sigma_{\Delta q}^2 = \frac{1}{\pi} \int_0^{\infty} G_{\Delta q}(\omega) d\omega. \quad (10)$$

3. Influence of Vehicle and Track Parameters on Wheel-Rail Dynamic Forces

In the present section wheel-rail dynamic forces are numerically evaluated on the basis of the previously described model and the influence of track and vehicle various parameters on the dynamic overloads is investigated. It is to be noted that, taking into account the vehicle unsprung mass natural frequencies domain and the validity (in terms of frequency range) of the model of track irregularities, the upper limit of the integral in equation (10) is restricted to a value corresponding to a frequency of 100 Hz. The following parameter basic values have been adopted in the calculation: $m_w=1700$ kg, $m_t=300$ kg, $m_b=1600$ kg, $k_t=80$ MN/m, $k_1=1.4$ MN/m, $\zeta_t=0.2$, $\zeta=0.2$. In order to evaluate the influence of various parameters, variations of their values have to be done around the basic ones above mentioned. Also, calculations are made for a range of vehicle speeds, up to 300 km/h.

In figures 2, 3 and 4 can be seen the influence of track parameters on wheel-rail dynamic forces level.

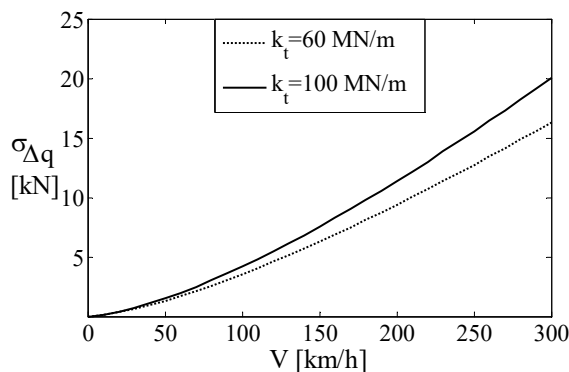


Figure 2. Influence of track stiffness on wheel-rail dynamic forces.

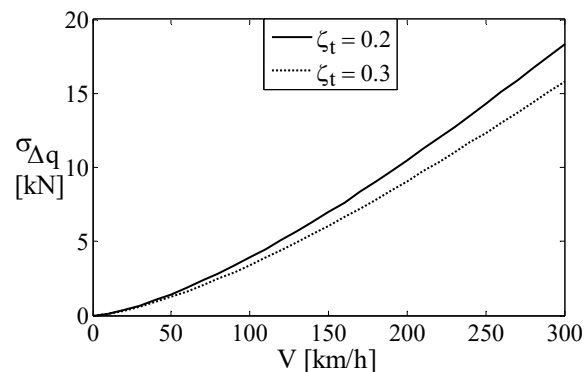


Figure 3. Influence of track damping ratio on wheel-rail dynamic forces.

First of all it is to be noted that the dynamic overloads level is highly influenced by vehicle speed in the sense that it increases with speed. This fact confirms that wheel-rail dynamic forces represent an important issue especially for high-speed trains. In figure 2 is highlighted the influence of track stiffness. It can be seen that a higher elasticity of the track has a positive effect, leading to a decrease of wheel-rail dynamic mutual forces. A similar positive effect – of diminishing the dynamic overloads - has an increased track damping ratio – see figure 3.

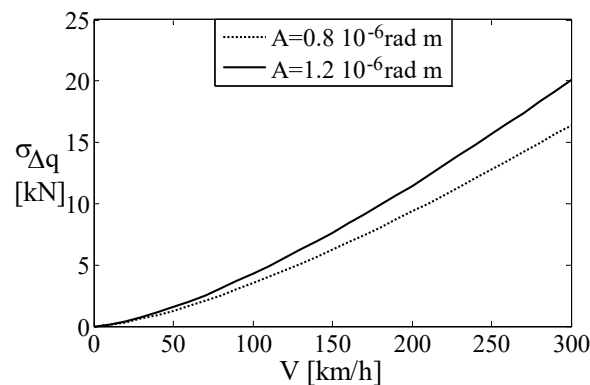


Figure 4. Influence of track quality on wheel-rail dynamic forces.

As expected, the track quality – expressed by constant A – has a major influence on vehicle – track dynamic interaction. A higher level of track irregularities leads, obviously, to an increased level of wheel-rail dynamic forces – see figure 4.

In figures 5, 6, 7 and 8 can be seen the influence of vehicle parameters on wheel-rail dynamic forces level. Concerning the characteristics of vehicle primary suspension, it can be seen that it has little influence on wheel-rail dynamic mutual forces. If the effect of damping ratio is somewhat significant, in the sense that a reduced (lower) damping ratio has some beneficial effect (see figure 5), the influence of vehicle primary suspension elasticity is insignificant (figure 6).

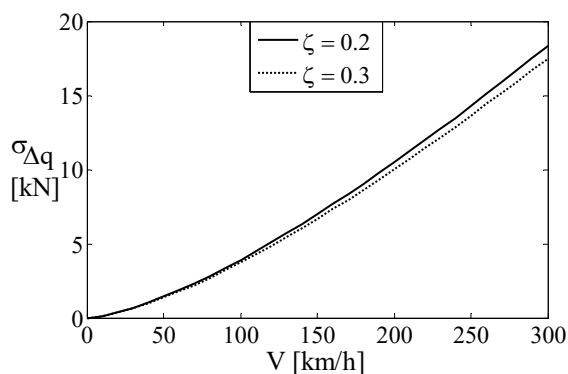


Figure 5. Influence of primary suspension damping ratio on wheel-rail dynamic forces.

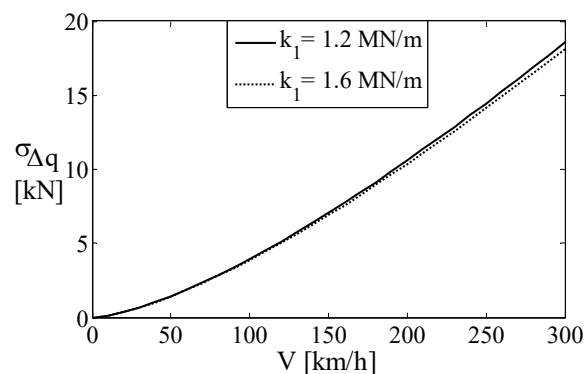


Figure 6. Influence of primary suspension stiffness on wheel-rail dynamic forces.

In figure 7 is shown the effect on wheel-rail dynamic forces of bogie sprung mass. The chart confirms the reduced (almost null) influence of vehicle sprung masses on vehicle – track dynamic interaction. On the other hand, the wheelset mass proves to be an essential factor of influence on the wheel-rail dynamic overloads – see figure 8. Increasing unsprung mass of the vehicle leads to considerably higher wheel-rail dynamic forces.

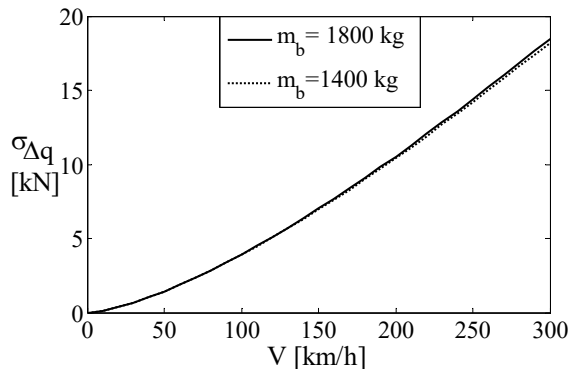


Figure 7. Influence of bogie sprung mass on wheel-rail dynamic forces.

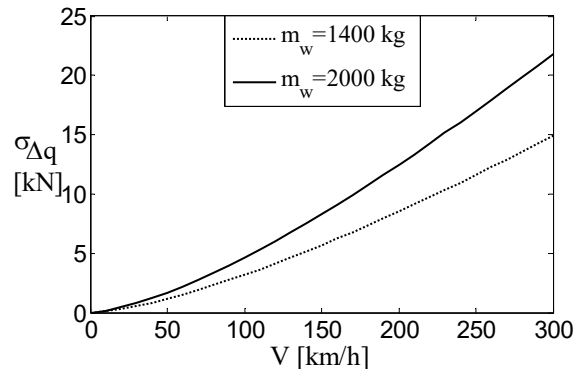


Figure 8. Influence of vehicle unsprung mass on wheel-rail dynamic forces.

4. Conclusion

The aim of present paper was to investigate the wheel-rail dynamic forces, an important issue especially for the high-speed rail transport, for which it may constitute a speed limitation by its effects on vehicles, on track and on the land in the vicinity of railway. The research was carried out using a model which included track system, vehicle unsprung mass, vehicle primary suspension and the bogie sprung mass and also assuming random vertical irregularities of the track. The influence of track and vehicle various parameters on the wheel-rail dynamic forces was investigated considering a range of vehicle speeds - up to 300 km/h.

The analysis showed that the vehicle speed and unsprung mass have the highest influence on the magnitude of wheel-rail dynamic overloads. The track characteristics are also important, especially its quality, quantified by the level of rail unevenness. Track elasticity and damping are, as well, significant influence factors on wheel-rail dynamic interaction. On the other hand, vehicle sprung mass and its suspension characteristics proved to have very little influence on wheel-rail dynamic forces. This was not an unexpected result, since the natural frequencies of the vehicle are much lower than those of the unsprung masses, thus the two systems can be considered decoupled in frequency.

It can be concluded that in order to ensure traffic safety, ride quality and to minimize the undesirable effects on vehicle and track, the main actions - with regard to wheel-rail dynamic overloads - are to reduce vehicle unsprung masses and to improve track quality in terms of elasticity and damping, but especially in terms of geometry i.e. of the level of its vertical irregularities.

References

- [1] Kouroussis G, Connolly D P and Verlinden O 2014 Railway-induced ground vibrations – a review of vehicle effects *Int. J. of Rail Transp.* **2** pp 69–110
- [2] Sebeșan I 2011 *Dynamics of Railway Vehicles* (in Romanian) (Bucharest: MatrixRom)
- [3] Spiroiu M 2012 *High-speed technique in rail transport* (in Romanian) (Bucharest: MatrixRom)
- [4] Steenbergen M 2008 Quantification of dynamic wheel-rail contact forces at short rail irregularities and application to measured rail welds *J. of Sound and Vibrations* **312** pp 606–29
- [5] Sun Y Q and Dhanasekar M 2002 A dynamic model for the vertical interaction of the rail track and wagon system *Int. J. of Solids and Structures* **39** pp 1337–59
- [6] Claus H and Schiehlen W 1998 Modeling and simulation of railway bogie structural vibrations *Veh. Sys. Dyn. Suppl.* **28** pp 538–52