

Computation of macro-fiber composite integrated thin-walled smart structures

S Q Zhang^{1,3}, S Y Zhang¹, M Chen², J Bai¹ and J Li¹

¹ School of Mechanical Engineering, Northwestern Polytechnical University, 710072 Xi'an, China

² Department of Industrial Design, Xi'an Jiaotong - Liverpool University, 215123 Suzhou, China

E-mail: sqzhang@nwpu.edu.cn

Abstract. Due to high flexibility, reliability, and strong actuation forces, piezo fiber based composite smart material, macro-fiber composite (MFC), is increasingly applied in various fields for vibration suppression, shape control, and health monitoring. The complexity arrangement of MFC materials makes them difficult in numerical simulations. This paper develops a linear electro-mechanically coupled finite element (FE) model for composite laminated thin-walled smart structures bonded with MFC patches considering arbitrary piezo fiber orientation. Two types of MFCs are considered, namely, MFC-d31 in which the d_{31} effect dominates the actuation forces, and MFC-d33 which mainly uses the d_{33} effect. The proposed FE model is validated by static analysis of an MFC bonded smart plate.

1. Introduction

Thin-walled metal structures integrated with smart materials, like piezoelectric, magnetostrictive, and shape memory alloys, called smart structures, have been attracted by many researchers and engineers in various fields for vibration suppression, acoustic control, and health monitoring. Due to high flexibility, reliability, and high actuation forces, macro-fiber composite (MFC) materials are more attractive in the applications of smart structures instead of piezoceramics and piezopolymers. MFC was first invented by NASA Langley Research Center [1] and now is produced by Smart Material Corporation [2]. The MFC material has very complex structural arrangement integrated with many different materials. MFC patches have five major layers with an active layer sandwiched between layers of adhesive, electrodes, and polyimide film. The active layer is manufactured by rectangular piezoceramic rods embedded in an epoxy matrix with interdigitated electrodes. Due to different polarization of piezo rods, there have two different types of MFCs, MFC-d31 and MFC-d33. For the patches of MFC-d31, the d_{31} effect dominates the actuation forces, while MFC-d33 patches mainly use the d_{33} effect.

The complexity of MFC materials increases the difficulty in modeling and computation of structures integrated with those materials. After the literature survey, it can be found that very few publications are available concerning the numerical analysis of MFC bonded smart structures. However, most of them were analyzed by commercial software, e.g. ANSYS [3, 4], ABAQUS [5, 6], with modified constitutive equations and using equivalent actuation forces. Beyond using the commercial software, Park and Kim [7], Bilgen et al. [8] developed analytical models respectively for MFC bonded active twist rotor blades and benders. Furthermore, based on the first-order shear



deformation (FOSD) hypothesis, Azzouz and Hall [9] developed a von Kármán type nonlinear finite element model for frequency analysis of a rotating MFC actuator.

Based on the author's previous work [10], this paper presents a computational FE model using the FOSD hypothesis for MFC bonded structures with considering various piezo-fiber orientations. Using the proposed FE model, a thin plate integrated with either MFC-d31 or MFC-d33 patches is analyzed and compared.

2. Mathematical models

As mentioned before, there are two types of MFCs, in which MFC-d31 has the polarization in the thickness direction and MFC-d33 has the polarization parallel to the piezo fiber direction, as shown in Figure 1.

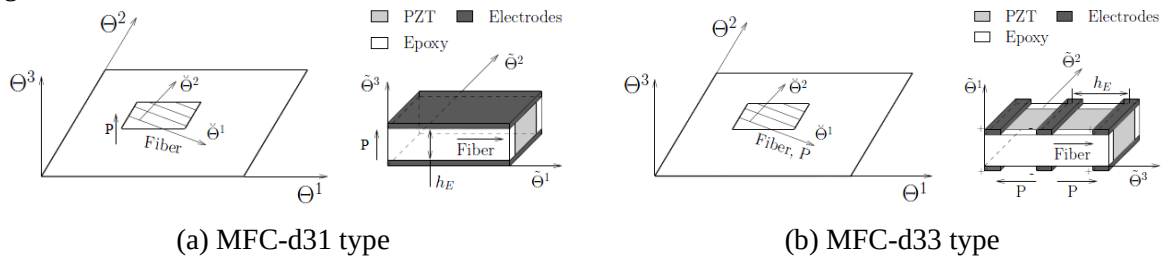


Figure 1. Two types of MFC.

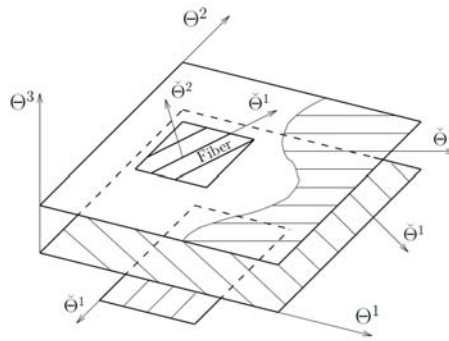


Figure 2. Multi-layer laminated composite structures with MFCs.

Using the homogenization method presented in [11, 12], MFC can be finalized as orthotropic materials including both elastic fiber and piezo fiber angles. Considering multi-layer laminated structures, as Figure 2 shown, the constitutive equations in the fiber coordinate system for each layer can be expressed as [10]

$$\tilde{\sigma} = \tilde{c}\tilde{\varepsilon} - \tilde{e}^T \tilde{E} \quad (1)$$

$$\tilde{D} = \tilde{e}\tilde{\varepsilon} + \tilde{\chi} \tilde{E} \quad (2)$$

Here $\tilde{\sigma}$, $\tilde{\varepsilon}$, $\tilde{E}$ and $\tilde{D}$ are respectively the stress, strain, electric field, and electric displacement vectors, the overhead $\tilde{}$ represents the quantities in the fiber coordinate system. Furthermore, $\tilde{c}$, $\tilde{e}$ and $\tilde{\chi}$ denote the elastic constant matrix, the piezoelectric constant matrix, and the dielectric constant matrix. The components in the piezoelectric constant matrix can be obtained by

$$\tilde{e}_{p1} = \tilde{d}_{p1} \frac{\tilde{Y}_1}{1 - \tilde{\nu}_{12}\tilde{\nu}_{21}} + \tilde{d}_{p2} \frac{\tilde{\nu}_{12}\tilde{Y}_2}{1 - \tilde{\nu}_{12}\tilde{\nu}_{21}} \quad (3)$$

$$\tilde{e}_{p2} = \tilde{d}_{p1} \frac{\tilde{\nu}_{12}\tilde{Y}_2}{1 - \tilde{\nu}_{12}\tilde{\nu}_{21}} + \tilde{d}_{p2} \frac{\tilde{Y}_2}{1 - \tilde{\nu}_{12}\tilde{\nu}_{21}} \quad (4)$$

where $\bar{Y}_1$ and $\bar{Y}_2$ are the Young's moduli, $\bar{\nu}_{12}$ and $\bar{\nu}_{21}$ are the Poisson's ratios. Furthermore, $\bar{d}_{p1}$ and $\bar{d}_{p2}$ are the piezoelectric coefficients, in which $p=1$ is for MFC-d33, $p=3$ is for MFC-d31.

In order to compute the elastic potential energy of each layer with various fiber orientations, the constitutive equations must be transformed to the structural coordinate system as [10]

$$\boldsymbol{\sigma} = \mathbf{c}\boldsymbol{\varepsilon} - \mathbf{e}^T \mathbf{E} \quad (5)$$

$$\mathbf{D} = \mathbf{e}\boldsymbol{\varepsilon} + \boldsymbol{\chi} \mathbf{E} \quad (6)$$

with

$$\mathbf{c} = \mathbf{T}^T \bar{\mathbf{c}} \mathbf{T}, \quad \mathbf{e} = \bar{\mathbf{e}} \mathbf{T}, \quad \boldsymbol{\chi} = \bar{\boldsymbol{\chi}} \quad (7)$$

Due to the assumption of no thickness change after deformation, the stress, strain, electric field, and electric displacement vectors are obtained as

$$\boldsymbol{\sigma} = \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \\ \sigma_{23} \\ \sigma_{13} \end{Bmatrix}, \quad \boldsymbol{\varepsilon} = \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ 2\varepsilon_{12} \\ 2\varepsilon_{23} \\ 2\varepsilon_{13} \end{Bmatrix}, \quad \mathbf{E} = \begin{Bmatrix} E_p^{(1)} \\ E_p^{(2)} \\ \vdots \\ E_p^{(N)} \end{Bmatrix}, \quad \mathbf{D} = \begin{Bmatrix} D_p^{(1)} \\ D_p^{(2)} \\ \vdots \\ D_p^{(N)} \end{Bmatrix} \quad (8)$$

Assuming that an electric voltage is applied on two neighboring electrodes in the direction of polarization, the piezoelectric constant and the dielectric constant matrices are respectively expressed as [10]

$$\mathbf{e} = \begin{bmatrix} e_{p1}^{(1)} & e_{p2}^{(1)} & 0 & 0 & 0 \\ e_{p1}^{(2)} & e_{p2}^{(2)} & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ e_{p1}^{(N)} & e_{p2}^{(N)} & 0 & 0 & 0 \end{bmatrix}, \quad \boldsymbol{\chi} = \begin{bmatrix} \chi_{pp}^{(1)} & 0 & \cdots & 0 \\ & \chi_{pp}^{(2)} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \chi_{pp}^{(N)} \end{bmatrix} \quad (9)$$

Here superscript N stands for the number of layers.

Using the principle of virtual work one obtains the static equilibrium equations

$$\mathbf{K}_{uu} \mathbf{q} + \mathbf{K}_{u\phi} \boldsymbol{\phi}_a = \mathbf{F}_{ue} \quad (10)$$

in which $\mathbf{K}_{uu}$ and $\mathbf{K}_{u\phi}$ are the elastic stiffness and the piezoelectric coupled stiffness matrices, respectively, $\mathbf{q}$, $\mathbf{F}_{ue}$, and $\boldsymbol{\phi}_a$ represent the nodal displacement vector, the external force vector, and the actuation voltage vector. More details are referred to [10].

3. Numerical simulations

First, the present FE model is tested by a clamped plate bonded with MFC patches proposed by Bowen et al. [4]. The MFC plate is comprised of one aluminum layer in the middle as the host structure and two MFC-d33 (or MFC-d31) patches respectively bonded on the top and bottom surfaces acting as actuators, as shown in Figure 3. The dimensions of the aluminum plate are $300 \times 75 \times 2 \text{ mm}^3$ and those of MFC patches (only active part) are $85 \times 57 \times 0.3 \text{ mm}^3$. The material constants of the aluminum plate, MFC-d31, and MFC-d33 are shown in Table 1.

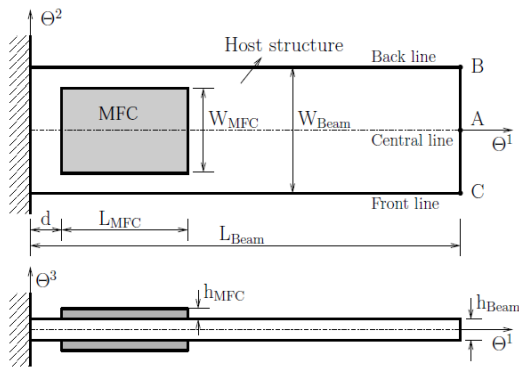


Figure 3. Clamped plate integrated with two MFC patches.

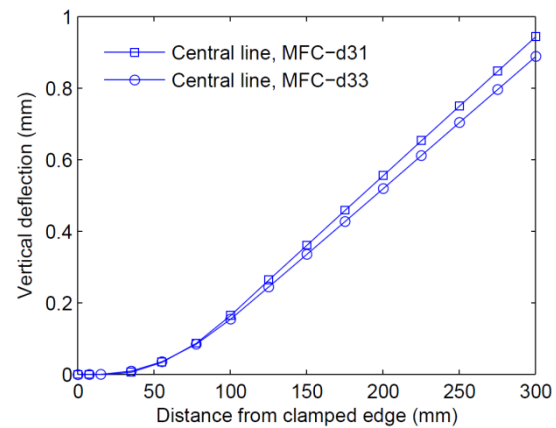


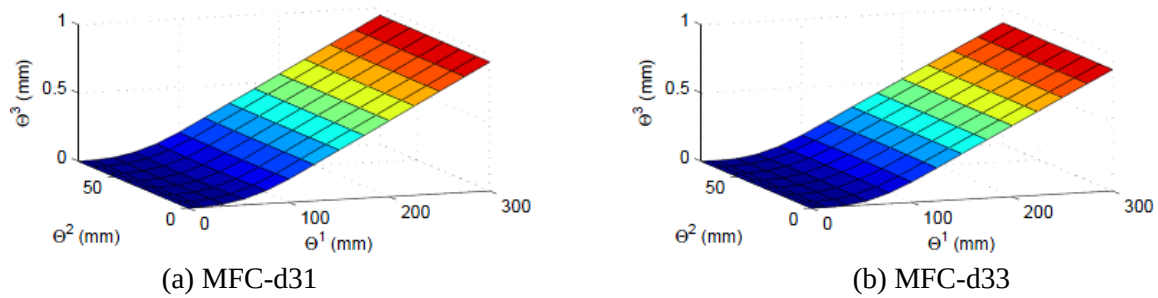
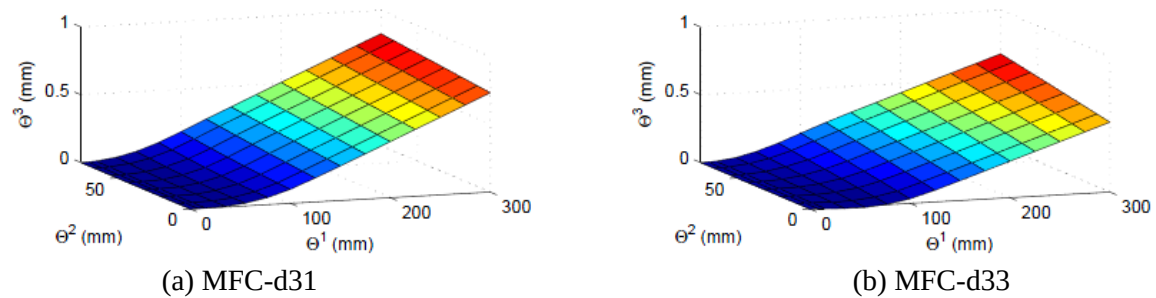
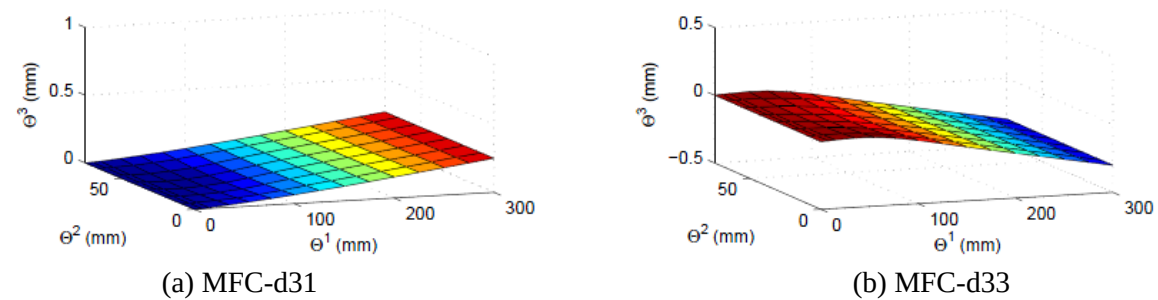
Figure 4. Central line displacements for MFC-d31 and MFCd33 bonded plates.

Table 1. Material Constants of aluminum and MFCs [2, 10].

Aluminum	MFC-d33	MFC-d31
$\bar{Y}=70$	$\bar{Y}_1 = 29.4$ GPa	$\bar{Y}_1 = 30.336$ GPa
$\bar{\nu} = 0.3$	$\bar{Y}_2 = \bar{Y}_3 = 15.2$ GPa	$\bar{Y}_2 = \bar{Y}_3 = 15.857$ GPa
	$\bar{G}_{12} = \bar{G}_{13} = 6.06$ GPa	$\bar{G}_{12} = \bar{G}_{13} = 5.515$ GPa
	$\bar{G}_{23} = 5.79$ GPa	$\bar{G}_{23} = 5.515$ GPa
	$\bar{\nu}_{12} = \bar{\nu}_{13} = 0.312$	$\bar{\nu}_{12} = \bar{\nu}_{13} = 0.31$
	$\bar{\nu}_{23} = 0.312$	$\bar{\nu}_{23} = 0.438$
	$\bar{d}_{11} = 467 \times 10^{-12}$ m/V	$\bar{d}_{31} = -170 \times 10^{-12}$ m/V
	$\bar{d}_{12} = -210 \times 10^{-12}$ m/V	$\bar{d}_{32} = -100 \times 10^{-12}$ m/V
	$h_E = 0.5$ mm	$h_E = 0.18$ mm

An electric voltage of 400 V is applied on the top MFC patch for both cases of MFC-d31 and MFC-d33 bonded structures. The distances of two neighboring electrodes of MFC-d33 and MFC-d31 are respectively $h_E = 0.5$ mm and $h_E = 0.18$ mm, which leads to the electric fields along the polarization being equal to $E_1 = 400 / 0.5$ V/mm for MFC-d33 patches and $E_3 = 400 / 0.18$ V/mm for MFC-d31 patches. The vertical deflections at the central line of both cases are presented in Figure 4, where the piezo fiber angle is zero. From the figure, we can see that the central line deflection of the MFC-d31 bonded structure are quite close to those of the MFC-d33 bonded structure, even though the coefficients d_{11} and d_{31} are very different. This is because the elongation strains $\varepsilon_{11} = d_{11} E_1 = 3.736 \times 10^{-4}$ for MFC-33 and $\varepsilon_{11} = d_{31} E_3 = 3.778 \times 10^{-4}$ for MFC-31 are very close to each other, with the former one being slightly smaller than the latter one.

Under the same electric loading for the above two cases, considering MFC piezo fiber orientations of 0° , 30° , and 90° , results in different deformed shapes of the MFC plate, as can be seen in Figures 5-7. The results first show that the deformation of the MFC-d31 bonded plate does not change the sign, while the deformation of the MFC-d33 bonded plate changes the sign. This can be explained by the fact that the piezoelectric constants d_{31} and d_{32} of MFC-d31 have the same sign. Obviously, we can see that the twists of the MFC-d33 bonded plate are much larger than that of the MFCd31 bonded plate, due to the change of piezo fiber angles.

**Figure 5.** MFC piezo fiber orientation 0°.**Figure 6.** MFC piezo fiber orientation 30°.**Figure 7.** MFC piezo fiber orientation 90°.

4. Conclusions

This paper has developed a linear piezoelectric coupled finite element model for MFC smart structures using a 2-dimensional FE method based on the FOSD hypothesis. An aluminum plate bonded with different MFCs is modeled and analyzed, namely MFC-d31 and MFC-d33. The present model has been applied to MFC-d31 or MFC-d33 patches bonded aluminum plates with different piezoelectric fiber orientations. The displacements and the deformation shapes have been simulated and compared with each other. The results showed that the MFC-d33 bonded plate has larger twist compared to the MFC-d31 plate. Furthermore, due to the different sign of $\tilde{d}_{11}$ and $\tilde{d}_{12}$ coefficients for MFC-d33, the aluminum plate bonded with MFC-d33 patches changes the sign of the displacements, while that with MFC-d31 does not.

Acknowledgements

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