Prediction of thermal strains in fibre reinforced plastic matrix by discretisation of the temperature exposure history

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Abstract. Prediction of environmental effects on fibre reinforced plastics habitually is made difficult due to the complex variability of the natural service environment. This paper suggests a method to predict thermal strain distribution over the material lifetime by discretisation of the exposure history. Laboratory results show a high correlation between predicted and experimentally measured strain distribution

1. Introduction

Due to the need for energy efficient materials, there has been a considerable increase of interest in light and green materials such as fibre reinforced plastics (FRP) in the aerospace industry. However, environmental effects have shown to be a hindrance to extensive use of FRP. A reliable method to quantify and predict environmental effects on FRP is required for safe and efficient use of these materials.

Stresses resulting from environmental factors such as cycling temperature and humidity have been recognised as one of the most deleterious effect on FRP materials [1]. Hygrothermal stresses are known to be the cause for crack initiation and cause the material to become brittle or to deform [2, 3]. They can approach or exceed the design load and acceptable material limits [4].

A number of works on the modelling of environmental stress effect on FRP material have been reported [5-12]. These reports deal mostly with the characterisation of the effect of temperature and humidity on the stress state of composite laminates. In this regard, models and computation algorithms of various performance levels have been made available. However, most of the time complication results from the material heterogeneity and the complex variability of the environment. In addition, only the effect of the physical degradation is taken into account explicitly, while the material undergoes a chemical degradation also.

This analysis focuses on the determination of environmental stresses without involving the resulting modification on the stress state of the material. Attention is on the prediction of a long-term effect, comprehensively taking into account the physical and the chemical degradation, as well as the viscoelastic, time-dependent effect. In the present paper, the thermal strain history is predicted based on the knowledge of temperature exposure. In a subsequent paper, strains will be used to compute stresses taking into account the time dependent relaxation and environmental degradation.

For the purpose of this paper, only a one-dimensional problem in plate laminate is considered. This is because it is demonstrated experimentally that the degradation of the mechanical strength of materials used in aviation varies in a layered way after exposure to a natural environment [13]. The degradation process can be considered as a progressing damaged front that moves from the outer

surface through the laminate [14]. A gradient is generated through the laminate thickness and because of such a gradient effects on mechanical strength depend strongly on the laminate thickness [13, 15].

The suggested method is based on the environmental resistance of the matrix. This is because in practical applications, the environmental resistance of a laminate is determined by that of its barrier coat, made of a resin rich layer. It is also observed, in general, that the degradation mechanism of FRP initiates in the matrix and primarily affects the matrix dominated properties [14, 16-17]. Therefore, the investigation focuses on the matrix as the latter appears to be the most sensitive part of the composite laminate structure.

In order to resolve its complexity, the natural environment is discretised into a succession of constant environmental states over time. The time intervals of a constant environment can be minimised sufficiently in such a way as to obtain a discrete pattern as illustrated in Figure 1.
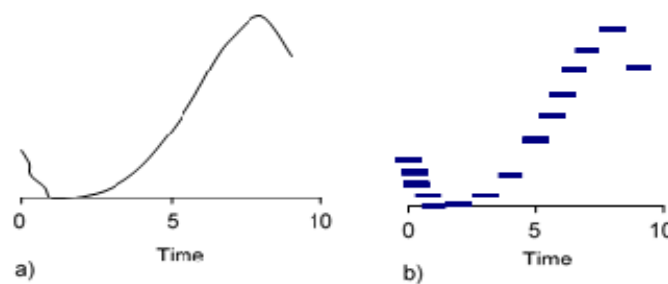

Figure 1. Schematic representation of a transformation from a continuous variable environment (a) to a succession of constant environments (b).

2. Mathematical Formulation

2.1. Assumptions
From all the above the following assumptions are introduced:
- The material is homogeneous and isotropic. Thermal strains result only from through the thickness gradient of the temperature.
- The environment to be considered is constant in function of time.
- The material structure is a plate laminate.

2.2. Thermal strains
Figure 2 represents a plate laminate of thickness l exposed to environmental temperature $f(x,t)$, which is a function of time and the material thickness.
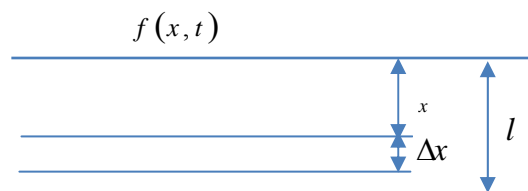

Figure 2. Schematic representation of a plate laminate.

At a given distance x from the material surface and fixed time t_i, the gradient of environmental temperature can be expressed as

$$grad f(x,t_i) = \frac{\partial f(x,t_i)}{\partial x} dx \qquad (1)$$

Then, based on thermal expansion laws, the strain between two layers respectively situated at distances x and $x + \Delta x$ in the material can be evaluated as

$$\Delta\varepsilon(x,t_i) = \lambda \int\limits_{x}^{x+\Delta x} \frac{\partial f(x,t_i)}{\partial x} dx \tag{2}$$

where, ε and λ represent the strain and the material coefficient of thermal expansion respectively.

In equation (2), the determination of strain is straightforward if the temperature field inside the material is known. However, Fourier's law does not always allow for the handling of the complex variability of the external environment, frequently resulting in the solution for complex variation not being available. The method using constant environment, hereby, is introduced to determine thermal strain distribution over time.

2.3. Resultant strain at time t

Considering the succession of constant environmental states as in Figure 1b, the strain history in the laminate is given by a succession of strains such as the one represented in equation (2). Then, based on the Boltzmann superposition principle [18], the resultant strain over a period of time t, comprising a finite number of constant environmental states, can be estimated as:

$$\varepsilon(x,t) = \sum_{i=0}^{n} J(t-t_i)\Delta\sigma(x,t_i) \tag{3a}$$

or

$$\varepsilon(x,t) = \sum_{i=0}^{n} \Delta\varepsilon(x,t_i) \qquad \forall t_i \prec t \tag{3b}$$

In equation (3a), σ and $J(t)$ respectively represent the stress and the material compliance.
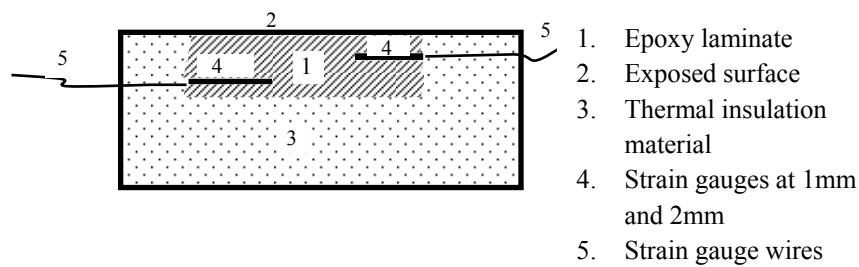
3. Experimental Method

Strain gauges installed inside the material at specific positions measure strains that develop in the material. The strain gauges used are made of constantan grid with one direction configuration, embedded in a phenolic resin. An epoxy adhesive is used to stick the strain gauge to the testing material. The outside temperature is recorded and used to compute and predict internal strains. Then the computed strains are compared to the measured strains.

3.1. Experimental material and test specimens

A 100% epoxy resin laminate is used. The formulation is shown in Table 1.
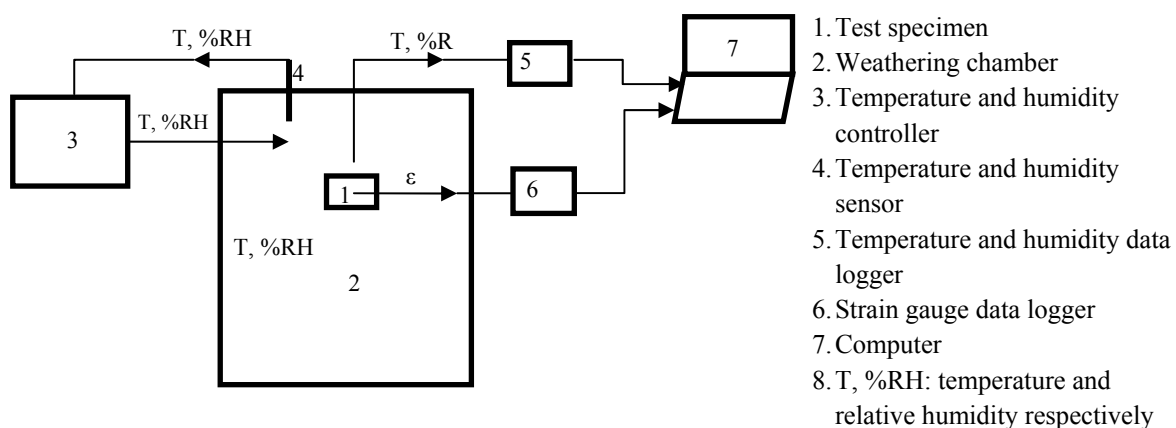
Table 1. Experimental material.

Epoxy resin: Ampreg 21	
Resin phr	100
Hardener	33
Curing cycle	48h/ 18°C

1. Epoxy laminate
2. Exposed surface
3. Thermal insulation material
4. Strain gauges at 1mm and 2mm
5. Strain gauge wires

Figure 3. Test specimen.

The test specimen is constructed in such a way as to obtain exposure on one face only, and strain gauges are enclosed at specific distances (1 mm and 2 mm) of the external surface (Figure 3).
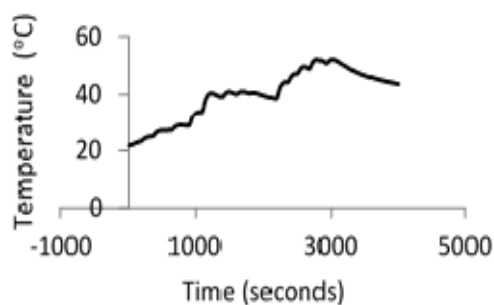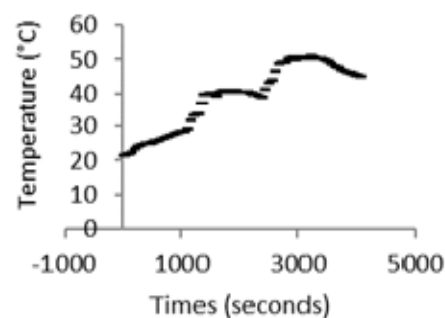
3.2. Experimental setup
The experimental set up is shown in Figure 4 here below.

1. Test specimen
2. Weathering chamber
3. Temperature and humidity controller
4. Temperature and humidity sensor
5. Temperature and humidity data logger
6. Strain gauge data logger
7. Computer
8. T, %RH: temperature and relative humidity respectively

Figure 4. Experimental set up.

3.3. Experimental procedure and results

3.3.1. A set of three material samples was exposed in the weather simulator following a predetermined temperature and humidity pattern. The chamber temperature was recorded with a specialised data logger and the exposure duration was set at 3800 seconds. Figure 5 shows the exposure pattern. The temperature distribution is than discretised in a 10-second constant temperature interval, as shown in Figure 6.

Figure 5. Temperature pattern.

Figure 6. Temperature discretization.

3.3.2. Internal free strains at 1 mm and 2 mm distances from the external surface of the laminate were measured using strain gauges and recorded with the strain data logger. Figure 7 presents the results.
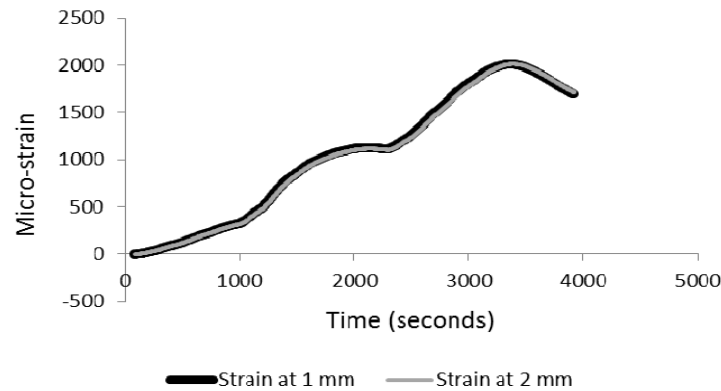

Figure 7. Measured free strain inside the laminate.

3.3.3. The temperature field at 1 mm and 2 mm distances were computed for the successive constant environmental temperature using Fourier's law. The specimens were considered to have semi-infinite, solid structure, for which an analytical solution is provided; see equation (4) where T_s and T_i respectively represent the constant temperature at the exposed surface and the initial temperature inside the material [19]. Symbols ρ, C, and k respectively, stand for the material density, specific heat capacity, and coefficient of thermal conductivity. The computed temperature profile is shown in Figure 8.

$$T(x,t) = (T_i - T_s)erf\left(\frac{x}{2}\sqrt{\frac{\rho c}{kt}}\right) + T_s \tag{4}$$
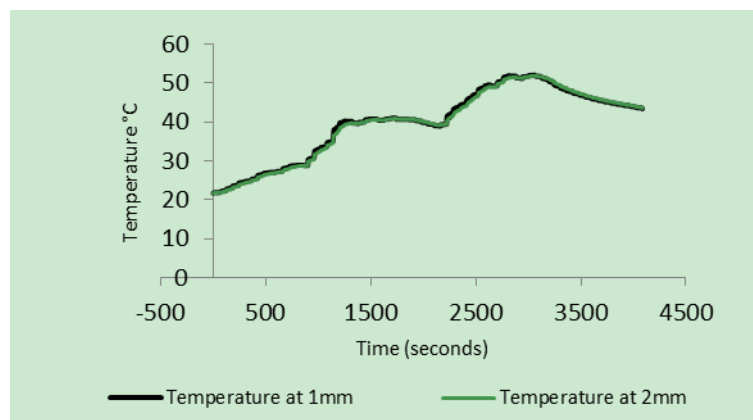

Figure 8. Computed internal temperature.

3.3.4. The temperature gradient between 1 mm and 2 mm distances from the outer surface was determined as per equation (1) and strains were computed as per equation (2). To this end, the material characteristics that were used are shown in Table 2. The exact value of the coefficient of thermal expansion for the material used in this experiment could not be found. However, a couple of values for the coefficient of thermal expansion of some epoxy resins were collected from various sources and averaged to provide the value displayed in Table 2. The use of that value is discussed in Section 4.

Table 2. Material thermal properties.

Epoxy resin Ampreg 21	
Density [Kg/m^3]	1100
Specific heat capacity [J/Kg. °K]	1670
Coefficient of thermal conductivity [W/m.°K]	0.17
Coefficient of thermal expansion [10^{-6}m/m]	30

Computation results are shown in Figure 9. Two different time intervals where used for the discretisation of the exposure history. At first, a 60-second time interval was used, and then the time interval was reduced to 10 seconds in order to improve the precision.
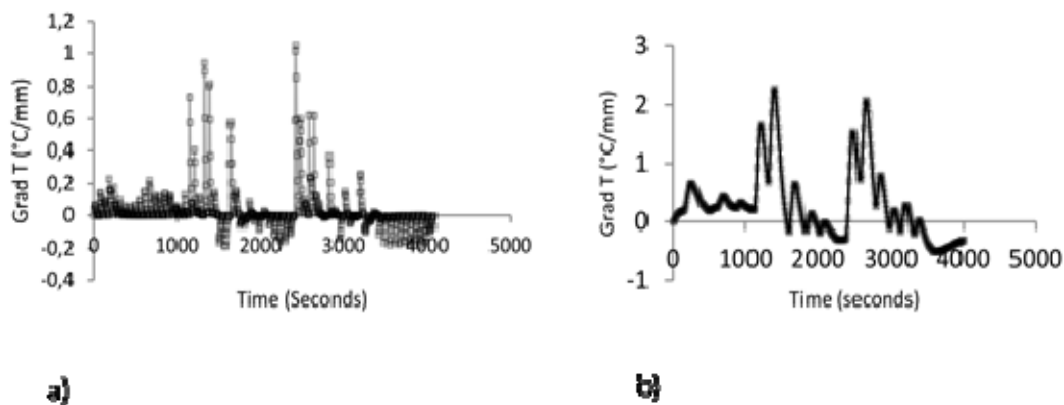

Figure 9. Distribution of temperature gradients computed a) per 60 seconds time interval, and b) per 10 seconds time interval.

3.3.5. Computed strains were then compared graphically to experimentally measured strains. The latter were obtained at each instant by differences in free strain of the material layers at 1 mm and 2 mm from the exposed surface; a comparison is shown in Figure 10.
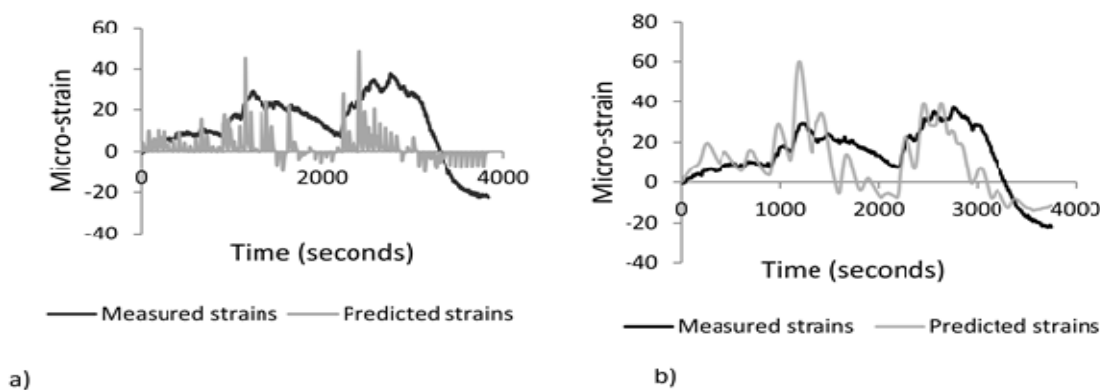

Figure 10. Measured strains compared to strains computed a) per 60 seconds time interval, and b) per 10 seconds time interval.

4. Discussion

The predicted strain history shows two major positive peaks, the first between 1000 and 1500 seconds and the second between 2000 and 2800 seconds. A negative peak is shown between 3100 and 3500 seconds. The positive peaks correspond to tension strain and the negative peak corresponds to a compression of the material. The pattern of predicted strain history consistently reflects what was experimentally recorded (Figure 10). However, as previously mentioned, the exact value of the

coefficient of thermal expansion was not found. There is uncertainty on the exact values of these peaks. Nevertheless, it can be seen from equation (2) that the computed strains should be linearly correlated to the real strains and the correlation curve should pass through the axe origin. Hence, resultant strains in function of time were computed as per the Boltzmann superposition principle for each loading period (tension and compression) and the linear correlation was assessed between the predicted strains and the measured strains. The results of the linear correlation assessment are presented in Table 3.

It is shown that for the 60-second time interval, the correlation is quite poor, while the reduction of the time interval to 10 seconds leads to a considerable improvement of the prediction. At the 10-second time interval, the correlation is excellent and the correlation equation shows that obtained values of peaks need to be corrected by a multiplicative factor of 1.4. Therefore, it can be concluded that the predicted strain history excellently reflects the real strain history. However, considerable differences can be observed right after tension periods. These differences are recorded more or less from 1500 to 2200 seconds and between 2800 to 3300 seconds. These periods correspond to tension release and it is shown that the recovery of measured strains lags behind, while the recovery for predicted strains is relatively fast. In reality, the computed strain history is reflective of the thermal loading only while the measured strain includes the material response. The observed differences show the material response to the release of the thermal load. The observed delayed response is typical of viscoelastic material.

Table 3. Correlation between measured strains and predicted strains.

Interval	60 sec time interval		10 sec time interval	
	R^2	Correlation equation	R^2	Correlation equation
0 to 1500 sec	0.920	0.3λ	0.981	1.5λ
2200 to 2800 sec	0.561	0.3λ	0.975	0.9λ
3300 3500 sec	0.120	0.2λ	0.991	1.7λ
Average		0.3λ		1.4λ

5. Conclusion

The analysis has demonstrated a possible method to predict the thermal strain history in a laminate matrix. It indicated that a variable temperature environment could be discretised in a succession of fixed-time intervals of constant temperature. Each constant temperature environment is treated individually as an incremental thermal load, to calculate the temperature field inside the laminate, based on Fourier's law for heat transmission. This leads to the determination of the internal strain distribution, over time, by using the law of thermal expansion. The analysis has shown sufficiently reducing the time interval of constant temperature leads to an acceptable prediction that excellently reflects the real thermal strain history.

References
[1]   Sachin Kishrsagar, Roberto A, Lopez-Anido and Gupta R K 2000 ACI Mater. J., title 97 M80, p 703
[2]   Mills N J 1993 *Environmental Effects*. In: 2nd ed. Plastics, Microstructure and Engineering Applications. (Edward Arnold, UK) p 228
[3]   Adam D S, Bowles D E and Herakovich C T 1988 *Thermally Induced Cracking in Graphite-Epoxy Cross-ply Laminate*. In: GS Springer ed. Environmental Effect on Composite Materials, vol.3. (Technomic, Westport CT)
[4]   Stone M A, Schwartz I F and Chandler H D 1997 Compos. Sci. Technol. 57, p 47

[5]   Benkheda A., Tounsi A., and Adda bedia E.A. 2008 Compos. Struct. 82, p 629

[6]   Gigliotti M, Jacquemin F, Moliard J, and Vautrin A, 2007 Macromol. Symp. 247, p 199

[7]   Pavankiran Vaddadi, Toshio Nakamura, and Raman P Singh, 2003 Comp. Part A 34, p 719

[8]   Upadhyay P C, Wayne W S, Prucz J, and Lyons D W 2009 J. Reinf. Plast. Compos.

[9]   Springer G S 1988 *Environmental Effects*. In: G S Springer ed. Environmental Effect on Composite Materials, vol. 3. (Technomic, Westport CT)

[10]   Guglieno Carra and Valter Carvelli 2014 Compos. Struct. 108, p 1019

[11]   Guzman E, Cugnoni J, and Gmur T 2014 Compos. Struct. 111, p 179

[12]   Silva M A G, Sena Da Fonseca B, Hugo Biscaia 2014 Compos. Struct. 116, p 377

[13]   Startsev O V, Krotov A S, and Startseva L T 1999 Polym. Degrad. Stab. 63, p 183

[14]   Sevostianov I, Sookay N K, Von Klemperer C J, and Verijenko V E 2003 Compos. Struct. 62, p 417

[15]   Larson F 1988 *The Effect of Ultraviolet Light on Mechanical Properties of Kevlar 49 Composites.* In: GS Springer ed. Environmental Effect on Composite Materials, vol. 3. (Technomic, Westport CT) p 132

[16]   Kumar B G, Singh R P, and Nakamura T 2002 J. Compos. Mater. 36(24), p 2713

[17]   Nakamura T, Singh R P, Vaddadi P 2006 Exp. Mech. 36, p 257

[18]   McCrum N G, Buckley C P, and Bucknal C B 1997 *Principles of Polymer Engineering* (Oxford University Press, Oxford)

[19]   Kreith F, Bohn M S, 2001 *Principles of heat transfer, 6th ed.*, (Thomson Learning, Australia)