# Modelling of uncertainness for a flow and level system

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**Abstract** This paper presents the identification of uncertainness that affects the dynamics of a flow and level system. Initially, flow a level system is descripted. Then, family of plants is determined from the identification of dynamic model for different operating conditions. The uncertain model reflects the changes for different operating conditions when the output flow and storage tank dimensions are varied. Finally, the maximum multiplicative uncertain is calculated to define the desired controller specifications to achieve a robust stability and performance of the closed loop system.

## 1. Introduction

System dynamics is affected by external and internal factors that do not allow obtain an accurate mathematical model of the system, which difficult the control system design and the desired operation conditions [1]. These factors might be due to a system not modelled dynamics, uncertainness it the model parameters, external perturbations, system nonlinearities among others. Because of this, the uncertainness modeling of the system is important since it provides information about unknown dynamics against different operating conditions and ease the control system design to reach the robust stability and performance conditions [2]-[7]. Therefore, this paper proposes the development of a methodology for the parametric uncertainness identification for a flow and level system to obtain the closed loop specifications to guarantee the robust stability and performance of the system. This paper is structured as follows. First the flow and level system is described. Second system identification is performed to determinate the family of plants in the presence of parametric uncertainness and external perturbations of the system. Third, multiplicative uncertain for the family of plants is calculated. Fourth, maximum uncertain condition is modeled and the operating conditions to reach robust stability and performance are stablished. Finally, conclusions are presented.

## 2. System description

Flow and level system is composed by three tanks (TK1, TK2 and TK3), a servo-valve (FV1), two solenoid valves (FV2 and FV3), two manual valves (HV1 and HV2), a pipeline system and a pump (P). Also, system has an ABB control module for the implementation of basic control strategies. Upper tanks TK1 and TK2 let develop of level control strategies and TK3 tank works as deposit tank. Servo-valve (FV1) is the system actuator and works with a 4-20 mA signal. In this paper, control variable is the level in the tanks TK1 and TK2. Figure 1a shows the flow and level system and Figure 1b shows the P&ID diagram of the system.
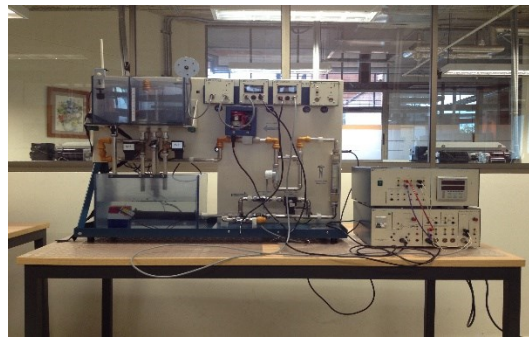
(a)
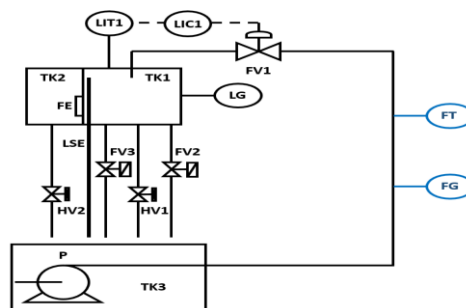


(b)

**Figure 1.** (a) Flow and level system and (b) P&ID diagram of the system

## 3. System identification

The identification of the mathematical model of the system is performed from the operation of the closed loop system. To do this, control loop was closed with a proportional controller with known gain. Data acquisition was performed with a NI DAQ 6008 and the model of the system was obtained using MATLAB identification toolbox, where input signal was the control action and the output signal belongs to the level in TK1.

## 4. Family of plants

Family of plants estimation was performed from the process identification for six operating conditions. These conditions using some combinations from the operation of solenoid valves FV2 and FV3 with TK1 and TK2 tanks. Table 1 shows the six operating conditions of the plant to be used for the identification process. ON condition in FV2 and FV3 indicates that solenoid valve is working, while ON condition in TK1 and TK2 indicates if the tank taken into account. Each condition in Table 1 is repeated five times for a total of 30 set of identification data.

**Table 1**. System operating conditions

| Operating condition | FV3 | FV2 | TK2 | TK1 |
|:---:|:---:|:---:|:---:|:---:|
| **1** | ON | OFF | OFF | ON |
| **2** | OFF | ON | OFF | ON |
| **3** | ON | ON | OFF | ON |
| **4** | ON | OFF | ON | ON |
| **5** | OFF | ON | ON | ON |
| **6** | ON | ON | ON | ON |

The family of plants for each operating conditions shown in Table 1 are identified with the transfer function model given by (1), which correspond to a second order with real poles system.

$$P(s) = k \frac{1 + T_z s}{(1 + T_{p1}s)(1 + T_{p2}s)} \tag{1}$$

Since for each operating condition data is taken five times. Table 2 shows an identification example of five models for the operating condition one.

**Table 2**. Family of plants for operating condition 1

| K | $Tp_1$ | $TP_2$ | $T_z$ | TK1 |
|---|---|---|---|---|
| 1,9829 | 55,079 | 25,248 | 8,1294 | 88,41 |
| 1,9955 | 74,975 | 11,028 | 2,8404 | 87,48 |
| 2,0005 | 87,785 | 4,3547 | -2,2075 | 84,48 |
| 1,9754 | 76,355 | 11,111 | 2,312 | 83,69 |
| 2,0024 | 93,055 | 0,16539 | -5,8687 | 81,84 |

Figure 2 shows the frequency response of the family of plants identified for the six operating conditions shown in Table 1. It shows that operating conditions variations affects the system dynamic at high frequencies. This variations in the system dynamics against operating conditions are modeled as parametric uncertainness of the model [8]. Nominal plant is calculated from the mean of parameters given by (1) for the 30 identified plants which is defined by (2) and is represented in Figure 2 by (*).

$$P_n(s) = 1.28 \frac{1 + 0.25s}{(1 + 80s)(1 + 0.13s)} \tag{2}$$
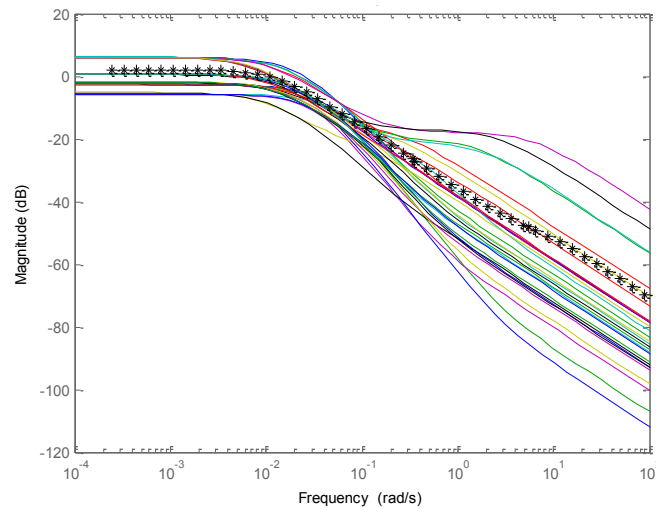


**Figure 2**. Frequency response of the family of plants and nominal plant.

## 5. Multiplicative uncertain modelling

From the family of plants, the multiplicative uncertain is modelled for each member of the family, which is defined by (3) and described in [9].

$$|\delta P_i(s)| = \left| \frac{\tilde{P}_i(s)}{P_n(s)} - 1 \right| \tag{3}$$

where $P_n(s)$ correspond to the nominal plant described by (2) and $\widetilde{P}_i(s)$ to the $i$-th plant of the family of plants. Frequency response for the multiplicative uncertain of the family of plants is presented in Figure 3.
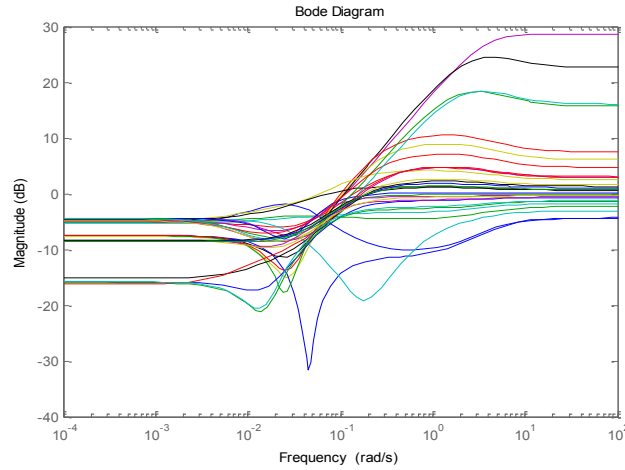


**Figure 3.** Multiplicative uncertain of the family of plants

Finally, the maximum multiplicative uncertain function is estimated according to the limit behaviors in low and high frequency giving the uncertain profile described by (4) and shown in Figure 4 represented by (*).

$$\delta_{pmax}(s) = 0.605 \frac{(\frac{s}{0.0055} + 1)(\frac{s}{0.105} + 1)}{(\frac{s}{0.01} + 1)(\frac{s}{3} + 1)} \tag{4}$$
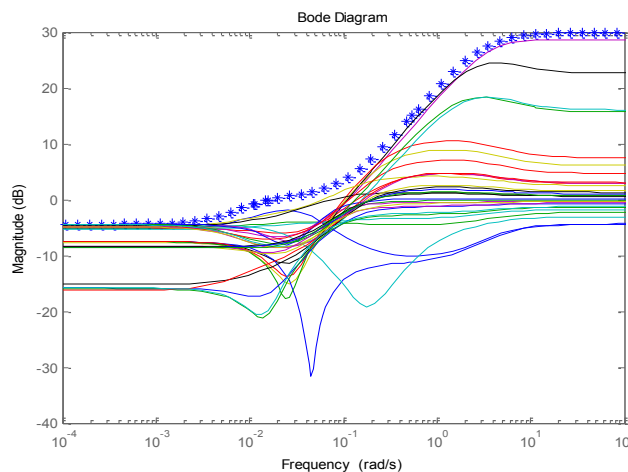


**Figure 4** Maximum multiplicative uncertain profile

## 6. Robust operation conditions

From (4), closed operating conditions to achieve robust stability and performance are calculated according to (5).

$$|T(s)| \leq \left| \frac{1}{\delta P_{max}(s)} \right| \tag{5}$$

where $|T(s)|$ is the magnitude of the complementary sensibility function in closed loop and $\left|1/_{\delta P_{max}(s)}\right|$ belongs to the magnitude inverse of the maximum uncertain profile of the family of plants given by (4). Closed loop system specifications are limited by the bandwidth $\beta\omega$ given by (5). Operating specifications comes from the desired characteristic polynomial defined by (6). The values for $\zeta$ and $w_n$ are calculated from $\beta\omega \cong 0.64w_n$ for $\zeta = 0.7$ or $\beta\omega \cong w_n$ for $\zeta = 1$ [10]. As show in Figure 5, for the flow and level system the bandwidth correspond to $\beta\omega = 0.01$ rad/s, which guarantee that feedback system has a robust stability and performance [9].

$$s^2 + 2\zeta w_n s + w_n{}^2 \qquad\qquad (6)$$
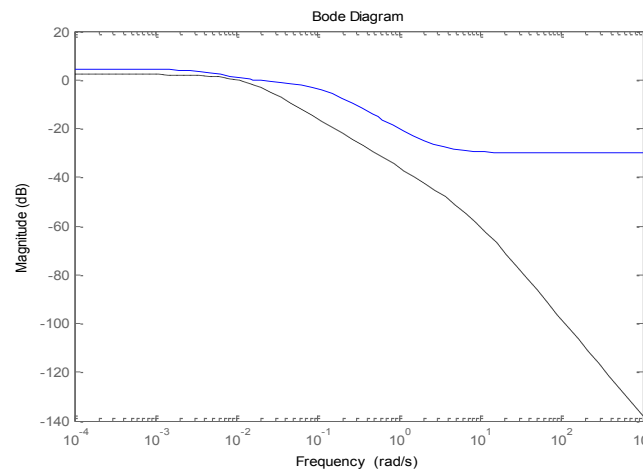


**Figure 5** Bandwidth to achieve the conditions of robust stability and performance.

## 7. Conclusions

This paper presented a methodology for parametric uncertain experimental identification that affects the dynamics of a flow and level system. The family of plants was identified for different operating conditions of the system and the maximum multiplicative uncertain was calculated. Frequency response of the inverse maximum multiplicative uncertain allows to find the maximum system bandwidth to define the closed loop operation conditions of the system that met the condition of robust stability and performance. The obtained results show the dynamic behavior changes of the system when external perturbation appears. Thus, applying the proposed methodology is possible to modeling this changes through a multiplicative uncertain function to define the desired robust performance conditions in the controller design stage.

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