Parametric uncertain identification of a robotic system

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Abstract. This paper presents the parametric uncertainties identification of a robotic system of one degree of freedom. A MSC-ADAMS / MATLAB co-simulation model was built to simulate the uncertainties that affect the robotic system. For a desired trajectory, a set of dynamic models of the system was identified in presence of variations in the mass, length and friction of the system employing least squares method. Using the input-output linearization technique a linearized model plant was defined. Finally, the maximum multiplicative uncertainty of the system was modelled giving the controller desired design conditions to achieve a robust stability and performance of the closed loop system.

1. Introduction

Usually, a parametric identification process is performed for a system under ideal conditions. That is, it does not take into account the presence of parametric uncertainness and external perturbations that alter the system dynamics. For a robotic system, parametric uncertain and external perturbations identification presents a challenge because these systems are non-linear, multivariable and exhibit a high degree of interaction between its links [2]-[4]. This paper proposes a parametric uncertainness identification for a robotic system of one degree of freedom (DOF). A MSC-ADAMS/MATLAB co-simulation model is built to simulate the robotic system dynamics. Then, the non-linear dynamic model parametric identification of the robot is performed using the recursive least square method [1]. After this, a linearized dynamic model of the robotic system is obtained applying the input-output linearization technique. This linear model belongs to the nominal plant and it represents a model of the system without considering external disturbances and parametric uncertainness. Then, the family of plants for the robotic system is calculated varying the mass, width and friction in the co-simulation model of the robotic system, applying the linearization technique for each condition. From the family of plants, the multiplicative uncertainty is computed for each member of the family and the maximum multiplicative uncertain profile of the system is determined. This maximum uncertain profile allows to find the desired design conditions for the controller which ensure the stability and robust performance of the robotic system [5]. The paper is structured as follows. First, a MSC-ADAMS/MATLAB co-simulation model is presented. Second, the parametric identification using recursive least square method is performed. Third, a linearized model of the robotic system is obtained by input-output linearization. Fourth, the family of plants is determined to obtain the maximum multiplicative uncertain profile and find the desired design controller specifications to reach a robust stability and performance.

2. ADAMS-MATLAB co-simulation dynamic model

In order to study the real-time dynamic behaviour of the robotic system is necessary to build a simulation model able to emulate all the dynamic actions applied over the robotic system during its operation, such as friction, Coriolis forces, gravity effect, and more. For this, MSC-ADAMS software was used. The robotic articulation of one DOF can be represented with a simple pendulum that receives an applied torque in order to generate the movement. The pendulum is placed in the same direction as the gravity, which must be compensated later for the development of the controller. The robotic system rotates around Z-axis, and the applied torque is perpendicular to the movement axis. Then, the MSC-ADAMS model is exported to MATLAB to modelling the parametric uncertain of the robotic system [6]. Figure 1a shows the MSC-ADAMS model of the robotic system.

3. Identification of the robotic-joint dynamic model

According to [1], the dynamic model for a robotic joint of 1DOF is given by (1)

$$\tau = [I_{r1} + m_1 {l_{cl}}^2]\ddot{q} + b_1\dot{q} + f_{c1}sign(\dot{q}) + [m_1 g l_{cl}]sin(q) \quad (1)$$

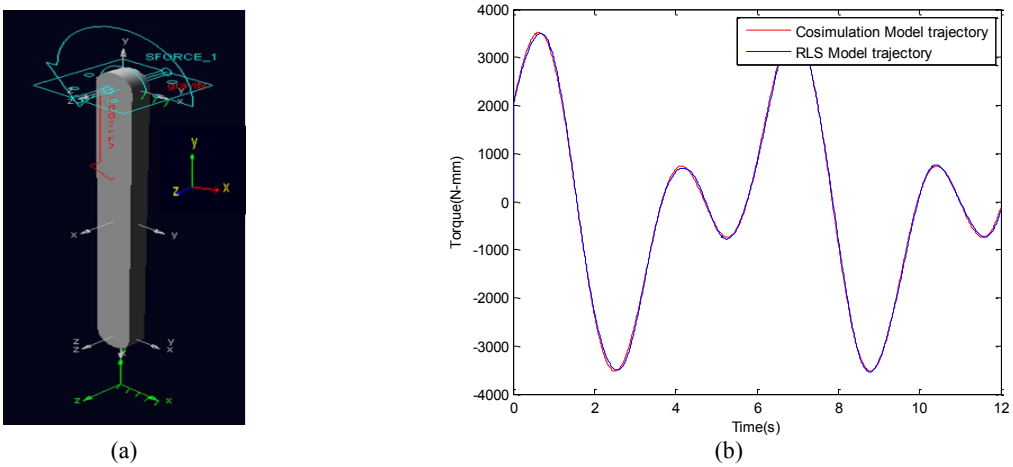
where, τ is the applied torque, m_1 is the mass of the pendulum, l_{cl} is the gravity centre, g is the gravity force, $q, \dot{q}$ and $\ddot{q}$ are the position, speed and acceleration of the pendulum respectively, and b_1 and f_{c1} represent the viscous and Coulomb friction. (1) can be expressed in parametric form as is defined in (2).

$$\tau = \theta_1\ddot{q} + \theta_2\dot{q} + \theta_3 sign(\dot{q}) + \theta_4 sin(q) \quad (2)$$

where $\theta_1 = I_{r1} + m_1 {l_{cl}}^2$, $\theta_2 = b_1$, $\theta_3 = f_{c1}$, $\theta_4 = m_1 g l_{cl}$. The identification of the dynamic model constants $\theta_1, \theta_2, \theta_3$ and θ_4 is performed using the linear regression method based on the recursive least-squares (RLS) algorithm. Based on the co-simulation model and the applied torque defined in (3) the identification process is implemented using the experimental data from position, speed, acceleration and applied torque of the joint.

$$Z = [2000\cos(2t) + 2000sin(10t)] \text{ N-mm} \quad (3)$$

Applying the recursive least-squares algorithm, the constants of the dynamic model proposed in (2) corresponds to θ_1=150.0136, θ_2= -41.69, θ_3=-4.69, θ_4=5274.7. Figure1b shows the co-simulation torque trajectory Z against to the RLS model with torque trajectory Z with a correlation of 99.7%.



(a) (b)

Figure 1. (a) MSC-ADAMS model, (b) identification torque trajectory

4. Robotic system linearization using the input-output technique

To obtain the nominal plant and the family of plants, is necessary a linear model of the system. Since the robot dynamic is non-linear system as is described in (2), it requires a linearization technique to obtain its linear model. In this case, the input-output linearization technique was used [7]. The representation of the system in state variables is shown in (4), with $x_1 = q$ and $x_2 = \dot{q}$.

$$\begin{bmatrix} \dot{x_1}(t) \\ \dot{x_2}(t) \end{bmatrix} = \begin{bmatrix} x_2(t) \\ -\frac{\theta_2}{\theta_1}x_2(t) - \frac{\theta_3}{\theta_1}sign(x_2(t)) - \frac{\theta_4}{\theta_1}\sin(x_1(t)) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{\theta_1} \end{bmatrix}\tau(t) \tag{4}$$

$$\text{If, } f(x) = \begin{bmatrix} x_2(t) \\ -\frac{\theta_2}{\theta_1}x_2(t) - \frac{\theta_3}{\theta_1}sign(x_2(t)) - \frac{\theta_4}{\theta_1}\sin(x_1(t)) \end{bmatrix} \text{ and } g(x) = \begin{bmatrix} 0 \\ \frac{1}{\theta_1} \end{bmatrix}$$

The system (4) can be expressed as:

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= h(x) = x_1 \end{aligned} \tag{5}$$

The linearization control law is given by (6)

$$u = \frac{1}{L_g L_{f(x)}}[-\beta_0 h_{(x)} - \beta_1 L_{f(x)} - {L_{f(x)}}^2 + v] \tag{6}$$

where v is the new entry of the system, β_0 and β_1 are the constants of the desired polynomial that represents the open-loop behavior of the linearized system. $L_{f(x)}, {L_{f(x)}}^2$ and $L_g L_{f\,(x)}$, are given by (7)-(10) using Lie's algebra.

$$l_f h(x) = \frac{\partial h(x)}{\partial x} f(x) = x_2 \tag{7}$$

$$l_g h(x) = \frac{\partial h(x)}{\partial x} g(x) = 0 \tag{8}$$

$${l_f}^2 h(x) = \frac{\partial l_f h(x)}{\partial x} f(x) = -\frac{\theta_2}{\theta_1}x_2(t) - \frac{\theta_3}{\theta_1}sign(x_2(t)) - \frac{\theta_4}{\theta_1}\sin(x_1(t)) \tag{9}$$

$$l_g l_f h(x) = \frac{\partial l_f h(x)}{\partial x} g(x) = \frac{1}{\theta_1} \tag{10}$$

The desired characteristic polynomial for the linearized system is described in (11):

$$s^2 + \beta_1 s + \beta_0 = 0 \tag{11}$$

Parameters β_1, β_0, are computed so that the frequency response of linearized system $(P(s))$ be comparable to a second order linear model presented in (12).

$$P(s) = \frac{y(s)}{v(s)} = \frac{0.49}{1 + 1.30s + 0.496s^2} \tag{12}$$

5. Parametric uncertain identification and determination of robust operation conditions

Linearized model of the system presented in (12) correspond to the nominal plant. To obtain the family of plants, the robotic system is tested against parametric uncertainness in the width, mass and joint

friction. Table 1 shows the original parameter of the robotic system and its percentage of variation due to parametric uncertainness described above.

Table 1 Range of variation of parametric uncertainness

Parameter	Nominal Value	Perturbation (%)
Mass (M)	2kg	± 10
Width (W)	4 cm	± 5
Joint Friction (F)	0.3 N	± 15

From each condition presented in Table 1, co-simulation model is modified to obtain the parameters θ_1 to θ_4 of the dynamic model (2) against each set of perturbations as show in Table 2. The + and – signs in Table 2 indicates the maximum and minimum values of the parameters mass (M), width (W) and joint friction (F). As shown in Table 2, the parameters of the dynamic model (2) varying according to the uncertain set presented. This indicates that the robotic system dynamics is different for each uncertain set applied. From the dynamic model parameters (2), the robotic system is linearized as a linear second order system descripted in (13) with ζ, T_w and k_p calculated for each condition as shown in Table 3. Frequency response of the family of plants is presented in Figure 2.

$$\widetilde{P}_l(s) = \frac{k_p}{1 + 2\zeta T_w s + (T_w s)^2} \tag{13}$$

Table 2. Robotic system dynamic model parameters variation for each parametric uncertainness.

Parameters	Nominal	F M W - - -	F M W + - -	F M W + - +	F M W - - +	F M W + + -	F M W + + +	F M W - + -	F M W - + +
$\boldsymbol{\theta_1}$	150.0136	139.7957	160.0754	160.03	138.95	160.11	160.07	139.08	138.91
$\boldsymbol{\theta_2}$	-13.6251	-18.872	-18.6263	-17.98	-17.74	-18.56	-17.82	-18.33	-17.69
$\boldsymbol{\theta_3}$	4.9657	30.5345	-0.0071	2.35	22.41	0.13	2.07	23.5	22.09
$\boldsymbol{\theta_4}$	5275	4759	5786	5784	4725	5785	5785	4725	4725

Table 3 Linearized family of plants for the robotic system.

Parameters	Nominal	F M W - - -	F M W + - -	F M W + - +	F M W - - +	F M W + + -	F M W + + +	F M W - + -	F M W - + +
$\boldsymbol{k_p}$	0.49	1.7	0.193	0.19	1.69	0.193	0.1911	1.71	1.69
$\boldsymbol{Tw}$	0.7	1.26	0.365	0.36	1.27	0.365	0.368	1.25	1.27
$\boldsymbol{\zeta}$	0.92	0.35	0.66	0.67	0.355	0.666	0.673	0.3624	0.355

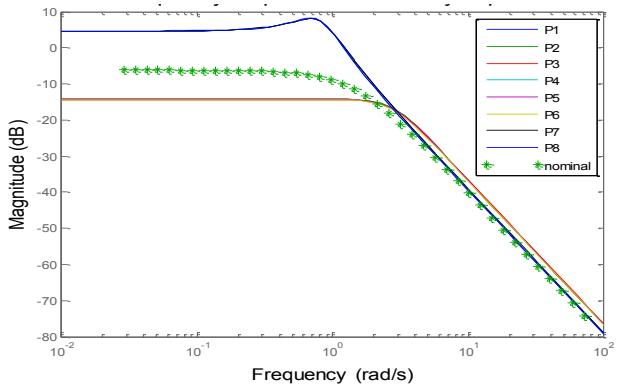


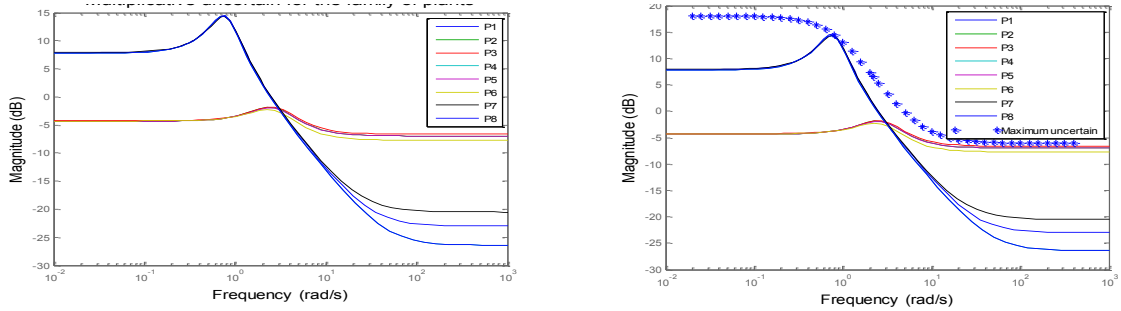
Figure 2. Frequency response of the family of plants of the robotic system

As shown in Figure 2, the nominal plant represented by * is placed between the variation range presented by the uncertainness descripted in Table 1. Once the family of plants is defined, the uncertain of each member of the family of plants is calculated using the multiplicative uncertain model [8] showed in (14).

$$|\delta P_i(s)| = \left|\frac{\tilde{P}_i(s)}{P_n(s)} - 1\right| \tag{14}$$

where $P_n(s)$ correspond to the nominal plant descripted by (12) and $\tilde{P}_i(s)$ is the ith member of the family of plants. Frequency response of the multiplicative uncertain profile of each member of the family of plants is shown in Figure 3a. To find the conditions for robust performance and stability is necessary to determinate the maximum multiplicative uncertain profile. For this, the maximum uncertain profile is estimated according to the frequency response of the multiplicative uncertain for the family of plants shown in Figure 3. The maximum multiplicative uncertain profile reached is given by (15) and its frequency response is shown in Figure 3b.

$$\delta_{pmax}(s) = \frac{0.5s^2 + 5s + 8}{s^2 + 2s + 1} \tag{15}$$



(a) (b)

Figure 3. (a) Multiplicative uncertain profile for the family of plants of the robotic system and (b) Maximum multiplicative uncertain profile of the family of plants.

As observed in Figure 3b, the maximum profile of multiplicative uncertain represented by * covers all the multiplicative uncertainness descripted by the family of plants. From [8], the closed loop desired specifications are calculated to achieve a robust stability and performance according to (16).

$$|T(s)| \leq \left|\frac{1}{\delta P_{max}(s)}\right| \tag{16}$$

where $|T(s)|$ is the magnitude of the complementary sensibility function and $|\delta P_{max}(s)|$ correspond to the magnitude of maximum profile of multiplicative uncertain. Figure 4 shows the function $|1/\delta P \max\ (s)\ |$ which restrict the closed loop bandwidth of the system to 0.7 rad/s. The bandwidth is related to the natural frequency w_n by (17).

$$w_b = w_n\sqrt{1 - 2\zeta^2 + \sqrt{(1 - 2\zeta^2)^2 + 1}} \tag{17}$$

Assuming $\zeta = 0.6$ which is equivalent to an 10% overshoot, from (16) $w_n = 0.61\ rad/s$, the desired closed loop function is (18). The frequency response of (18) is shown in Figure 5 which bandwidth is limited by the maximum inverse uncertain profile.

$$T(s) = \frac{0.037}{s^2 + 0.73s + 0.37} \tag{18}$$

The closed loop specifications to reach a robust stability and performance are given by (19) and (20) where (19) is the phase margin and (20) the gain crossover frequency. Solving (19) and (20), the design conditions for the controller are $pm = 60°$ and $w_c = 0.43\ rad/s$.

$$pm = 100\zeta \tag{19}$$

$$w_c = w_n\sqrt{\sqrt{4\zeta^4 + 1} - 2\zeta^2} \tag{20}$$

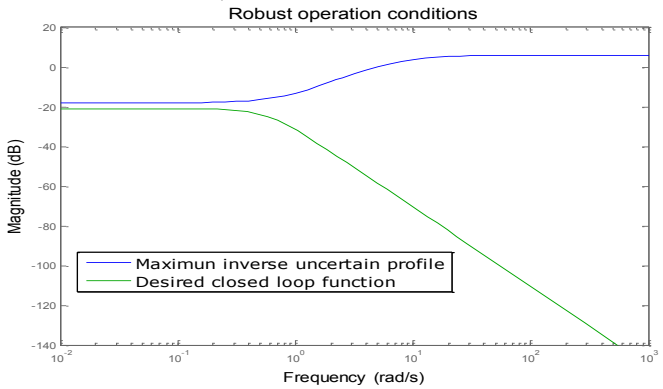


Figure 4. Desired robust operation conditions for the robotic system

6. Conclusions

This paper presented a methodology for the parametric uncertainness identification of a robotic system. A MSC-ADAMS/MATLAB co-simulation model was built for the parametric identification of the dynamic model using least square algorithm. The dynamic model was linearized using input-output linearization, obtaining the nominal plant. After that, a set of parametric uncertainness is defined to obtain the family of plants for the robotic system. Then the multiplicative uncertain of the family of plants is determined to find the maximum uncertain profile, which one establishes the closed loop system specifications to reach a robust stability and performance. Results show that when a robotic system is affected by external disturbances and parametric uncertainness, it results in many dynamical behaviours of the system, which must be covered for a controller to get a robust performance of the system. Employing the proposed methodology the desired design controller specifications to achieve a robust stability and performance against the presence of external disturbances a parametric uncertainness can be found. As well, this methodology may be applied to any non-linear system in order to achieve a robust stability and performance.

References

[1] Reyes Cortes F 2010 *MATLAB aplicado a Robotica y mecatronica* (Madrid, Alfaomega grupo editor) 460p
[2] Salloum R, Arvan M R and Moaveni B Identification 2013 uncertainty modelling, and robust controller design for an electromechanical actuator *International conference on Control, Automation, Robotics and Embedded Systems*
[3] Sadeghzadeh A 2014 Performance analysis for uncertain multivariable systems obtained by system identification *International Journal of Systems Science* **45** pp 547-555
[4] Sanchez L, Alazard D, Michon G and Beriloz A 2014 Robustness Analysis of Helicopter Ground Resonance with Parametric Uncertainties in Blade Properties *Journal of Guiadance Control and Dynamics* **36** pp 272-281
[5] Astrom K and Hagglund T 1995 *Advanced PID control* (USA: ISA) **1** p 480
[6] Ángel L, Pérez M P, Díaz-Quintero C R and Mendoza C 2012 ADAMS/MATLAB co-simulation: Dynamic systems analysis and control tool *Applied Mechanics and Materials* **232**, pp 527-531
[7] Viola J and Angel L 2015 Identification, Control and robustness analysis of a robotic system using fractional control *IEEE Latin America Transactions* **13** (5) pp 1294-1302
[8] Doyle J Francis and B Tannenbaum A 1990 *Feedback Control Theory* (USA: Macmillan Publishing Co) **1** p 224