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Method in calculating own vibration frequencies of open sections bars with thin walls

N Mihuţ[1]
"Constantin Brâncuşi" University of Tg-Jiu, Faculty of Engineering, Romania,
Department of Engineering and Management of Technological Systems,
Eroilor 30 st, 210135, Tg-Jiu, Romania

E-mail: mihut.nicoleta@gmail.com

Abstract. Dynamic stability of thin-walled bars of open sections, as well as the stability of elastic systems dynamics in general, is studying closely with their vibrations. This, because, areas of dynamics instability is around twice the frequency of free vibration of the bar or elastic system in all cases excitation parametric, on the one hand, and on the other hand matrices involved in the matrix equation of free vibration are matrices of matrix equation of dynamic stability.
In this paper we settled differential equations of parametric vibrations of thin-walled straight bars open sections constant as a system with a triple infinity of second order differential equations, linear coefficients homogeneous and periodicals. In the end of work, by customizing differential equations of forced vibration parameters have been obtained differential equations of own vibration of bars with thin wall and open sections as a system with a triple infinity of differential equations of second order, linear, homogeneous with constant coefficients and, using it, the algebraic equation of own vibrations pulsations.

1. Introduction

The basic problem that arises for the own vibrations of thin walled bars of open sections, as in all cases of own vibration, is the pulsations calculation.

Starting from the own vibration system equations of bars with thin wall of open sections and appropriate algebraic equation for free pulsations calculation are chosen as functions of displacement decomposition functions of cross-sections of the bars in the vibration process, bending and twisting functions frame bars in base system of displacement method, unitary displacement action in the direction of further ties [1]. In this case, the pulsations equation coefficients gets interpretation of the dynamic reaction which is calculated in the same way as the influence coefficients from displacement method of solving frames in the static building.

The equations are general because it contain, by means of own vibration and the functions of static stability loss functions, the conditions of abutment ends, being valid for any bearing way of the bar not only for those with homogeneous type articulated at the ends [3]. The obtained equations are valid for excitations by regular forces steering and variable pursuers passing through a fixed point [4]. Also, equations obtained are available for distribution in any manner along the axis of the external rod. In this way, the pulsations coefficients equation get interpretation of dynamic reactions which are

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calculated in the same manner as influential coefficients from displacement method of solving frameworks in static constructions.

2. Dynamic reaction method in calculating own vibration frequencies of open sections bars with thin walls

It is known [1], [5] that the own vibration differential equations of bars with thin wall of opened sections are presented as a system of differential equations of the second degree with known coefficients.

$$A\frac{d^2T}{dt^2} + BT = 0 \tag{1}$$

Generating the next higher grade algebraic equation to determine the free vibration pulsations as, [1]:

$$[B - \omega^2 A] = 0 \tag{2}$$

Equations number (2) can be written as, [1]:

$$|R| = 0 \tag{3}$$

And the R hyper-matrix elements are square matrices of three order, [1], [6]:

$$R_{ik} = \begin{pmatrix} r_{ik}^{uu} & 0 & r_{ik}^{u\varphi} \\ 0 & r_{ik}^{vv} & r_{ik}^{v\varphi} \\ r_{ik}^{\varphi u} & r_{ik}^{\varphi v} & r_{ik}^{\varphi u} \end{pmatrix} \tag{4}$$

With the elements, [7]:

$$\begin{aligned} r_{ik}^{uu} &= b_{ik}^{uu} - \omega^2 a_{ik}^{uu}; \\ r_{ik}^{vv} &= b_{ik}^{vv} - \omega^2 a_{ik}^{vv}; \\ r_{ik}^{\varphi\varphi} &= b_{ik}^{\varphi\varphi} - \omega^2 a_{ik}^{\varphi\varphi}; \\ r_{ik}^{u\varphi} &= -\omega^2 a_{ik}^{u\varphi}; \\ r_{ik}^{v\varphi} &= -\omega^2 a_{ik}^{v\varphi}; \\ r_{ik}^{\varphi u} &= -\omega^2 a_{ik}^{\varphi u}; \\ r_{ik}^{\varphi v} &= -\omega^2 a_{ik}^{\varphi v}; \end{aligned} \tag{5}$$

The elements *a*, *b* of number (5) relationships have the following expressions, [4]:

$$\begin{aligned} a_{ik}^{uu} &= \sum m\int_0^l Z_i^u(z)Z_k^u(z)\,dz + \sum m r_y^2 \int_0^l r_y^2 \int_0^l \frac{d\,Z_i^n(z)}{dz}\frac{d\,Z_k^n(z)}{dz}dz \\ a_{ik}^{vv} &= \sum m\int_0^l Z_i^v(z)Z_k^v(z)\,dz + \sum m r_x^2 \int_0^l \frac{dZ_{i(t)}^v}{dz}\frac{dZ_{k(t)}^v}{dz}dz; \end{aligned} \tag{6}$$

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$$a_{ik}^{\varphi\varphi} = \sum m r_0^2 \int_0^l Z_i^{\varphi}(z) Z_k^{\varphi}(z)\, dz + \sum m r_{\omega}^4 \int_0^l \frac{dZ_i^{\varphi}(z)}{dz} \frac{dZ_k^{\varphi}(z)}{dz} dz;$$

$$a_{ik}^{u\varphi} = \sum m y_0 \int_0^l Z_i^u(z) Z_k^{\varphi}(z)\, dz; \; a_{ik}^{v\varphi} = -\sum m x_0 \int_0^l Z_i^v(z) Z_k^{\varphi}(z)\, dz;$$

$$a_{ik}^{\varphi u} = \sum m y_0 \int_0^l Z_i^{\varphi}(z) Z_k^u(z)\, dz; \; a_{ik}^{\varphi v} = -\sum m x_0 \int_0^l Z_i^{\varphi}(z) Z_k^v(z)\, dz;$$

$$b_{ik}^{uu} = \sum EI_y \int_0^l \frac{d^2 Z_i^u(z)}{dz^2} \frac{d^2 Z_k^u(z)}{dz^2} dz$$

$$b_{ik}^{vv} = \sum EI_x \int_0^l \frac{d^2 Z_i^v(z)}{dz^2} \frac{d^2 Z_k^v(z)}{dz^2} dz;$$

$$b_{ik}^{\varphi\varphi} = \sum EI_{\omega} \int_0^l \frac{d^2 Z_{i(t)}^{\varphi}}{dz^2} \frac{d^2 Z_{k(t)}^{\varphi}}{dz^2} dz + \sum GI_z \int_0^l \frac{dZ_{i(t)}^{\varphi}}{dz} \frac{dZ_{k(t)}^{\varphi}}{dz} dz;$$

In relations number (6) are contained all of the geometric and elastic characteristic of the bars, *m* being the mass units length of the bar, $r_x, r_y, r_0, r_{\omega}$, inertia axial rays, polar and sectorial, and EI_x, EI_y, GI_z and EI_{ω} bending stiffness to the main central axis of inertia section of the twisting and gliding respectively.

Also in relations (6) appears the function of the *z* abscissa of bar section $Z_k^u(z), Z_k^v(z), Z_k^{\varphi}(z)$ which has to fulfill the conditions at the ends of each bar.

In the most advantageous for practical calculations the $Z_k^u(z), Z_k^v(z), Z_k^{\varphi}(z)$ functions can take the form of bending and twisting functions of terminals framework in the basic system of the displacements method from constructions static under the action of unit displacement in the additional direction as is done for cast bars, [3].

If "*n*" is the number of unknowns of the bars, in the calculation with displacements method as the main $Z_k^u(z), Z_k^v(z), Z_k^{\varphi}(z)$, *n* functions we can take bending and twisting static functions of the bars under the action of unit displacement applied in a time to the frame in the based system after the displacement method. Elements of A_{ik} and B_{ik} matrixes get in this case a simple static interpretation allowing their calculation rather easily, [2], [8]:

Writing:

$$\begin{aligned} \frac{d^2 Z_i^n(z)}{dz^2} &= -\frac{M_{y,i}(z)}{EI_y}; \\ \frac{d^2 Z_i^v(z)}{dz^2} &= -\frac{M_{x,i}(z)}{EI_x}; \\ \frac{d^2 Z_i^{\varphi}(z)}{dz^2} &= -\frac{B_{\omega,i}(z)}{EI_{\omega}}; \\ \frac{dZ_i^{\varphi}(z)}{dt} &= \frac{M_{z,i}(z)}{GI_z}; \end{aligned} \tag{7}$$

$M_{y,i}(z), M_{x,i}(z), B_{\omega,i}(z), M_{z,i}(z)$, being the bending moment in the main planes of inertia, flexural-torsion bi-moment and torsion moment produced by the unit *i* movement acting in based system, $b_{ik}^{uu}, b_{ik}^{vv}, b_{ik}^{\varphi\varphi}$ coefficients become:

$$b_{ik}^{uu} = \sum \int_0^l \frac{M_{y,i}(z) M_{y,k}(z)}{EI_y} dz;$$
$$b_{ik}^{vv} = \sum \int_0^l \frac{M_{x,i}(z) M_{x,k}(z)}{EI_x} dz; \tag{8}$$
$$b_{ik}^{\varphi\varphi} = \sum \int_0^l \frac{B_{\omega,i}(z) B_{\omega,k}(z)}{EI_\omega} dz + \sum \int_0^l \frac{M_{t,i}(z) M_{t,k}(z)}{GI_z} dz;$$

This way you can see how b_{ik} coefficients get the interpretation of unit reactions or influence coefficients obtained in displacement method. Thus b_{ik}^{uu} will represent the reaction amount on the direction of extra "*i*" connections due to bending of bars in xz plan under the influence of the k unit movement acting in the based system (the same for b_{ik}^{vv}) and $b_{ik}^{\varphi\varphi}$ will be the amount of the reactions on the additional connections direction "*i*" caused by the frame twisting under the action of unit k movement acting in the basic system.
On the reciprocity theorem, mechanical work will be:

$$Z_i^u(z) = -R_i^u(z); \quad Z_i^v(z) = -R_i^v(z); \quad Z_i^\varphi(z) = -R_i^\varphi(z);$$
$$\frac{dZ_i^u(z)}{dz} = -R_i^{*u}(z); \frac{dZ_i^v(z)}{dz} = -R_i^{*v}; \frac{dZ_i^\varphi(z)}{dz} = -R_i^{*\varphi} \tag{9}$$

$R_i^u(z), R_i^v(z)$ si $R_i^\varphi(z)$ representing reactions in *i* direction of additional links, due to the bending bars in *xz* and *yz* planes and of twisting of these products by a unitary force applied on the bending-torsion center axis *x* of a bar which is perpendicular in *z* coordinate point in *x* and y direction and of a unit torsion moment applied also in *z* coordinate point.
$R_i^u(z), R_i^v(z)$ and $R_i^\varphi(z)$ are reactions in the direction of *i* additional links due to bars bending in *xz* and *yz* planes and also due to torsion caused by a centered unit moment applied in z coordinate point on the torsion centers axis of a bar in parallel plans with *xz* and *yz* planes and by a double unit moment of torsion applied in the section of *z* abscissa axis.
Will result on this basis:

$$a_{ik}^{uu} = -\sum \int_0^l m R_i^u(z) Z_k^u(z)\, dz - \sum \int_0^l m r_y^2 R_i^{*u}(z) \frac{dZ_k^u(z)}{dz} dz;$$
$$a_{ik}^{vv} = -\sum \int_0^l m R_i^v(z) Z_k^v(z)\, dz - \sum \int_0^l m r_x^2 R_i^{*v}(z) \frac{dZ_k^v(z)}{dz} dz;$$
$$a_{ik}^{\varphi\varphi} = -\sum \int_0^l m R_i^\varphi(z) Z_k^\varphi(z)\, dz - \sum \int_0^l m r_\omega^4 R_i^{*\varphi}(z) \frac{dZ_k^\varphi(z)}{dz} dz; \tag{10}$$
$$a_{ik}^{u\varphi} = -\sum \int_0^l m y_0 R_i^u(z) Z_k^\varphi(z)\, dz; \quad a_{ik}^{v\varphi} = \sum \int_0^l m x_0 R_i^v(z) Z_k^\varphi(z)\, dz;$$
$$a_{ik}^{\varphi u} = -\sum \int_0^l m y_0 R_i^\varphi(z) Z_k^u(z)\, dz; \quad a_{ik}^{\varphi v} = \sum \int_0^l m x_0 R_i^\varphi(z) Z_k^v(z)\, dz;$$

And so the terms:

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$$a_{ik}^{uu}\frac{d^2T_k^u}{dt^2};\, a_{ik}^{vv}\frac{d^2T_k^v}{dt^2};\, a_{ik}^{\varphi\varphi}\frac{d^2T_k^\varphi}{dt^2};\, a_{ik}^{u\varphi}\frac{d^2T_k^v}{dt^2};\, a_{ik}^{\varphi u}\frac{d^2T_k^\varphi}{dt^2};\, a_{ik}^{v\varphi}\frac{d^2T_k^v}{dt^2};\, a_{ik}^{\varphi v}\frac{d^2T_k^\varphi}{dt^2}$$

contained in equation (2) of free vibrations pulsations, get the interpretation of reactions on the i additional links directions due to inertial forces caused by k unit displacements action, acting on the basis system frame.

It was shown that decomposing the displacements and torsion functions of the frame bars in process of vibration in the basic system of deformations method under the action of displacement unit, the prime terms of elements on the main array diagonals (4) R_{ik} represent unit reaction or influence coefficients in displacement method in ordinary sense of construction statics, the remaining terms represent the influence of inertial forces.

Elements of matrices R_{ik} will be called dynamic reaction and matrices R_{ik}, element of R hyper-matrix the equation (3), will be named dynamic reactions arrays.

From all these presented elements it is shown that the approximate equation of own frame vibrations is achieved using frame based system in displacement method by which is determined the dynamic reactions in the same way as is determined the unit reactions in the displacement method.

For this, obviously, we need to know for the two types of bars that appear in the displacements method, double recessed-articulated bar, the dynamics reactions produced by unit displacements on the main direction.

3. Conclusions

Starting from the own vibration system equations of thin wall bars of opened sections and from algebraic equation appropriate for calculating free vibration pulsations are obtained as decomposition displacement functions of bars cross-sections in the vibration process, bending and twisting functions of frame bars in based system of displacement method under the unit action on additional links direction. All these displacement functions are shown in this paper where the pulsations coefficients equation get interpretation of dynamic reactions which are calculated in the same manner as influential coefficients from displacement method of solving frameworks in static constructions. The vibration characteristics are of fundamental importance in the design of thin-walled structures. In general, the shear centre and the cross-section centre for thin walled beams are not coincident. Thus, the bending and torsional vibrations are coupled and together calculated. The approach presented in this paper is quite general and can be applied also for treating beams with non-uniform cross-sections and also non-classical boundary conditions.

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