

Numerical investigation of a two – degrees – of – freedom ship model for pitch – roll motion

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Abstract. This paper numerically investigates a two degrees of freedom harmonically excited pendulum system, known in the literature to be a good model for the coupling between the roll and the pitch ship motions. We concentrate mainly on the dangerous situation for the ship where the pitch frequency is almost twice the roll frequency and the excitation period is near the pitch period. In this case, although only the pitch excitation is taken into consideration, part of the energy in the pitch mode is transferred to the roll mode through the non – linear coupling leading to excessive resonant rolling amplitudes and, as a consequence, to ship capsizing. The stability of the system is studied using both frequency response curves and bifurcation diagrams. It was proven that for small external forcing often co-exist two periodic attractors and that the jump phenomena accomplish the transition from one attractor to the other. Periodic oscillations take place with the external frequency for pitch mode and half of the external frequency for roll mode. Mostly, the transient towards the steady state is slowly, stage in which the model behaves chaotically. For moderate and high forcing the system evolves quasi-periodically or chaotically, especially in the neighbourhood of the jump points and for resonant frequencies. The effects of damping on dynamics are also illustrated.

1. Introduction

A ship has six degrees of free movements in waves: Rolling, pitching, yawing, swaying, surging, and heaving. The full equations of motion may be derived either by using Newton's second principle or by means of the Lagrange's formalism [1 - 3]. Accurate determination of ship - wave - wind dynamical interactions leads to a strongly non – linear coupled system with six equations whose analysis is very cumbersome and has a computational cost remarkably high.

This is the reason why the researchers in the field have proposed low dimensional models which may predict the main features of the ship behaviour for certain operational conditions. These models have focussed mainly on rolling, pitching and heaving. *Roll*, which is an oscillatory angular motion of the ship about its longitudinal axis, has received much interest because, with a typical hull – form, it is the least damped of the six motions. Consequently, the roll angle amplitudes can be large enough to affect the ship's stability and even to capsize it, especially when the wave frequency matches the ship's natural roll frequency [4 - 5]. *Pitch*, an oscillatory angular motion of the ship about its transversal axis, is also important because in heavy sea conditions it can determine the bow to come out of the water, then slam it hard in the water, with the immediate consequence of introducing enormous efforts in the ship's structure. *Heave*, an oscillatory linear motion of the ship in the direction



of its vertical axis, is taken into account mainly because it tends to be more closely coupled with the pitch mode [6 - 8].

The spring pendulum system is one of the most used two degrees of freedom dynamical system simulating the coupled pitch – roll ship motion. It has a very complex behaviour including jump phenomena, Hopf bifurcations, multiple regular attractors with fractal boundaries, or chaotic responses and boundary crises [9 - 11].

In this contribution, we perform an extended numerical investigation of the above – mentioned model for pitch and roll ship motions. We focus mainly on the dangerous situation for the ship where the pitch frequency is almost twice the roll frequency and the excitation period is near the pitch period. In this case, part of the energy in the pitch mode is transferred to the roll mode through the non – linear coupling leading to excessive resonant rolling amplitudes.

2. Short description of the ship model

The mechanical model used in the paper consists in a mass attached to the end of a massless linear spring which may rotate in a plane like a pendulum. The system is much more complex than a simple pendulum, as the spring adds a second degree of freedom. The spring and the pendulum motions simulate the pitch and roll modes, respectively.

For brevity, we refer to some existing papers on this ship model for the equations of motion. They are written in non-dimensional form as follows

$$\ddot{x} + c_1 \dot{x} + \omega_1^2 x - (1+x)\dot{\phi}^2 + \omega_2^2(1-\cos\phi) = f_1 \cos\Omega t \quad (1a)$$

$$(1+x)^2 \ddot{\phi} + c_2 \dot{\phi} + 2(1+x)x\dot{\phi} + \omega_2^2(1+x)\sin\phi = f_2 \cos\Omega t \quad (1b)$$

where x and ϕ represent the spring extensional and pendulum angular displacements, $c_i, i=1,2$, are the linear damping coefficients, $\omega_i, i=1,2$, the natural frequencies of the spring and pendulum modes, $f_i, i=1,2$, the forcing amplitudes, and Ω the excitation (encounter) frequency. Finally, the dots denote the differentiation with respect to the non-dimensional time. These equations have been studied with the method of multiple scales in the neighbourhood of the static equilibrium by assuming that the damping coefficients and the forcing amplitudes are on the order of ε^2 , with $0 < \varepsilon \ll 1$ a small parameter, while the involved frequencies are on the order of unity. Multiple regular or chaotic attractors with fractal boundaries between them may co-exist for some parameters values and the high-order approximations give better results than the first-order approximation [10, 11].

In the next section, we numerically investigate mainly the response of the system (1) in the proximity of the internal resonance $\omega_1 = 2\omega_2$ and external resonance $\Omega = \omega_1$.

3. Numerical results

In this section the ship equations (1) are numerically analysed with Runge – Kutta of fourth order method to determine the pitch and roll amplitudes for certain values of different model parameters. The system (1) exhibits a long transition towards the steady state response, thus we have integrated it for $T = 3000$ u.t. and used the last tenth of this time interval to compute the amplitudes. For non-periodic motions, by amplitudes we have understood the maximum value of the response in the above-mentioned time period. Our study is focussed only on the spring mode excitation such that the values $f_2 = 0$ and $\omega_1 = 1$ will be maintained unchanged throughout the paper.

Figure 1 shows the steady state amplitudes $a_i, i=1,2$, of the pitch and roll modes as a function of the roll natural frequency ω_2 for different forcing amplitudes f_1 . The other parameters were selected to be $c_1 = c_2 = 0.005$ and $\Omega = 1.04$. The black and red dots correspond to initial

conditions $\left(x(0), \dot{x}(0), \phi(0), \dot{\phi}(0)\right) = (0.2, 0.04, 0.1, 0.02)$, while the black and red circles describe the system behaviour for $\left(x(0), \dot{x}(0), \phi(0), \dot{\phi}(0)\right) = (0.02, -0.04, 0.03, 0.02)$. It is observed from figure 1a that for small f_1 and $\omega_2 \cong 0.45$ there co – exist at least two regular attractors. For example, if $\omega_2 = 0.46$, the stable attractors are characterized by $a_2 = 0$ and 0.2785. As f_1 increases, the basin of attraction for the first attractor diminishes (see figure 2).

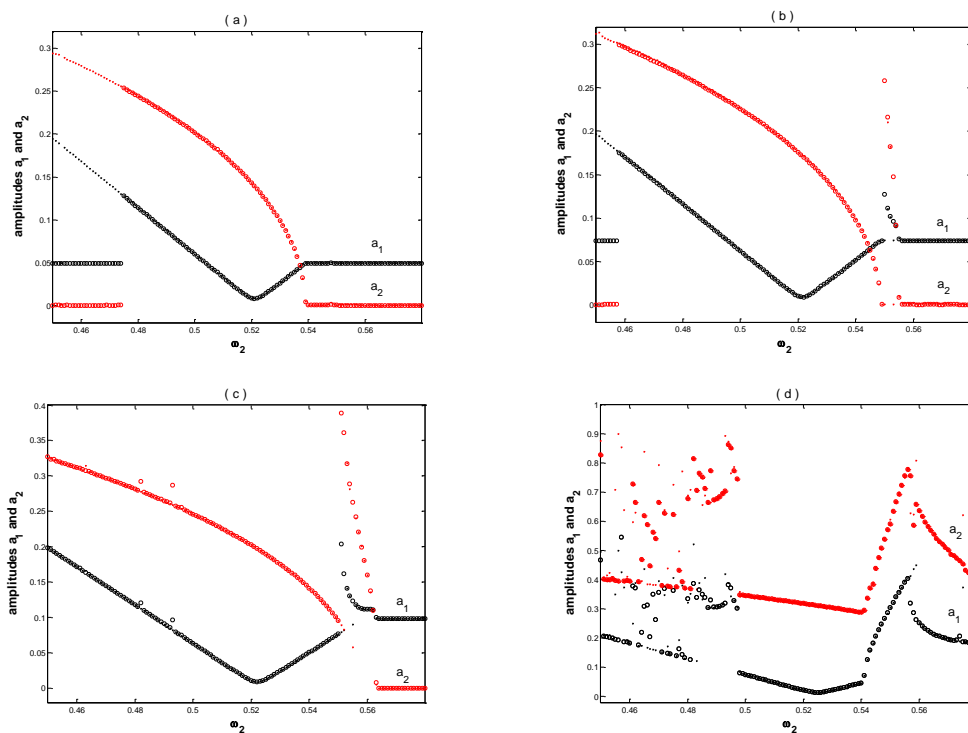


Figure 1. Pitch and roll amplitudes as a function of the roll natural frequency, for parameters values $c_1 = c_2 = 0.005$, $f_2 = 0$, $\omega_1 = 1$, $\Omega = 1.04$ and:

a) $f_1 = 0.004$; b) $f_1 = 0.006$; c) $f_1 = 0.008$; d) $f_1 = 0.02$.

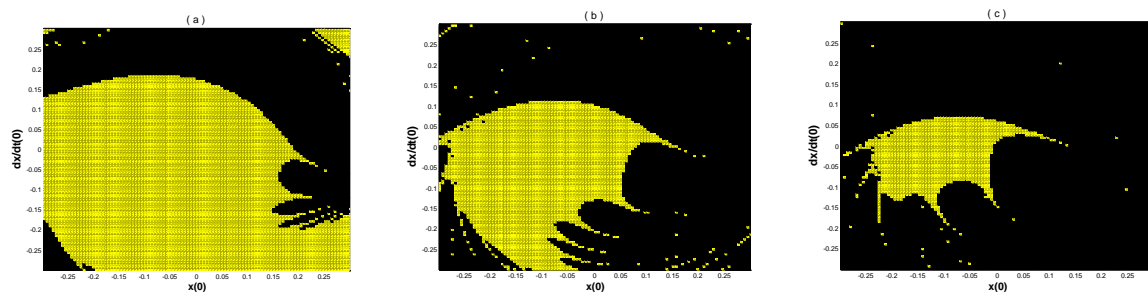


Figure 2. Basin of attraction for two periodic attractors obtained for parameters values

$c_1 = c_2 = 0.005$, $f_2 = 0$, $\omega_1 = 1$, $\omega_2 = 0.46$, $\Omega = 1.04$ and:

a) $f_1 = 0.002$; b) $f_1 = 0.004$; c) $f_1 = 0.008$. Yellow (light) colour stands for solution with $a_2 = 0$.

Figure 1a displays too an obvious jump between the two steady state responses determined by Lee and Park with multiple scales method [10]. This jump moves toward lower values of ω_2 as the force amplitude f_1 increases. It is worth noting also that the roll amplitudes have higher values to those of pitch mode, meaning that an important part of the energy introduced into the system in the pitch mode is transferred to the roll mode. Starting with $f_1=0.006$, another event comes into play: the appearance of quasi-periodic and chaotic dynamics. First it appears at the right border between the two sets of steady states but, for larger f_1 , it affects other and other ranges for ω_2 , as illustrated in figure 1d. The amplitudes $a_i, i=1,2$ corresponding to these behaviours are significantly larger than those for periodic motions, with immediate consequences on the stability and safety of the ship. Three different types of behaviours, obtained for the system's parameters used above, are reported in figures 3 to 5, after the transients die out. They represent a periodic oscillation (figure 3), a very slowly decreasing beating motion (figure 4) or a chaotic dynamics (figure 5).

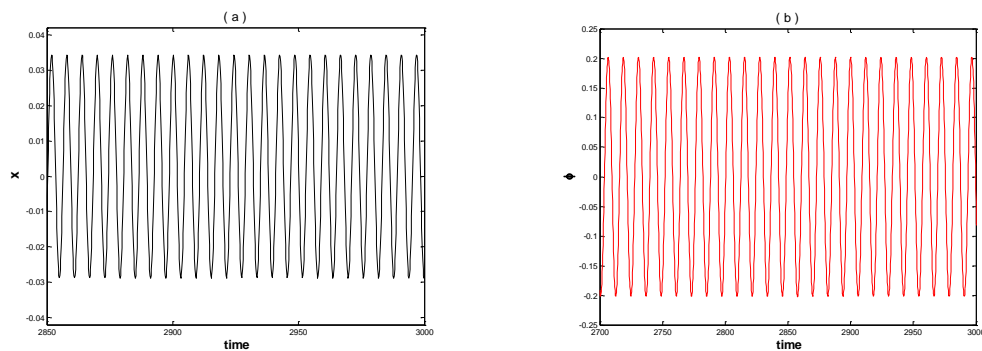


Figure 3. An example of periodic response of the system (1) at small amplitude of excitation and selected parameters $c_1=c_2=0.005$, $\omega_1=1$, $\omega_2=0.51$, $\Omega=1.04$, $f_1=0.004$, $f_2=0$.
a) pitch mode; b) roll mode.

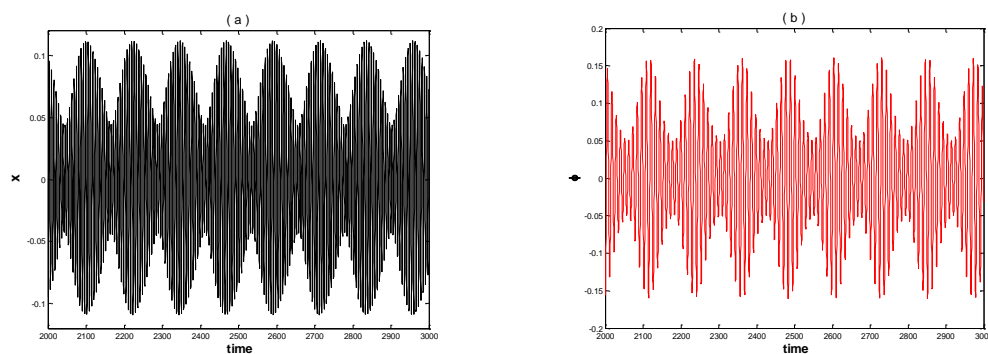


Figure 4. An example of beating motion of the system (1) at moderate amplitude of excitation and selected parameters $c_1=c_2=0.005$, $\omega_1=1$, $\omega_2=0.54$, $\Omega=1.04$, $f_1=0.008$, $f_2=0$.
a) pitch mode; b) roll mode.

The next simulation results aim at the effect of external frequency on the pitch and roll amplitudes. Figure 6 shows the bifurcation response curves of system (1) for an extended range of excitation frequency Ω and different forcing amplitudes f_1 . The plots contain the numerical results for the same initial conditions as those used in figure 1. From figure 6a, it is observed that both curves possess two jumps and a minimum. Also, it is noteworthy that a change in the initial conditions may shift the amplitudes from small to large values for certain Ω .

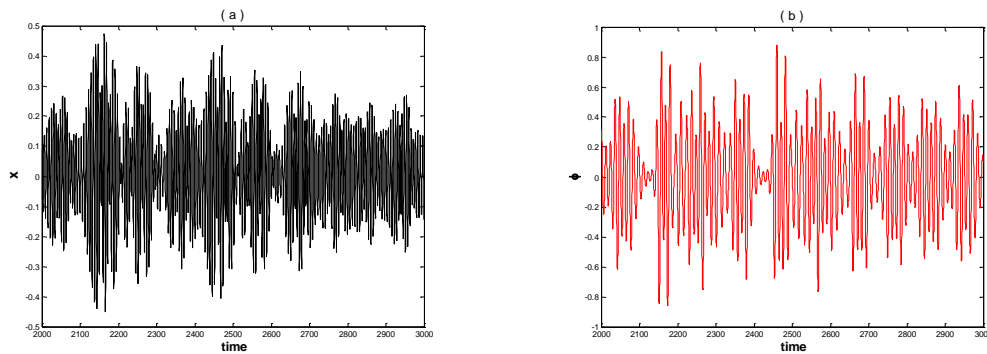


Figure 5. An example of chaotic response of the system (1) at large amplitude of excitation and selected parameters $c_1=c_2=0.005$, $\omega_1=1$, $\omega_2=0.47$, $\Omega=1.04$, $f_1=0.02$, $f_2=0$.
a) pitch mode; b) roll mode.

Again, as f_1 increases, the basin of attraction for the steady motions with small and null amplitudes diminishes continuously until its disappearance and the two jumps move farther with respect to the resonance location $\Omega=1$. Finally, note that even for small forcing f_1 in the nearness of $\Omega=1$ the system's behaviour is far to be periodic. For moderate and large forcing amplitudes there exist many regions where the solutions are bounded but not periodic (see panels 6c and 6d). Here, the maxima for pitch and roll angles are larger enough to present a real risk for the ship stability. New types of chaotic motions are detected, including that shown in figure 7. The integration time was increased at 20000 u.t, but no sign of regularity was recognized.

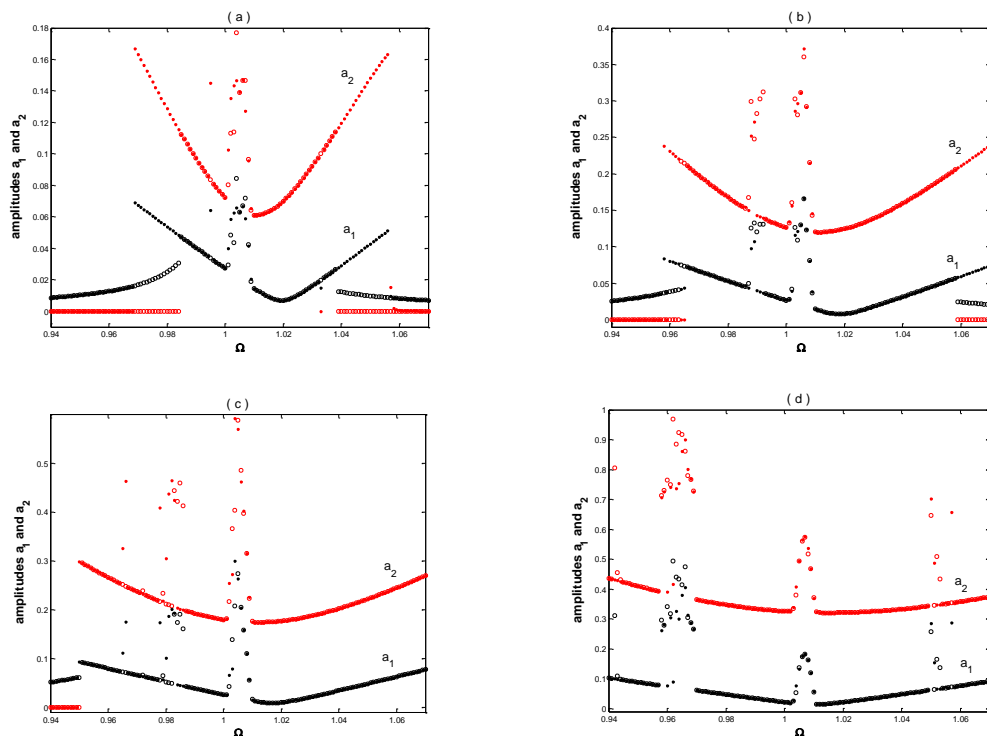


Figure 6. Pitch and roll amplitudes as a function of the external frequency, for parameters values $c_1=c_2=0.005$, $f_2=0$, $\omega_1=1$, $\omega_2=0.51$ and:
a) $f_1=0.001$; b) $f_1=0.003$; c) $f_1=0.006$; d) $f_1=0.02$.

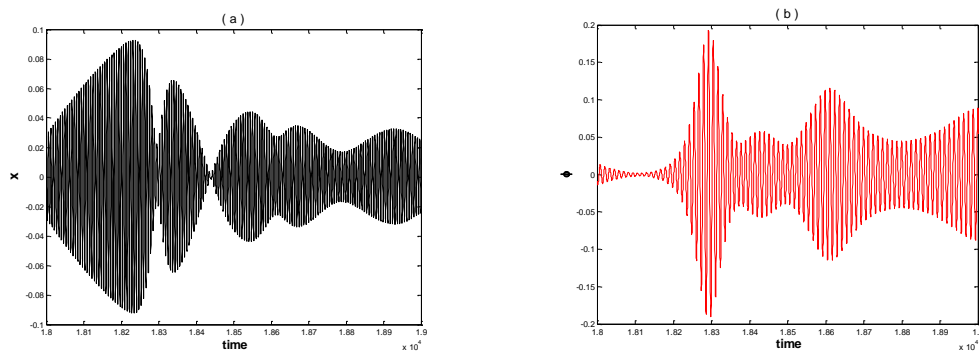


Figure 7. Chaotic response of the system (1) for the selected parameters $c_1 = c_2 = 0.005$, $\omega_1 = 1$, $\omega_2 = 0.51$, $\Omega = 1.007$, $f_1 = 0.02$, $f_2 = 0$: a) pitch mode; b) roll mode.

The analytical solutions derived in [10] suggest that the periodic oscillations for the pitch and roll modes are characterized by the frequencies Ω and $\Omega/2$. To check this, the bifurcation diagrams of x and ϕ versus f_1 have been plotted. The sampled time was chosen to be the forcing period $2\pi/\Omega$. The results displayed in figure 8 confirm, on the one hand, that for low excitation the system behaves periodically as Lee and Park stated [10] and, on the other hand, for strong excitation the system oscillates chaotically or quasi-periodically with large amplitudes.

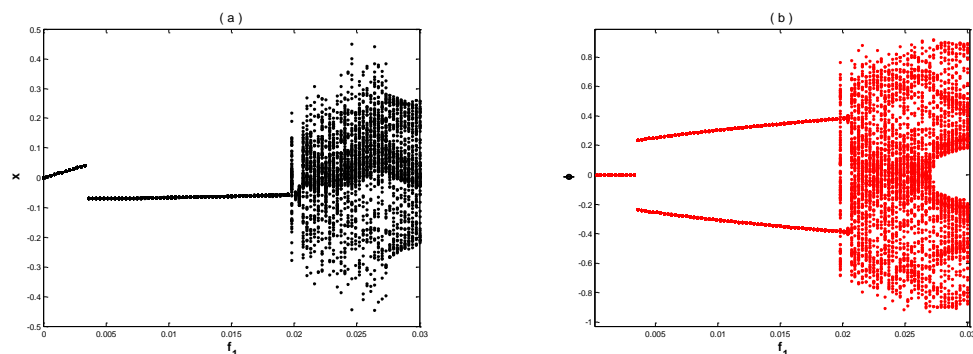


Figure 8. Bifurcation diagrams $x - f_1$ and $\phi - f_1$ for selected parameters $c_1 = c_2 = 0.005$, $\omega_1 = 1$, $\omega_2 = 0.51$, $\Omega = 0.96$ and $f_2 = 0$.

In order to examine the effects of other important parameters, in the third stage of simulations we have determined the amplitudes of oscillations for various damping coefficients. Figure 9 presents some of our findings. Thus, for fixed parameters excepting c_1 , the system may evolve periodically with amplitudes almost independent on c_1 and proportional to f_1 (see figure 9a) or may exhibit a complex behaviour including jumps in the amplitudes' values and a high concentration of chaotic motions for large forcing f_1 (see figures 9b and 9c). In principle, for low damping and high excitation we may expect to chaotic oscillations.

The last set of numerical investigations have concerned the ship model's dynamics if only one of the resonant conditions $\omega_1 = 2\omega_2$ and $\Omega = \omega_1$ is fulfilled. Thus, figure 10 illustrates the frequency-amplitude curves for $\Omega = 1.1$. As seen in the figure, for the same values of the other parameters, the steady-state solution with $a_2 = 0$ covers a larger range of ω_2 frequencies when compared with the case $\Omega = \omega_1$ (see also figure 1).

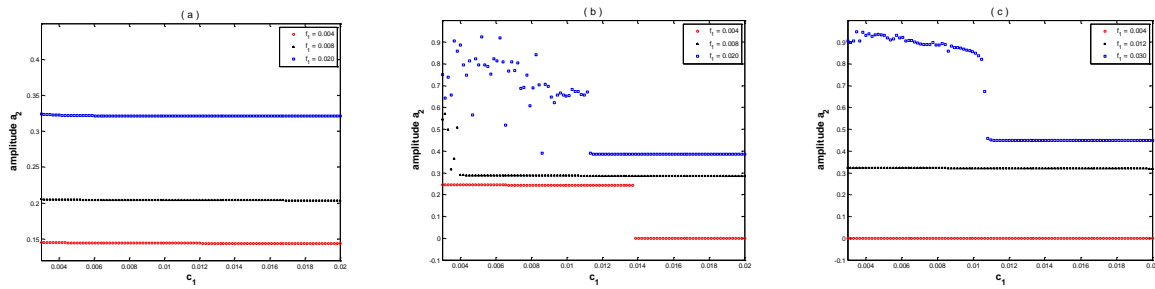


Figure 9. The influence of damping on roll amplitudes for system (1) with $\omega_1=1$, $\omega_1=1$, $f_2=0$ and different forcing f_1
a) $\Omega=1.02$, $c_2=0.003$; b) $\Omega=0.96$, $c_2=0.003$; c) $\Omega=0.96$, $c_2=0.03$.

Chaotic motions are still possible but they require high levels of forcing or small damping. Moreover, x and ϕ maxima for the non-periodic responses are not so scattered as for external resonant conditions ($\Omega=\omega_1$). If the external frequency Ω is taken far away from ω_1 , then the solution with constant amplitudes $a_i, i=1,2$, prevails on the entire range of natural roll frequencies used in figure 10.

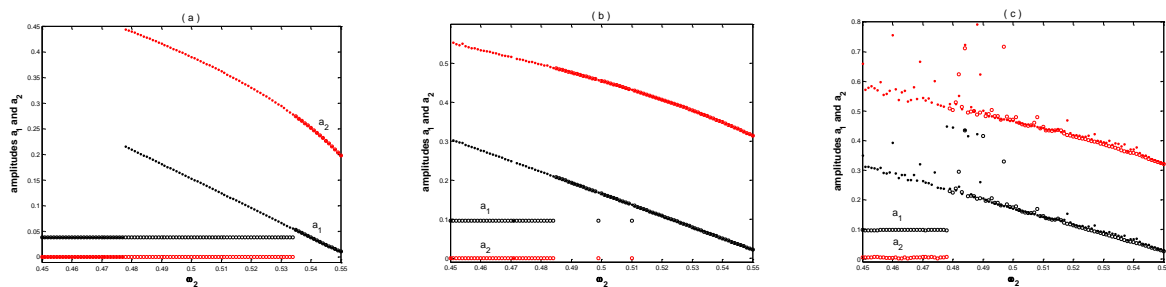


Figure 10. Pitch and roll amplitudes as a function of the roll natural frequency, for parameters values, $f_2=0$, $\omega_1=1$, $\Omega=1.1$ and:
a) $c_1=c_2=0.005$, $f_1=0.008$; b) $c_1=c_2=0.005$, $f_1=0.02$; c) $c_1=c_2=0.002$, $f_1=0.02$.

The other studied case is for $\omega_1 \neq 2\omega_2$. In this situation, as shown in figure 11, the roll mode is excited only for moderate and large forcing f_1 and just in the proximity of $\Omega=1$.

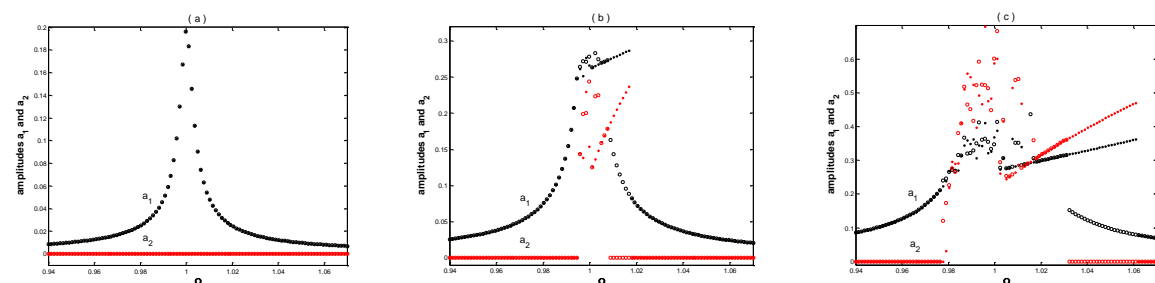


Figure 11. Pitch and roll amplitudes as a function of the external frequency, for parameters values $c_1=c_2=0.005$, $f_2=0$, $\omega_1=1$, $\omega_2=0.4$ and:
a) $f_1=0.001$; b) $f_1=0.003$; c) $f_1=0.01$.

4. Conclusions

The nonlinear responses of two degrees of freedom model for pitch and roll ship motions have been studied numerically for the particular case in which only the pitch mode is excited. The effort was focussed mainly on the simultaneous resonance case where the pitch frequency is almost twice the roll frequency and the external excitation period is near the pitch period, but some attention was given to the situations where only one of these conditions is fulfilled.

From the study the following may be concluded:

- For small forcing there co-exist two periodic attractors for many combinations of system's parameters. This was proven by plotting the frequency response curves and determining the basins of attraction for a given set of initial conditions. Jump phenomena accompany the transition from one attractor to another;
- Periodic oscillations take place with the external frequency for pitch mode and half of external frequency for roll mode. In many cases the roll amplitudes are much larger than the pitch ones, meaning that an important part of the energy introduced in the system in the pitch mode is transferred to the roll mode;
- The non-zero steady state amplitudes seem to be monotonic increasing functions in the forcing amplitude;
- For moderate and high forcing the periodic behaviour is replaced by quasi-periodic or chaotic dynamics, mainly in the proximity of the jump points or resonant frequencies. The maxima for roll and pitch amplitudes become extremely dangerous for ship stability;
- By decreasing the damping in the system, the probability of non-periodic solutions gets higher;
- The transient towards the steady state solutions is lengthy. The system behaves chaotically during this stage.

Our numerical results are in good agreement with the available published analytical studies.

References

- [1] Nayfeh A H, Mook D T and Marshall L R 1974 Perturbation - energy approach for the development of the nonlinear equations of ship motion *Journal of Hydrodynamics* **8** 130
- [2] Suleiman B M 2000 Identification of finite-degree-of freedom models for ship motions *Ph. D. thesis* Virginia Polytechnic Institute and State University, Blackburg, Va, USA
- [3] Ibrahim R A and Grace I M 2011 Modelling of ship roll dynamics and its coupling with heave and pitch *Mathematical Problems in Engineering* ID 934714
- [4] Deleanu D 2015 A numerical investigation into the capsize phenomenon of a vessel *International Journal of Modern Manufacturing Technologies* **7** 12
- [5] Deleanu D 2015 Comparative approximate studies on the ship's rolling motion *Journal of Marine Technology and Environment* **1** 29
- [6] Haddara M R and Xu J 1999 On the identification of the ship coupled heave – pitch motions using neural networks *Ocean Engineering* **26** 381
- [7] Kamel M M 2007 Bifurcation analysis of a nonlinear coupled pitch – roll ship *Mathematics and Computers in Simulation* **73** 300
- [8] Pan R and Davies H G 1996 Non-stationary response of a non-linear coupled pitch-roll ship model under modulated excitation *Journal of Sound and Vibration* **192** 669
- [9] Eissa M, El-Sera S, El-Sheikh M and Sayeda M 2003 Stability and primary simultaneous resonance of harmonically excited non-linear spring pendulum system *Applied Math. And Computation* **145** 421
- [10] Lee W K and Park H D 1997 Chaotic dynamics of a harmonically excited spring pendulum system with internal resonance *Nonlinear Dynamics* **14** 211
- [11] Alasty A and Shabani R 2006 Chaotic motions and fractal basin boundaries in spring pendulum system *Nonlinear Analysis: Real World Applications* **7** 81