

# The Accounting for Materials with Strength Differential Effect in Calculating the Durability in Order to Raise the Exploitation Effectiveness

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**Abstract.** The article is devoted to the mathematical model of bending of machines and constructions. The fact that they are made of multimodulus materials is taken into consideration. The priority is given to the multilayer qualities of such materials. The peculiarity of their deformation under real conditions due to the reciprocal slip of layers is taken into account. The redistribution of forces between the layers is achieved through the variable joint rigidity which is also represented in the given article. The construction of the mathematical model is given in details. The calculation method of bending of composite plates with layers of material with strength differential effect is described. While calculating the stress-strained state, the influence of rigidity of the multilayer bonding was taken into consideration.

## 1. Introduction

To decrease the material consumption and to preserve the durability and safety at the same time is an important task of raising effectiveness while creating any constructions and mechanisms, which can be solved due to the use of composite materials. Multilayer plates and shells are widely used at the construction sites in the oil and gas industry, in the chemical industry and in the atomic energetics, in creating complex fortified transport mechanisms. The use of such elements is stipulated by their high durability and rigidity while their mass is relatively low, excellent thermal and sound insulation qualities. Their high radiation protection should be mentioned separately.

New and traditional construction materials in use demonstrate different mechanical qualities depending on the main stress signs. In the majority of cases these materials possess different multimodulus qualities, that is they have different elasticity modules when they are stretched and compressed, for example concrete. Its multimodulus degree depends on the size of the specimen, the kind of its filler, humidity and other factors. For example, the multimodulus degree (Young's modulus) of fine-grain concrete on quartz sand and portland cement equals  $E^+ = 0.65 \cdot 10^3$  MPa,  $E^- = 1.82 \cdot 10^3$  MPa. The multimodulus quotient (Young's modulus) of heavy-weight concrete ( $E^+/E^-$ ) may vary from 1.07 to 1.82. The multimodulus quotient (Young's modulus) of light-weight concrete may be higher, lower or may equal 1.

When constructing block-boxes, multilayer plates, in which the concrete layer is connected with steel sheets with the help of anchors and glue, are used. This bonding has the finite shear rigidity.



Thus, it is necessary to solve the task of durability of such constructions taking into account the interlayer bonding qualities and multimodulus qualities of the layers.

## 2. Materials and Method

As in [1] we understand two or more plates when we use the term "a composite plate". Every single plate is an i-layer, the number of which equals  $n+1$ . The number of intervals between them (joints) equals  $n$ . We will adopt the upper-case numeration. The layers in the composite construction are connected between themselves with elastic-compliance connections, which allow the shear of one layer towards another one. The cross-binding bonds are absolutely rigid. In accordance with it, all the layers have the identical deflection (buckling)  $W(x,y)$ . The Kirchhoff–Love theory of plates is applied to one separate layer, not for the whole packet of layers. The packet load is directed along the normal and is distributed along the surface in a general-independent way. It is important to take into consideration the rigidity of the interlayer connections and multimodulus qualities of separate layers to solve the task of defining the stress-strained state of such constructions.

The integral characteristics of rigidity for the i-layer of the composite multi-layer plate should be written with the account of the multimodulus qualities of the material. The main force sign is the most important factor in the multimodulus elasticity theory [2]. The points and areas of the body where all main forces have the same signs (that is the body is compressed and stretched along the main forces) will be called the points and areas of the first kind, and we will not concentrate our attention on them in the given article. Those points and areas of the body where one of the main forces has a different sign from the other two forces, will be called the points or areas of the second kind. The physical proportions defining the dependence of the material deformation on the force size, will be written only for the area of the second kind [3].

To represent the integral characteristics of rigidity in Cartesian coordinates, it is necessary to make a transition from the direction of the main platforms to the x-y coordinate system. As being different from other known formulae, we suggest performing this transition through the transformation of the compliance matrix [4].

We will write the compliance matrix in the coordinate system, defining the position of the main platforms for the planar-stressed state as:

$$[a] = \begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & 0 \\ 0 & 0 & a_{66} \end{bmatrix}, \quad (1)$$

Where the elements  $a_{ik}$  are calculated according to the proportions [5].

As being different from other existing models, it is necessary to possess a complete set of the rigidity characteristics in the main coordinate system to make a transition from the coordinate system of the main platforms to the rigidity matrix in Cartesian coordinates. At the same time, when  $\sigma_1 > 0$ ,  $\sigma_2 < 0$  at the two orthogonal platforms we have characteristics of the material which correspond to stretching and compressing; that is why it is necessary to differentiate between the shear compliance (shear modulus)  $1/G^+$  and  $1/G^-$ . There sultings hear compliance value was calculated by averaging the values which were present at one point on two orthogonal X, Y platforms:

$$a_{66} = \frac{G^+ + G^-}{2G^+G^-} = \frac{E^-(1+\nu^+) + E^+(1+\nu^-)}{E^+E^-}, \quad (2)$$

Using [6], let us make a transition from the compliance matrix (1) in the coordinate system of the main platforms to the matrix in the X-Y coordinate system:

$$A_{11} = a_{11} \cos^4 \varphi + a_{22} \sin^4 \varphi + (2a_{12} + a_{66}) \sin^2 \varphi \cdot \cos^2 \varphi;$$

$$\begin{aligned}
 A_{22} &= a_{11} \sin^4 \varphi + a_{22} \cos^4 \varphi + (2a_{12} + a_{66}) \sin^2 \varphi \cdot \cos^2 \varphi; \\
 A_{12} &= (a_{11} + a_{22} - a_{66}) \sin^2 \varphi \cdot \cos^2 \varphi + a_{12} (\sin^4 \varphi + \cos^4 \varphi); \\
 A_{16} &= [2(a_{22} \sin^2 \varphi - a_{11} \cos^2 \varphi) + (2a_{12} + a_{66})(\cos^2 \varphi - \sin^2 \varphi)] \sin \varphi \cdot \cos \varphi; \\
 A_{26} &= [2(a_{22} \cos^2 \varphi - a_{11} \sin^2 \varphi) - (2a_{12} + a_{66})(\cos^2 \varphi - \sin^2 \varphi)] \sin \varphi \cdot \cos \varphi; \\
 A_{66} &= (4a_{11} - 8a_{12} + 4a_{22}) \sin^2 \varphi \cdot \cos^2 \varphi + (\cos^2 \varphi - \sin^2 \varphi)^2 a_{66}
 \end{aligned} \tag{3}$$

where  $\varphi$  - is an angle, defining the position of the main platforms towards the X, Y coordinates.

Let us write the forces through the deformation components in the following form [7]:

$$\begin{aligned}
 \sigma_x &= b_{11} \varepsilon_x + b_{12} \varepsilon_y + b_{16} \gamma_{xy}; \quad \sigma_y = b_{21} \varepsilon_x + b_{22} \varepsilon_y + b_{26} \gamma_{xy}; \\
 \tau_{xy} &= b_{61} \varepsilon_x + b_{62} \varepsilon_y + b_{66} \gamma_{xy},
 \end{aligned} \tag{4}$$

$$\text{where } b_{11} = \frac{A_{22}A_{66} - A_{26}^2}{\Omega}; \quad b_{22} = \frac{A_{11}A_{66} - A_{16}^2}{\Omega}; \quad b_{12} = \frac{A_{16}A_{26} - A_{12}A_{66}}{\Omega} \quad \text{and so on.};$$

$$\Omega = (A_{11}A_{22} - A_{12}^2)A_{66} + 2A_{12}A_{16}A_{26} - A_{11}A_{26}^2 - A_{22}A_{16}^2.$$

The forces in the i-layer of the plate with the account of its multimodulus qualities are written in the accordance with [8]. Having written the deformations within the Kirchhoff-Love theory of plates, we have:

$$\begin{aligned}
 \sigma_x^i &= b_{11}^i \left( \varepsilon_x^i - Z^i \frac{\partial^2 w}{\partial x^2} \right) + b_{12}^i \left( \varepsilon_y^i - Z^i \frac{\partial^2 w}{\partial y^2} \right) + b_{16}^i \left( \gamma_{xy}^i - 2Z^i \frac{\partial^2 w}{\partial x \partial y} \right); \\
 \sigma_y^i &= b_{21}^i \left( \varepsilon_x^i - Z^i \frac{\partial^2 w}{\partial x^2} \right) + b_{22}^i \left( \varepsilon_y^i - Z^i \frac{\partial^2 w}{\partial y^2} \right) + b_{26}^i \left( \gamma_{xy}^i - 2Z^i \frac{\partial^2 w}{\partial x \partial y} \right); \\
 \tau_{xy}^i &= b_{61}^i \left( \varepsilon_x^i - Z^i \frac{\partial^2 w}{\partial x^2} \right) + b_{62}^i \left( \varepsilon_y^i - Z^i \frac{\partial^2 w}{\partial y^2} \right) + b_{66}^i \left( \gamma_{xy}^i - 2Z^i \frac{\partial^2 w}{\partial x \partial y} \right).
 \end{aligned} \tag{5}$$

Here  $b_{mn}^i$  - is a rigidity characteristic at the point of the i-layer material.  $Z^i \in (-h^i/2, +h^i/2)$ ;  $\varepsilon_x^i, \varepsilon_y^i, \varepsilon_z^i$  - are the deformations in the mid-surface plane of the i-layer.

In contradistinction to the existing expressions, summands, which take into consideration the multimodulus qualities of the material, have appeared in (5)-(6).

Integrating in the thickness, let us move to the recording of the linear force ( $N_x^i, N_y^i, S^i$ ) and moments ( $M_x^i, M_y^i, M_{xy}^i$ ) in the i-layer from the forces at the arbitrary point integrating in the thickness:

$$\begin{aligned}
 N_x^i &= \int_{-h/2}^{h/2} \sigma_x^i dz; \quad N_y^i = \int_{-h/2}^{h/2} \sigma_y^i dz; \quad S^i = \int_{-h/2}^{h/2} \tau_{xy}^i dz; \\
 M_x^i &= \int_{-h/2}^{h/2} \sigma_x^i z dz; \quad M_y^i = \int_{-h/2}^{h/2} \sigma_y^i z dz; \quad M_{xy}^i = \int_{-h/2}^{h/2} \tau_{xy}^i z dz,
 \end{aligned} \tag{6}$$

where  $-h/2, h/2$  - is the greatest distance from the mid-surface plane to the outermost fibres of the i-layer of the plate.

Plugging the expression for the forces (5) in (6), after integrating we get an expression for the forces and moments in the i-layer:

$$\begin{aligned}
 N_x^i &= B_{11}^i \varepsilon_x^i + B_{12}^i \varepsilon_y^i - C_{11}^i \frac{\partial^2 w}{\partial x^2} - C_{12}^i \frac{\partial^2 w}{\partial y^2} + B_{16}^i \gamma_{xy}^i - 2C_{16}^i \frac{\partial^2 w}{\partial x \partial y} \\
 N_y^i &= B_{21}^i \varepsilon_x^i + B_{22}^i \varepsilon_y^i - C_{21}^i \frac{\partial^2 w}{\partial x^2} - C_{22}^i \frac{\partial^2 w}{\partial y^2} + B_{26}^i \gamma_{xy}^i - 2C_{26}^i \frac{\partial^2 w}{\partial x \partial y}; \\
 S^i &= B_{61}^i \varepsilon_x^i + B_{62}^i \varepsilon_y^i - C_{61}^i \frac{\partial^2 w}{\partial x^2} - C_{62}^i \frac{\partial^2 w}{\partial y^2} + B_{66}^i \gamma_{xy}^i - 2C_{66}^i \frac{\partial^2 w}{\partial x \partial y}; \\
 M_x^i &= -D_{11}^i \frac{\partial^2 w}{\partial x^2} - D_{12}^i \frac{\partial^2 w}{\partial y^2} + C_{11}^i \varepsilon_x^i + C_{12}^i \varepsilon_y^i - 2D_{16}^i \frac{\partial^2 w}{\partial x \partial y} + C_{16}^i \gamma_{xy}^i; \\
 M_y^i &= -D_{21}^i \frac{\partial^2 w}{\partial x^2} - D_{22}^i \frac{\partial^2 w}{\partial y^2} + C_{21}^i \varepsilon_x^i + C_{22}^i \varepsilon_y^i - 2D_{26}^i \frac{\partial^2 w}{\partial x \partial y} + C_{26}^i \gamma_{xy}^i \\
 M_{xy}^i &= -D_{61}^i \frac{\partial^2 w}{\partial x^2} - D_{62}^i \frac{\partial^2 w}{\partial y^2} + C_{61}^i \varepsilon_x^i + C_{62}^i \varepsilon_y^i - 2D_{66}^i \frac{\partial^2 w}{\partial x \partial y} + C_{66}^i \gamma_{xy}^i
 \end{aligned} \tag{7}$$

Here the integral characteristics of stretching, compression, shear, bending and torsion are calculated according to the following formulae:

$$(B_{mn}^i)_x = \int_{-\frac{h}{2}}^{\frac{h}{2}} (b_{mn}^i)_x dz \quad (C_{mn}^i)_x = \int_{-\frac{h}{2}}^{\frac{h}{2}} (b_{mn}^i)_x z dz \quad (D_{mn}^i)_x = \int_{-\frac{h}{2}}^{\frac{h}{2}} (b_{mn}^i)_x z^2 dz, \tag{8}$$

where  $m, n = 1, 2, 6$ .

On the other hand, the forces existing in the mid-surface plane of the  $i$ -layer can be written in accordance with [9]:

$$\begin{aligned}
 N_x^i &= \frac{\partial^2 \varphi^i}{\partial y^2} - \int_0^x \tau_x^i dx + \int_0^x \tau_x^{i-1} dx; \\
 N_y^i &= \frac{\partial^2 \varphi^i}{\partial x^2} - \int_0^y \tau_y^i dy + \int_0^y \tau_y^{i-1} dy; \\
 S^i &= -\frac{\partial^2 \varphi}{\partial x \partial y}.
 \end{aligned} \tag{9}$$

Let us define  $\int_0^x \tau_x^i dx = T_x^i$ ,  $\int_0^y \tau_y^i dy = T_y^i$ , where  $T_x^i$ ,  $T_y^i$  are the shear forces in the interlayer bonding of the  $i$ -joint.

Equating the forces  $N_x^i$ ,  $N_y^i$ ,  $S^i$  in accordance with (7) and (9), let us write the deformation values in the mid-surface plane of the  $i$ -layer for the multimodulus materials in the following form:

$$\begin{aligned}
 \varepsilon_x^i &= B_{11}^{*i} \frac{\partial^2 \varphi^i}{\partial y^2} - B_{12}^{*i} \frac{\partial^2 \varphi^i}{\partial x^2} - B_{11}^{*i} (T_x^i - T_x^{i-1}) + B_{12}^{*i} (T_y^i - T_y^{i-1}) - B_{16}^{*i} \frac{\partial^2 \varphi^i}{\partial x \partial y} - \varepsilon_1^i(w); \\
 \varepsilon_y^i &= B_{22}^{*i} \frac{\partial^2 \varphi^i}{\partial x^2} - B_{21}^{*i} \frac{\partial^2 \varphi^i}{\partial y^2} - B_{22}^{*i} (T_y^i - T_y^{i-1}) + B_{21}^{*i} (T_x^i - T_x^{i-1}) - B_{26}^{*i} \frac{\partial^2 \varphi^i}{\partial x \partial y} - \varepsilon_2^i(w); \\
 \gamma_{xy}^i &= B_{61}^{*i} \frac{\partial^2 \varphi^i}{\partial y^2} + B_{62}^{*i} \frac{\partial^2 \varphi^i}{\partial x^2} - B_{61}^{*i} (T_x^i - T_x^{i-1}) - B_{62}^{*i} (T_y^i - T_y^{i-1}) - B_{66}^{*i} \frac{\partial^2 \varphi^i}{\partial x \partial y} + 2\gamma_{12}^i(w).
 \end{aligned} \tag{10}$$

where  $B_{11}^{*i} = [B_{22}^i B_{66}^i - (B_{26}^i)^2] / (B_0^i)^3$ ;  $B_{12}^{*i} = B_{21}^{*i} = (B_{12}^i B_{66}^i - B_{16}^i B_{62}^i) / (B_0^i)^3$ ;  
 $B_{16}^{*i} = B_{61}^{*i} = (B_{12}^i B_{26}^i - B_{16}^i B_{22}^i) / (B_0^i)^3$ ;  $B_{22}^{*i} = [B_{11}^i B_{66}^i - (B_{16}^i)^2] / (B_0^i)^3$ ;  
 $B_{26}^{*i} = B_{62}^{*i} = (B_{16}^i B_{21}^i - B_{26}^i B_{11}^i) / (B_0^i)^3$ ;  $B_{66}^{*i} = [B_{11}^i B_{22}^i - (B_{12}^i)^2] / (B_0^i)^3$ ;  
 $(B_0^i)^3 = (B_{11}^i B_{22}^i - (B_{12}^i)^2) B_{66}^i + 2 B_{12}^i B_{26}^i B_{16}^i - B_{11}^i (B_{26}^i)^2 - B_{22}^i (B_{16}^i)^2$ .

The values  $\varepsilon_1^i(w)$ ,  $\varepsilon_2^i(w)$  and  $\gamma^i(w)$  are calculated according to the formulae:

$$\begin{aligned}\varepsilon_1^i(w) &= \bar{C}_{11}^i \frac{\partial^2 w}{\partial x^2} + \bar{C}_{12}^i \frac{\partial^2 w}{\partial y^2} + 2\bar{C}_{16}^i \frac{\partial^2 w}{\partial x \partial y}; \\ \varepsilon_2^i(w) &= \bar{C}_{21}^i \frac{\partial^2 w}{\partial x^2} + \bar{C}_{22}^i \frac{\partial^2 w}{\partial y^2} + 2\bar{C}_{26}^i \frac{\partial^2 w}{\partial x \partial y}; \\ 2\gamma_{12}^i(w) &= \bar{C}_{61}^i \frac{\partial^2 w}{\partial x^2} + \bar{C}_{62}^i \frac{\partial^2 w}{\partial y^2} + 2\bar{C}_{66}^i \frac{\partial^2 w}{\partial x \partial y}.\end{aligned}\quad (11)$$

We showed in details how to define the  $\bar{C}_{mn}^i$  values in [1].

Bearing in mind the expressions (7), (10) and (11), let us write the formulae for the moments:

$$\begin{aligned}M_x^i &= -D_{11}^{*i} \frac{\partial^2 w}{\partial x^2} - D_{12}^{*i} \frac{\partial^2 w}{\partial y^2} - 2D_{16}^{*i} \frac{\partial^2 w}{\partial x \partial y} + m_1^i(\varphi, T_x, T_y); \\ M_y^i &= -D_{21}^{*i} \frac{\partial^2 w}{\partial x^2} - D_{22}^{*i} \frac{\partial^2 w}{\partial y^2} - 2D_{26}^{*i} \frac{\partial^2 w}{\partial x \partial y} + m_2^i(\varphi, T_x, T_y); \\ M_{xy}^i &= -D_{61}^{*i} \frac{\partial^2 w}{\partial x^2} - D_{62}^{*i} \frac{\partial^2 w}{\partial y^2} - 2D_{66}^{*i} \frac{\partial^2 w}{\partial x \partial y} + m_{12}^i(\varphi, T_x, T_y).\end{aligned}\quad (12)$$

Here  $D_{11}^{*i} = D_{11}^i + C_{11}^i \bar{C}_{11}^i + C_{12}^i \bar{C}_{21}^i - C_{16}^i \bar{C}_{61}^i$ ;  $D_{21}^{*i} = D_{21}^i + C_{21}^i \bar{C}_{11}^i + C_{22}^i \bar{C}_{21}^i - C_{26}^i \bar{C}_{61}^i$ , and so on.

The values  $m_1^i(\varphi, T_x^i, T_y^i)$ ,  $m_2^i(\varphi, T_x^i, T_y^i)$ ,  $m_{12}^i(\varphi, T_x^i, T_y^i)$  are calculated according to the formulae:

$$\begin{aligned}m_1^i(\varphi, T_x^i, T_y^i) &= C_{11}^{*i} \frac{\partial^2 \varphi^i}{\partial y^2} + C_{12}^{*i} \frac{\partial^2 \varphi^i}{\partial x^2} - C_{11}^{*i} (T_x^i - T_x^{i-1}) - C_{12}^{*i} (T_y^i - T_y^{i-1}) + C_{16}^{*i} \frac{\partial^2 \varphi^i}{\partial x \partial y}; \\ m_2^i(\varphi, T_x^i, T_y^i) &= C_{21}^{*i} \frac{\partial^2 \varphi^i}{\partial y^2} + C_{22}^{*i} \frac{\partial^2 \varphi^i}{\partial x^2} - C_{22}^{*i} (T_y^i - T_y^{i-1}) - C_{21}^{*i} (T_x^i - T_x^{i-1}) + C_{26}^{*i} \frac{\partial^2 \varphi^i}{\partial x \partial y}; \\ m_{12}^i(\varphi, T_x^i, T_y^i) &= C_{61}^{*i} \frac{\partial^2 \varphi^i}{\partial y^2} + C_{62}^{*i} \frac{\partial^2 \varphi^i}{\partial x^2} - C_{61}^{*i} (T_x^i - T_x^{i-1}) + C_{62}^{*i} (T_y^i - T_y^{i-1}) + C_{66}^{*i} \frac{\partial^2 \varphi^i}{\partial x \partial y},\end{aligned}\quad (13)$$

where  $C_{11}^{*i} = C_{11}^i B_{11}^{*i} - C_{12}^i B_{21}^{*i} + C_{16}^i B_{61}^{*i}$ ;  
 $C_{16}^{*i} = -C_{11}^i B_{16}^{*i} - C_{12}^i B_{26}^{*i} - C_{16}^i B_{66}^{*i}$ ;  
 $C_{22}^{*i} = -C_{21}^i B_{12}^{*i} + C_{22}^i B_{22}^{*i} + C_{26}^i B_{62}^{*i}$ ;  
 $C_{61}^{*i} = C_{61}^i B_{11}^{*i} - C_{62}^i B_{21}^{*i} + C_{66}^i B_{61}^{*i}$ ;  
 $C_{66}^{*i} = -C_{61}^i B_{16}^{*i} - C_{62}^i B_{26}^{*i} - C_{66}^i B_{66}^{*i}$ ;  
 $C_{12}^{*i} = -C_{11}^i B_{12}^{*i} + C_{12}^i B_{22}^{*i} + C_{16}^i B_{62}^{*i}$ ;  
 $C_{21}^{*i} = C_{21}^i B_{11}^{*i} - C_{22}^i B_{21}^{*i} + C_{26}^i B_{61}^{*i}$ ;  
 $C_{26}^{*i} = -C_{21}^i B_{16}^{*i} - C_{22}^i B_{26}^{*i} - C_{26}^i B_{66}^{*i}$ ;  
 $C_{62}^{*i} = -C_{61}^i B_{12}^{*i} + C_{62}^i B_{22}^{*i} + C_{66}^i B_{62}^{*i}$ ;

Using the equation of equilibrium, following the methodology [9] for the composite plate, the proportions (12) taking into consideration (13), having summed in all the layers, we will get the equation which will be represented in the operator form:

$$L_1(D^*, w) = q - L_2(D^*, w) + \Lambda(\varphi, T_x^i, T_y^i) + L(c, T_x^i, T_y^i); \quad (14)$$

$$L_1(D^*, w) = D_{11}^* \frac{\partial^4 w}{\partial x^4} + 2(D_{16}^* + D_{61}^*) \frac{\partial^4 w}{\partial x^3 \partial y} + (D_{12}^* + D_{21}^* + 4D_{66}^*) \frac{\partial^4 w}{\partial x^2 \partial y^2} + 2(D_{26}^* + D_{62}^*) \frac{\partial^4 w}{\partial x \partial y^3} + D_{22}^* \frac{\partial^4 w}{\partial y^4}; \quad (15)$$

$$L_2(D^*, w) = \frac{\partial^2 D_{11}^*}{\partial x^2} \frac{\partial^2 w}{\partial x^2} + 2 \frac{\partial D_{11}^*}{\partial x} \frac{\partial^3 w}{\partial x^3} + \frac{\partial^2 D_{12}^*}{\partial x^2} \frac{\partial^2 w}{\partial y^2} + 2 \frac{\partial D_{12}^*}{\partial x} \frac{\partial^3 w}{\partial x \partial y^2} + 2 \frac{\partial^2 D_{16}^*}{\partial x^2} \frac{\partial^2 w}{\partial x \partial y} + 2 \frac{\partial D_{16}^*}{\partial x} \frac{\partial^3 w}{\partial x^2 \partial y} + \frac{\partial^2 D_{21}^*}{\partial y^2} \frac{\partial^2 w}{\partial x^2} + 2 \frac{\partial D_{21}^*}{\partial y} \frac{\partial^3 w}{\partial x^2 \partial y} + \frac{\partial^2 D_{22}^*}{\partial y^2} \frac{\partial^2 w}{\partial y^2} + 2 \frac{\partial D_{22}^*}{\partial y} \frac{\partial^3 w}{\partial y^3} + 2 \left( \frac{\partial^2 D_{26}^*}{\partial y^2} \frac{\partial^2 w}{\partial x \partial y} + 2 \frac{\partial D_{26}^*}{\partial y} \frac{\partial^3 w}{\partial x \partial y^2} \right) + 2 \left( \frac{\partial^2 D_{61}^*}{\partial x \partial y} \frac{\partial^2 w}{\partial x^2} + \frac{\partial D_{61}^*}{\partial x} \frac{\partial^3 w}{\partial x^2 \partial y} + \frac{\partial D_{61}^*}{\partial y} \frac{\partial^3 w}{\partial x^3} \right) + 2 \left( \frac{\partial^2 D_{62}^*}{\partial x \partial y} \frac{\partial^2 w}{\partial y^2} + \frac{\partial D_{62}^*}{\partial x} \frac{\partial^3 w}{\partial y^3} + \frac{\partial D_{62}^*}{\partial y} \frac{\partial^3 w}{\partial x \partial y^2} \right) + 4 \left( \frac{\partial^2 D_{66}^*}{\partial x \partial y} \frac{\partial^2 w}{\partial x \partial y} + \frac{\partial D_{66}^*}{\partial x} \frac{\partial^3 w}{\partial x \partial y^2} + \frac{\partial D_{66}^*}{\partial y} \frac{\partial^3 w}{\partial x^2 \partial y} \right);$$

$$\Lambda(\varphi, T_x^i, T_y^i) = \sum_{i=1}^{n+1} \left( \frac{\partial^2 m_1^i}{\partial x^2} + \frac{\partial^2 m_2^i}{\partial y^2} + 2 \frac{\partial^2 m_{12}^i}{\partial x \partial y} \right);$$

$$L(c, T_x^i, T_y^i) = \sum_{i=1}^n \left( \frac{\partial c^i}{\partial x} \frac{\partial T_x^i}{\partial x} + \frac{\partial c^i}{\partial x} \frac{\partial T_y^i}{\partial y} \right) + \sum_{i=1}^n c^i \left( \frac{\partial^2 T_x^i}{\partial x^2} + \frac{\partial^2 T_y^i}{\partial y^2} \right);$$

$$D_{11}^* = \sum_{i=1}^{n+1} D_{11}^{*i}.$$

The summands  $m_1^i(\varphi, T_x^i, T_y^i)$ ,  $m_2^i(\varphi, T_x^i, T_y^i)$ ,  $m_{12}^i(\varphi, T_x^i, T_y^i)$  in the operator form  $\Lambda(\varphi, T_x^i, T_y^i)$  are found from (13).

Let us add a continuity deformation equation for the i-layer of the plate to the equation of equilibrium [9]. Let us write the deformation continuity equations for the mid-surface plane of each layer using the expressions for the deformations (10) and calculating the following derivatives:

$$L_1^i(B^*, \varphi) + L_2^i(B^*, \varphi) - L_1^i(B^*, T_x^i, T_y^i) - L_2^i(B^*, T_x^i, T_y^i) - \Lambda^i(c, w) = 0; \quad (16)$$

$$L_1^i(B^*, \varphi) = B_{22}^{*i} \frac{\partial^4 \varphi^i}{\partial x^4} + 2 \left( \frac{1}{2} B_{66}^{*i} - B_{12}^{*i} \right) \frac{\partial^4 \varphi^i}{\partial x^2 \partial y^2} + B_{11}^{*i} \frac{\partial^4 \varphi^i}{\partial y^4} - 2 B_{26}^{*i} \frac{\partial^4 \varphi^i}{\partial x^3 \partial y} - 2 B_{16}^{*i} \frac{\partial^4 \varphi^i}{\partial x \partial y^3};$$

$$L_2^i(B^*, \varphi) = 2 \left[ \frac{\partial B_{22}^{*i}}{\partial x} \frac{\partial^3 \varphi^i}{\partial x^3} + \frac{\partial}{\partial x} \left( \frac{1}{2} B_{66}^{*i} - B_{12}^{*i} \right) \frac{\partial^3 \varphi^i}{\partial x \partial y^2} \right] +$$

$$\begin{aligned}
 & + 2 \left[ \frac{\partial B_{11}^{*i}}{\partial y} \frac{\partial^3 \varphi}{\partial y^3} + \frac{\partial}{\partial y} \left( \frac{1}{2} B_{66}^{*i} - B_{12}^{*i} \right) \frac{\partial^3 \varphi^i}{\partial x^2 \partial y} \right] + \left( \frac{\partial^2 B_{22}^{*i}}{\partial x^2} - \right. \\
 & - \frac{\partial^2 B_{12}^{*i}}{\partial y^2} \left. \right) \frac{\partial^2 \varphi^i}{\partial x^2} + \left( \frac{\partial^2 B_{11}^{*i}}{\partial y^2} - \frac{\partial^2 B_{21}^{*i}}{\partial x^2} \right) \frac{\partial^2 \varphi^i}{\partial y^2} + \frac{\partial^2 B_{66}^{*i}}{\partial x \partial y} \frac{\partial^2 \varphi^i}{\partial x \partial y} - \\
 & - \frac{\partial}{\partial y} (2B_{16}^{*i} + B_{61}^{*i}) \frac{\partial^3 \varphi^i}{\partial x \partial y^2} - \frac{\partial}{\partial x} (2B_{26}^{*i} + B_{62}^{*i}) \frac{\partial^3 \varphi^i}{\partial x^2 \partial y} - \\
 & - \left( \frac{\partial B_{61}^{*i}}{\partial x} \frac{\partial^3 \varphi^i}{\partial y^3} + \frac{\partial B_{62}^{*i}}{\partial y} \frac{\partial^3 \varphi^i}{\partial x^3} \right) - \left( \frac{\partial B_{16}^{*i}}{\partial y^2} + \frac{\partial B_{26}^{*i}}{\partial x^2} \right) \frac{\partial^2 \varphi^i}{\partial x \partial y} - \\
 & - \left( \frac{\partial^2 B_{61}^{*i}}{\partial x \partial y} \frac{\partial^2 \varphi^i}{\partial y^2} + \frac{\partial^2 B_{62}^{*i}}{\partial x \partial y} \frac{\partial^2 \varphi^i}{\partial x^2} \right); \\
 L_1^i(B^*, T_x^i, T_y^i) &= B_{12}^{*i} \frac{\partial^2 (T_y^i - T_y^{i-1})}{\partial y^2} - B_{11}^{*i} \frac{\partial^2 (T_x^i - T_x^{i-1})}{\partial y^2} - B_{22}^{*i} \frac{\partial^2 (T_y^i - T_y^{i-1})}{\partial x^2} + B_{21}^{*i} \frac{\partial^2 (T_x^i - T_x^{i-1})}{\partial x^2} - \\
 & - B_{61}^{*i} \frac{\partial^2 (T_x^i - T_x^{i-1})}{\partial x \partial y} - B_{62}^{*i} \frac{\partial^2 (T_y^i - T_y^{i-1})}{\partial x \partial y}; \\
 L_2^i(B^*, T_x^i, T_y^i) &= (T_x^i - T_x^{i-1}) \left( \frac{\partial^2 B_{21}^{*i}}{\partial x^2} - \frac{\partial^2 B_{11}^{*i}}{\partial y^2} - \frac{\partial^2 B_{61}^{*i}}{\partial x \partial y} \right) - (T_y^i - T_y^{i-1}) \left( \frac{\partial^2 B_{12}^{*i}}{\partial x^2} - \frac{\partial^2 B_{22}^{*i}}{\partial y^2} - \frac{\partial^2 B_{62}^{*i}}{\partial x \partial y} \right) - \\
 & - \left[ \frac{\partial (T_x^i - T_x^{i-1})}{\partial y} \left( \frac{\partial B_{11}^{*i}}{\partial y} + \frac{\partial B_{61}^{*i}}{\partial x} \right) - \frac{\partial (T_x^i - T_x^{i-1})}{\partial x} \cdot \left( \frac{\partial B_{21}^{*i}}{\partial y} + \frac{\partial B_{61}^{*i}}{\partial x} \right) + \frac{\partial (T_y^i - T_y^{i-1})}{\partial y} \left( \frac{\partial B_{12}^{*i}}{\partial y} + \frac{\partial B_{62}^{*i}}{\partial x} \right) - \right. \\
 & - \left. \frac{\partial (T_y^i - T_y^{i-1})}{\partial x} \left( \frac{\partial B_{22}^{*i}}{\partial y} + \frac{\partial B_{62}^{*i}}{\partial x} \right) \right] \cdot 2. \\
 \Lambda^i(c, w) &= \frac{\partial^2 \varepsilon_1^i}{\partial y^2} + \frac{\partial^2 \varepsilon_2^i}{\partial x^2} + 2 \frac{\partial^2 \gamma_{12}^i}{\partial x \partial y}.
 \end{aligned}$$

The values  $\varepsilon_1^i(w)$ ,  $\varepsilon_2^i(w)$ ,  $\gamma^i(w)$  are calculated according to (11).

The equations describing the joint layer work adjoining the i-joint loop the system with regard to the sought functions  $W$ ,  $\varphi^i$ ,  $T_x^i$ ,  $T_y^i$ .

Let us single out a separating plane in each joint; longitudinal displacements for the i-layer and (i+1)-layer exist to the both sides of the separating plane. The subtractions of the longitudinal displacements  $\Delta u^i$  and  $\Delta v^i$  to the both sides of the separating plane can be written as [9]:

$$\Delta u^i = u^{i+1} - u^i + \frac{\partial w}{\partial x} c^i; \quad \Delta v^i = v^{i+1} - v^i + \frac{\partial w}{\partial x} c^i, \quad (17)$$

where  $u^i(x, y)$ ,  $v^i(x, y)$ ,  $w^i(x, y)$  - are longitudinal and transversal displacements of the points of the mid-surface plane of the i-layer;  $c^i(x, y)$  - is the distance between the mid-surface planes of the adjacent layers with the account of the variable layer thickness.

The connection between  $\Delta u^i$ ,  $\Delta v^i$  and the shear strain in the i-joint will be written as:

$$\tau_x^i = \eta_x^i \Delta u^i; \quad \tau_y^i = \eta_y^i \Delta v^i, \quad (18)$$

where  $\eta_x^i(x, y)$ ,  $\eta_y^i(x, y)$  - is the shear stiffness coefficient depending on the shear value of one layer towards another one. The shear strain is recorded through the functions  $T_x^i$  и  $T_y^i$  accounting for the work of the i-joint:

$$\tau_x^i = \frac{\partial T_x^i}{\partial x}; \quad \tau_y^i = \frac{\partial T_y^i}{\partial y}. \quad (19)$$

Let us express  $\Delta u^i$  and  $\Delta v^i$  from (19) plugging them in (17). Let us differentiate each equation in x, the second one in y. Let us write the work of the joint equation in the following way:

$$\begin{aligned} \frac{1}{\eta_x^i} \frac{\partial^2 T_x^i}{\partial x^2} + \frac{\partial}{\partial x} \left( \frac{1}{\eta_x^i} \right) \frac{\partial T_x^i}{\partial x} &= \frac{\partial}{\partial x} \left( c^i \frac{\partial w}{\partial x} \right) + \varepsilon_x^{i+1} - \varepsilon_x^i; \\ \frac{1}{\eta_y^i} \frac{\partial^2 T_y^i}{\partial y^2} + \frac{\partial}{\partial y} \left( \frac{1}{\eta_y^i} \right) \frac{\partial T_y^i}{\partial y} &= \frac{\partial}{\partial y} \left( c^i \frac{\partial w}{\partial y} \right) + \varepsilon_y^{i+1} - \varepsilon_y^i. \end{aligned} \quad (20)$$

The obtained system of equations enables to solve the tasks of bending of the multilayer plate whose layers are composed of materials with strength-differential effect considering the influence of the rigidity of the interlayer bonding on the stress-strained state of the construction and this system includes the differential equations of equilibrium (14), continuity (16) and the work of the joint (20). If the number of layers in a plate is  $n+1$ , then we have one equation of equilibrium for the whole packet,  $n+1$  compatibility of deformation equations or equations of continuity for each layer separately and  $2n$  equations considering the compatibility of the work of the layers due to the finite rigidity of the bonding connecting them. The total number of equations is  $4n$ . The number of the sought functions is also  $4n$ : the buckling/bending function  $W(x, y)$ , the same for the whole packet,  $n+1$  functions of forces  $\varphi^i(x, y)$  and  $2n$  - shear functions  $T_x^i(x, y)$ ,  $T_y^i(x, y)$ , and that means that the obtained system is looped.

The mathematical model of bending of composite multilayer plates made of materials with strength-differential effect (SD) includes not only the system of differential equations, but also borderline conditions.

As a result of integrating (14), (18), (20) the sought functions have to satisfy borderline conditions, which correspond to the specific outline fixing. The fact that differential equations are high-order (high-degree) ones allows to take into consideration complex and different kinematical and static conditions of fixing the plate layers.[10]. The borderline conditions for the function which reflects the conditions connected with the outer static indeterminacy, have to be written for every layer, for the function responsible for the inner static indeterminacy of the construction, for every joint. Two groups of borderline conditions were singled out in the calculations: on the outline support (for the whole packet) and on the end faces of the packet (for every i-layer). Based on it, the conditions on the outline are formulated independently from the conditions on the end faces of the layers and joints.

### 3. Results and Discussion

The authors have worked out the algorithm and methodology of solving tasks devoted to the bending of the composite multilayer plates whose layers are made of multimodulus materials. Hinge support based on the rectangular outline of the packet of the composite construction was considered to be borderline conditions. The end faces are tied with the band, which is absolutely rigid in its plane and absolutely flexible outside the plane, that means we have  $y=0$ ,  $b$  on the boundary; along the  $X$  axis the following conditions were fulfilled:

$$W = 0; \quad M_x = 0; \quad T_x^i = 0; \quad N_x = 0; \quad V^i = 0. \quad (21)$$

The sought functions have been approximated by Fourier two-fold series satisfying the borderline conditions (21).

The transition from the system of the differential equations to the system of algebraic ones has been realized with the help of Galerkin orthogonality. The system of the algebraic equations has been solved with the help of the Gauss method. The integral characteristics of rigidity of the layers and their derivatives have been calculated with the help of the numerical differentiation method. To achieve this aim, we introduced a net with the number of junctions  $I \times J = 21 \times 21$  on the plate surface. The rigidity values  $b_{mn}^i$  have been defined according to formulae (5) at the net corners and on the surface of the composite plate and on five layers in the thickness of the  $i$ -layer. Integrating according to (9), the transition to the integral values of rigidity  $B_{mn}^i C_{mn}^i D_{mn}^i$  has been realized. The integral values have been acquired according to Simpson's quadrature formula. The derivatives of the integral values of rigidity, stretching, compressing, bending and so on have been recorded in accordance with the finite-difference scheme.

#### 4. Conclusion

The given calculations have shown that the consideration of the finite shear rigidity specifies greatly the stress-strained state of the construction for the three-layer plate, where the middle layer possesses the strength-differential effect. If a multilayer plate is treated as a single-layer plate, then we will get underestimated values of bending and strains. If we consider the case when the layers slide on each other without any impediment, we will get overestimated bending values and, what is more important, overestimated strain values, that means that we unreasonably underestimate the structural capability of the construction. Multimodulus qualities of the material have the greatest influence on the stress-strained state when the rigidity of the interlayer bonding is low. Besides, the change of the stress-strained state with the increase of the coefficient, defining multimodulus qualities, is shown only to some extent.

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