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Development of material model for assessment of brittle cracking behavior of plexiglas

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Abstract. The objective of this study is to investigate the brittle cracking behavior of Plexiglas material when subjected to indentation loading. Indentation tests were conducted on Modified Vickers testing machine to acquire the experimental data in the form of load-displacement curve. Several mechanical properties such as hardness, yielding stress and fracture toughness have been ascertained from the analysis of the experimental data. The experimental data then used to create a mathematical model of Plexiglas for its brittle cracking behavior with indentation loading. Furthermore, a numerical simulation based study was carried out to simulate the brittle cracking in Plexiglas plate when subjected to indentation loading. The simulations were performed in the FE solver Abaqus. The brittle cracking model in Abaqus/Explicit is used which determines the required force and displacement to produce crack in Plexiglas. Finally a comparison of simulation results was made to the experimental data to validate the FEA procedures and accuracy of predictions. The numerical predictions of load-displacement curve found remarkably consistent with experimental results.

1. Introduction

Indentation tests have been commonly employed because they do not require elaborate sample preparation and are straightforward to execute. Initially they were used for determining the hardness of a material. Indentation techniques have also been used to find fracture toughness and other mechanical properties of material.

Finite element method (FEM) is the most powerful numerical technique for understanding mechanical behavior of materials and has become more common day by day because it's a time saving and cost effective alternative. It has been successfully used to solve various indentation problems with sufficient accuracy. Numerical simulations using finite element methods have also been employed to investigate the indentation fracture characteristics of brittle materials. Experiments and simulations have indicated that when brittle materials are subjected to indentation, various forms of cracking are detected [1-4] including median, radial, lateral cracking etc. The nature of cracking generally depends on the shape of the indenter and on the properties of the material.

Plexiglas is the common name for a type of polymer that is a cross between plastic and glass. It is a commercial name of PMMA (poly methyl methacrylate). It is also sometimes called acrylic glass and has many properties of glass. It has been widely used for residential, military, medical and commercial purposes as it possesses high toughness and transparency.

Various researchers and investigators have studied the behaviour of a Plexiglas under different mechanical and environmental conditions. R. Rikards et al. [5] developed a numerical and experimental method to determine elasto-plastic properties i.e. yield stress, tangential modulus and hardness of different polymers when subjected to Vickers indentation. Zhouhua Li et al. [6] investigated the thermo-mechanical behavior of polymethyl methacrylate (PMMA) when subjected to compressive dynamic loading. SaiSarva et al. [7]carried out experimental and analytical investigation to understand the high rate deformation and projectile impact behavior of polycarbonate and poly-mathylmethacrylate and develop a new constitutive model. Based on a chain of experiments and by using theoretical model (ZWT), Tao Suo et al. [8] investigate the temperature and strain rate effect on acrylic polymer during quasi-static and dynamic loading. N.K. Naik et al. [9] have carried out experimental studies and presented an analytical method to understand the effect of strain rate on tensile properties of acrylic. By using both experimental and simulation through commercial software Abaqus, Arjun Tekalur et al. [10] have studied the failure mechanism of PMMA plate when subjected to low velocity impact response. From the literature, it is ascertained that different analytical, experimental and numerical investigations are carried out in order to study the behavior of Plexiglas and other polymers under different mechanical and environmental conditions. However, in order to simulate the brittle cracking behaviour of Plexiglas, no appropriate model is available in literature. The study of brittle cracking in Plexiglas is of great interest due to its wide range of application.

In this paper, the failure mechanism of PMMA plate subjected to indentation loading is investigated. For this purpose indentation experiments were conducted on Vickers testing machine to acquire the data. This data is then used to derive various mechanical properties as well as to develop a mathematical model for Plexiglas. Furthermore a numerical simulation based study is made to predict the brittle cracking in Plexiglas plate when subjected to indentation loading. Finally a comparison of simulation result with the experimental result is given to validate the FEA procedures and accuracy of predictions.

2. Experimental

2.1 Procedure

Experiments of hybrid nature are performed in which a sharp tipindenter is pressed into a flat surface of Plexiglas under a specific load for a definite period of time. The indenter employed in the test is conical with self-similar geometry. The Plexiglas specimen has 25.4 mm diameter and 3 mm thickness. The time was few seconds to reach the maximum load regardless of its volume. These tests are based on technology that allows for either load-controlled or depth-controlled experiments. The applied load, penetration depth and cycle time are continuously measured during these tests. The mechanical properties are determined by analysing the geometrical dimensions of the indent and from the analysis of a load-depth curve. All the experiments were performed at room temperature. Our resulting data is in the form of load displacement curves. In our final output, we have both the plots and the pictures of cracking. The same tests were repeated thrice, and similar results were obtained repeatedly.

2.2 Indentation cracking in plexiglas plate

Figure 1 shows an image after a conical indenter was forced into the surface of Plexiglas. Two cracks are initiated from the edge of the indent. In the literature, these kinds of cracks are referred to as radial cracks [11]. Therefore it can be seen in these experiments that when a conical indenter is indented into Plexiglas materials, radial cracks develop normal to the surface after a critical load has been reached.

2.3 Analysis of indentation load displacement curve

Primary information about an investigated Plexiglas material in this test is described by the load-displacement curve. The load-displacement data obtained from the indentation process contain the information about hardness, fracture toughness, maximum crack load etc. Figure 2 displays the average load–displacement curve acquired from these indentation experiments on Plexiglas material. As shown, when the indentation force reaches to a value of 400 N, the crack is initiated in Plexiglas material. This P-h curve can be used to derive different mechanical properties of materials such as the hardness and fracture toughness of material.

Figure 1. Indentation in Plexiglas with the conical indenter showing well defined radial cracks

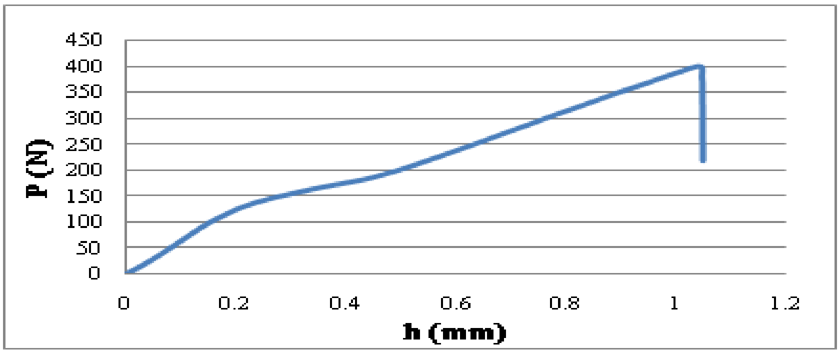

Figure 2. Average load–deflection curve

3. Data Processing and material modelling

3.1 Measurement of hardness and yield stress

Hardness is an important property of a material that enables it to resist deformation, usually by penetration of a hard indenter. The indentation hardness is calculated from the peak indentation load, P_{max}, divided by the projected contact area A_p at the contact depth h_c:

$$H = \frac{P_{max}}{A_P} \qquad (1)$$

For conical indenter as shown in Figure 3, the projected contact area under the maximum load is determined by

$$A_p = \pi {h_c}^2 tan^2\theta \qquad (2)$$

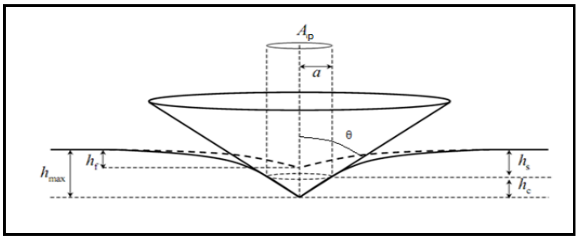

Figure 3. Geometry to characterize Indentation

The specimen's yield stress, σ_Y, is directly proportional to the hardness and can be expressed as

$$H \approx C \times \sigma_Y \tag{3}$$

Where C ≈ 1.5 for materials with a low ratio of $\frac{E}{\sigma_Y}$ such as polymers, glass etc. By substituting values in equations 1 and 3, the indentation hardness and yield stress can be calculated as shown in Table 1.

Table 1. Properties of Plexiglas material used in measurement of indentation hardness and yield stress.

Material	**h_c(mm)**	**Ap(mm^2)**	**P(N)**	**H(MPa)**	**σ_Y(MPa)**
Plexiglas	0.664	4.155	400	96	64

3.2 Measurement of fracture toughness

Accurately measuring the fracture toughness of brittle materials can often be challenging. Different indentation techniques could be used to determine the fracture toughness of brittle materials. Assessing fracture toughness by making direct measurements of cracks created using a sharp indenter can appear to be an attractive alternative to traditional fracture toughness testing techniques. As is evident from Figure 1, when a sharp tip such as conical indenter is indented into brittle materials, radial cracking usually occurs after a critical load is reached, which can be used to calculate fracture toughness based on the maximum indentation load and the crack length. The equation used to calculate the material fracture toughness Kc by indentation is

$$K_C = \alpha\left[\frac{E}{H}\right]^{1/2} + \left[\frac{P}{C_0^{3/2}}\right] \tag{4}$$

Where P is the indentation load, H is the hardness, and c is the average radial crack length as shown in Figure 4. α is a calibration constant and its is considered 0.04 for sharp tip indentation geometries.

By substituting the values of elastic modulus, E, hardness, H, crack length and maximum indentation load in equation 4, the fracture toughness value is obtained as shown in Table 2.

Table 2. Properties of Plexiglas material used in indentation cracking measurement of fracture toughness.

Material	**E(MPa)**	**H(MPa)**	**P(N)**	**C_o (mm)**	**K_c (MPa.mm$^{1/2}$)**
Plexiglas	3100	96	400	2.4	0.773

3.3 Strain graph for Conical Indentation

The stress and strain values are determined from the force displacement data by following calculation. The average stress can be calculated from the indentation force, P, divided by the true contact area A, as follow

Stress = Force/Contact Area

$$\sigma = \frac{P}{A} \tag{5}$$

For simple conical indenter as shown in Figure 5, the true contact area A at indentation depth h can be calculated as

$$A = \pi h^2 tan^2\alpha \tag{6}$$

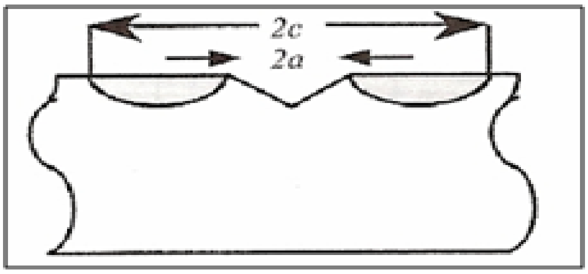

Figure 4. Radial crack (side view)

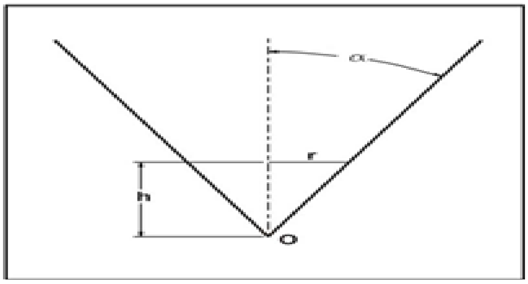

Figure 5. Cone indenter

The average strain in the direction of the indentation load can be calculated by dividing the change in depth, h, by the initial depth h_o:

Strain = change in depth/ initial depth

$$\varepsilon = \frac{h}{h_0} \tag{7}$$

After calculating the stress strain values from the load displacement data, the stress strain graph is generated as shown in Figure 6.

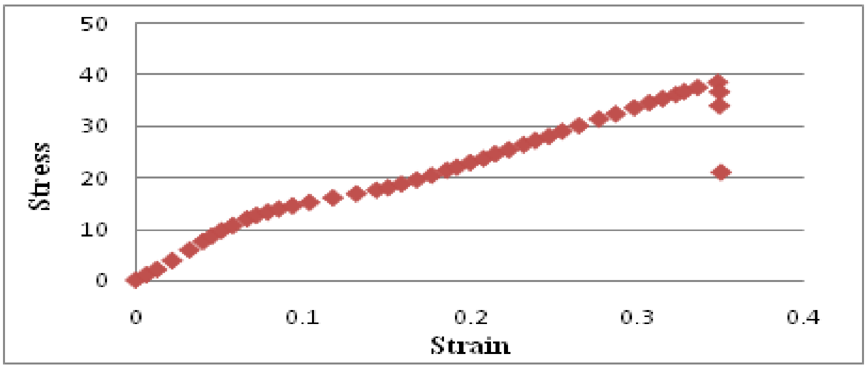

Figure 6. Stress strain curve

3.4 Mathematical model created using curve fitting technique

After getting the stress strain graph, a mathematical model is developed for this graph by curve fitting of the data. This model can be used for data visualization to infer values of a function where no data are available and to summarize the relationships among the variables. The model is created by using Fourier fit to the data points in curve fitting toolbox of Matlab as shown in Figure 7. The mathematical model is expressed as

$$\sigma = a_0 + a_1 \times cos(\varepsilon \times \omega) + b_1 \times sin(\varepsilon \times \omega) + a_2 \times cos(2 \times \varepsilon \times \omega) + b_2 \times sin(2 \times \varepsilon \times \omega) + a_3 \times cos(3 \times \varepsilon \times \omega) + b_3 \times sin(3 \times \varepsilon \times \omega) + a_4 \times cos(4 \times \varepsilon \times \omega) + b_4 \times sin(4 \times \varepsilon \times \omega) \tag{8}$$

The values of the model coefficients are shown in Table 3.

Table 3. Coefficients of mathematical model values

Coefficients	a_0	a_1	b_1	a_2	b_2	a_3	b_3	a_4	b_4	w
Values	1.491e6	-1.78e6	-2.294e6	2.101e4	2.055e6	3.703e5	-7.152e5	-1.025e5	8.268e4	1.664

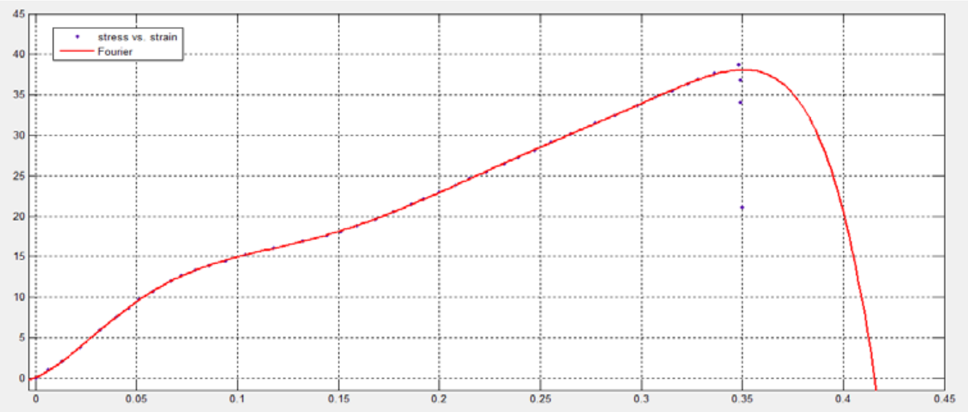

Figure 7. Stress strain curve with Fourier fit

4. Numerical simulation

The use of numerical simulations for understanding mechanical behavior of materials is becoming more popular due to its advantage of time saving and cost effectiveness. In this study acommercial FE package Abaqus 6.13 is used to simulate the brittle cracking in Plexiglas. Modeling of a conical indentation experiment can be accomplished by mean of an axi-symmetric model as the system is symmetric about an axis. The numerical model consists of two parts created separately: a plate and an indenter. Since the indenter is much stiffer than the plate, the indenter is considered to be perfectly rigid and is modeled as analytical rigid surface. The reference point is assigned to the indenter, which manipulates the rigid body translation of the indenter. The plate has been created as 2D axi-symmetric deformable with 25.4 mm diameter and 3 mm thickness. The plate was modeled using four noded bilinear axi-symmetric quadrilateral elements. The element type “CAX4R” is used. The contact interactions were defined as surface to surface contact, which can describe the contact between a deformable surface and a rigid body. The indenter surface is defined as the “master” surface since the indenter is rigid body. The top of the specimen is the “slave” surface. In this work it is assumed that the friction effect is negligible and μ=0 is defined in all the models assuming there is no slip between the surfaces in contact.

The indenter is constrained in all directions except in the vertical direction normal to the plate surface (y-axis). The bottom of the plate is constrained axially and YASYMM boundary conditions are applied to the left side of the plate. The boundary conditions of plate and indenter are shown in Figure 8. Slightly finer mesh is created around the indenter and the element size is increased making the mesh denser away from the indenter towards the boundary of the model as shown in Figure 9.The indentation test was simulated by applying a downward displacement of 1.045 mm to the indenter which causes the indenter to push into the material to be indented.

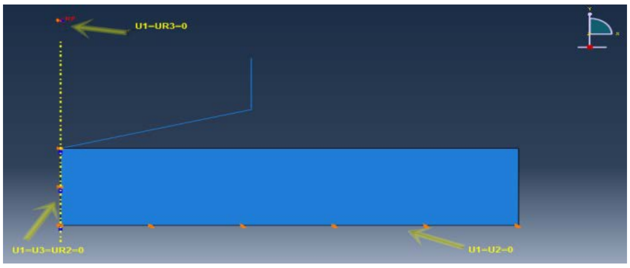

Figure 8. Boundary conditions applied in the model.

Figure 9. Meshed model of plate.

1.1. Brittle cracking model for plexiglas

The brittle cracking material model in Abaqus/Explicit is used to predict brittle cracking behavior of Plexiglas material when subjected to indentation loading. The brittle cracking parameters are extracted from the stress strain data obtained earlier and final values are fixed after an admissible result is obtained. The values of Plexiglas properties and the brittle cracking parameters are given in Table 4.

Table 4. Plexiglas mechanical and brittle cracking properties

Plexiglas Properties	Values	
Density	1.2x10-6 (kg/mm^3)	
Elasticity Modulus	3100 (MPa)	
Poisson ratio	0.37	
	Direct stress after cracking	**Direct cracking strain**
Brittle Cracking	36.8	0
	34.1	0.000333
	21.1	0.000667
Direct cracking failure strain	1x10-6	
	Shear retention factor	**Crack opening strain**
Brittle shear	1	0
	0.5	0.001
	0.25	0.002
	0.125	0.003

5. Results and discussion

5.1 Load displacement curve obtained from simulation

For brittle cracking material model, finite element simulation results gives a set of “P” and displacement “h” data for each loading step and then these data points are plotted on a graph. The load-depth curve obtained from simulation is shown in Figure 10.

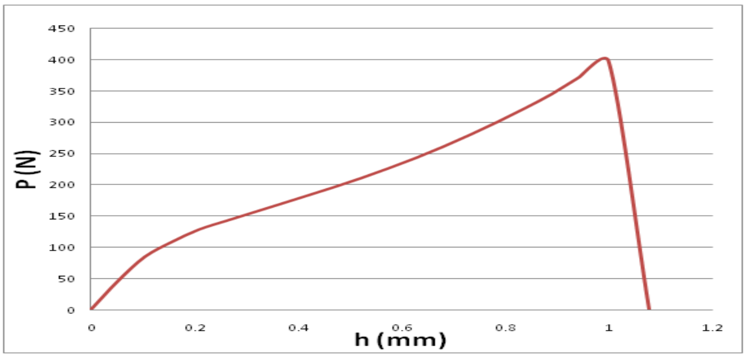

Figure 10. Load depth curve using brittle cracking model.

5.2 Comparison between FEM results and experimental results

Load displacement curve obtain by using brittle cracking model in current numerical simulation is compared with previously extracted experimental load displacement curve is shown in Figure 11. The Figure indicates that the model predictions of P-h relationship agree well with the experimental results.

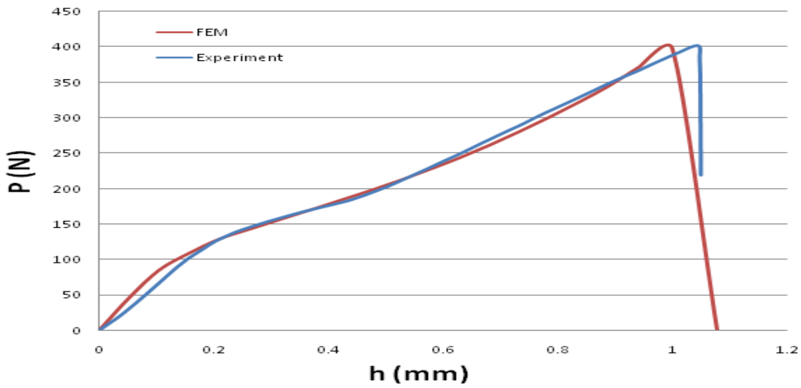

Figure 11. Load depth curve comparison between simulation and experimental results

As seen in Figure 11, the force displacement curve is linear when the indention force is small, and after that the curve shows non-linear behaviour as the magnitude of the indentation force increases. When the force reaches to critical valve of 400 N, the curve has a sudden drop which indicates that cracking started at this point, relieving the load. Therefore both experimental and numerical results provide evidence that radial cracks are initiated during the loading stage, when the indentation force reaches to a value of 400 N. Hence by using brittle cracking material model the crack initiation in Plexiglas plate during indentation process can be predicted.

6. Conclusion

The brittle cracking behavior of poly methyl methacrylate (PMMA) material, subjected to sharp indentation loading is investigated in this study. ModifiedVickers testing apparatus is used to acquire the experimental data in the form of load displacement curve. The mechanical properties such as hardness, yield stress and fracture toughness are determined by analyzing the geometrical dimensions of the indent and from the analysis of a load-displacement curve. In addition stress strain graph is generated from the force displacement data. After getting the stress strain graph, a mathematical model is created for this graph by doing curve fitting.

Furthermore, a numerical simulation based study is made and a brittle cracking model in Abaqus/Explicit is proposed in order to predict the crack initiation in Plexiglas material when subjected to indentation loading. The crack prediction in Plexiglas plate by simulation of the experimental data is achieved successfully and a good correlation is observed between experiments and simulations results.Therefore by using brittle cracking model the crack initiation in Plexiglas plate when subjected to sharp indentation loading can be predicted without doing series of experiments.

7. References

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