

Tearing analysis of a new airship envelope material under uniaxial tensile load

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Abstract. This paper experimentally investigated the tearing properties of a new kind of coated woven fabrics, GQ-6, made of ultra-high molecular weight polyethylene fiber. Such material can be used for the envelope materials of a stratospheric airship. First, the uniaxial tearing tests were carried out. Effects of the stretching rate, the initial crack length, and the initial crack orientation on the material's tearing tensile strength were investigated. Experimental results showed that the initial crack length and the initial crack orientation can be represented by the equivalent initial crack length while the stretching rate has a slight influence on tearing behavior of the uniaxial tensile specimens. Then analytical studies using three methods, i.e. Griffith energy theory, the stress intensity factor theory, and Thiele's empirical theory, among which, the stress intensity factor theory gives the best correlation with the test data. Finally, a 48mm threshold of the equivalent initial crack length was recommended to the envelope material in operation.

1. Introduction

There has been an increasing interest in stratospheric airships as an alternative to traditional air vehicles for aerial exploiting and environmental monitoring. The envelope of an airship is one of the most important structural components, which is usually made of flexible membranes perhaps with the following properties [1]: high strength, high tear resistance, light weight, low creep, low permeability, reliable joints and resistance to environmental degradation due to temperature, humidity, and ultraviolet light factors, etc. The envelope fabrics of airships are prone to be punctured by internal components or external sharp objects in manufacturing. There is no doubt that the presence of cracks can lead to either the reduction of mechanical fabric performance or catastrophic failure. Therefore,, investigations on the tearing behavior of fabrics can provide guidance for its design and industrial applications.

In recent decades, there have been some efforts made [2-5] to investigate the tearing behaviors of woven fabrics by carrying out experimental studies. As an airship envelope material, central crack tensile tearing test is the foremost method which is specified in the Airship Design Criteria FAA P-8110-2, [6]. In general, the uniaxial, bi-axial, multi-axial tensile tearing tests and pressurized cylinder tests are the most commonly applied approaches. For instance, Minami et al.[4] carried out uniaxial and bi-axial tearing tests on specimens containing a single crack, a bias crack or a circular defect. It was concluded that the tearing strength of uniaxial specimens with initial crack oriented in the yarn direction was the lowest. Rigaud et al. [7] investigated the effects of the load ratio and the crack orientation on the tearing strength of polyester fabrics under mono-axial and bi-axial tensile tests. They discovered that crack propagation in coated fabrics was triggered by specific mechanisms linked



to the matrix softness. Meng et al. [8] studied the orientation influence on a certain stratospheric airship envelope subjected to a bi-axial load, then compared the experimental data with four empirical equations. They have concluded the various initial damages can be modeled as the effect of warp yarns. However, their conceptual design was lack of analyzing specimens with slit in the weft direction. Luo et al. [9] studied the mechanical behavior of a PVC coated warp fabric subjected to multi-axial load. It was found that the mechanical performance decreased with an increase of initial crack length for a given crack orientation, and the initial crack orientation perpendicular to a tensile direction had a maximal effect on the reduction of the mechanical performance. Topping et al. [10] performed an experimental investigation on crack growth in pressurized cylinders made of Dacron neoprene fabric. They employed the Griffith theory to predict the critical slit length, which had a good correlation with the experimental results. Maekawa et al. [11] carried out a bi-axial tensile test and a pressurized cylinder test to measure the tear propagation properties of the Zylon envelope material.

Additionally, a number of theoretical methods have been developed to analyze the mechanical properties of woven fabrics. Robert et al.[12] developed a probabilistic method to convert the distribution of applied failure load to the distribution of critical energy release rate. According to Minami's[4] test data, Timothy et al.[13] presented three fracture models to evaluate the failure load in coated fabrics containing cracks and the models coincided well with the experimental data. Evans et al. [14]improved the stress intensity factors which were determined by four key geometric configurations as well as uniaxial tearing tensile experimental data. Based on the uniaxial test data, Chen [15] derived a theoretical formula to evaluate the tearing strength of envelope materials with central cracks and the inclined orientation, etc.

Recently, the 46th Research Institute, 6th Academy of China Aerospace Science & Industry Corp. developed a new kind of woven fabrics, GQ-6, for stratospheric airship envelope. Such new textile is an ultra high molecular weight polyethylene fiber (UHMWPF). Its average thickness is approximately 264 μm and the specified areal density is 170 g/m^2 . The ultimate tensile stresses on the warp and weft direction are 100.4kN/m and 80.44kN/m, respectively. And the elastic modulus in the warp and weft direction are 3625.55N/mm and 3652.61 N/mm, respectively. However, the tearing tensile mechanical properties of such ripped fabrics are yet to be explored and understood.

This paper aims to examine the tearing properties of GQ-6 through uniaxial tearing tensile tests. First, the effects of the stretching rate, crack orientation, and initial crack length on crack propagation were discussed thoroughly. Then, three analytical methods were utilized to predict the crack propagation. The fidelity of the analytical results were assessed by the test data. Finally, the allowable equivalent initial crack length of the airship envelope material, GQ-6, was recommended.

2. Experimental Program

2.1. Specimens and test set-up

The ripped specimen for uniaxial tests was prepared by introducing a center crack in accordance with the standard of American airship design FAA-8110-2[6]. Such strip specimen is 101.6mm wide and 152.4mm long. All specimens were stretched aligned to their long axis. For instance, the warp specimen is the warp yarns vertical to loading direction, and the intersection angle between the initial crack and the shorter edge is defined as the initial crack angle, see figure 1(a). The uniaxial tearing test was carried out by using a universal testing machine, MINEBEA-CHA-20KN (figure 1(b)), made in Shanghai Taiyang membrane structure co., LTD. Displacement-controlled testing was carried out on the uniaxial specimens.

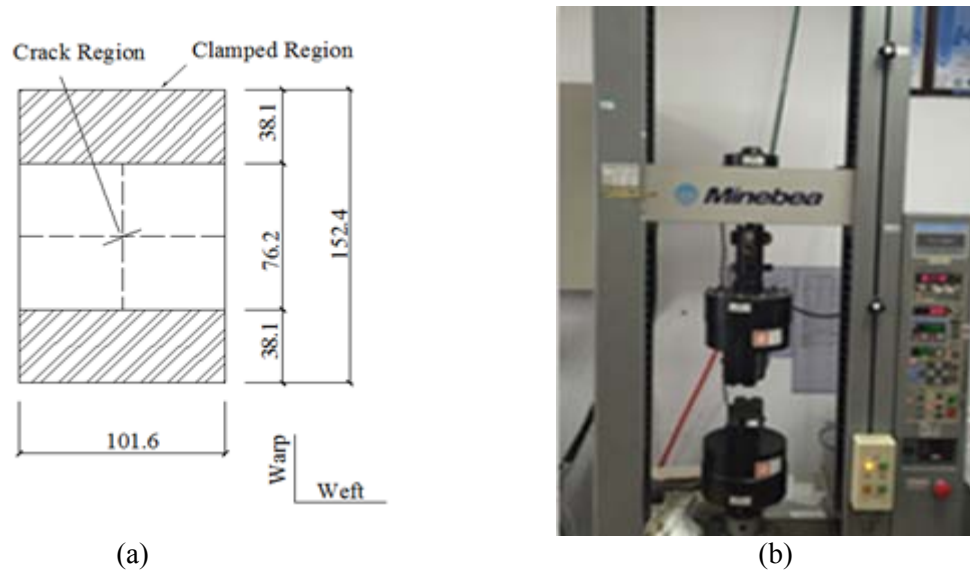


Figure 1. Test set-up. (a) schematic drawing of the warp specimen and (b) universal testing machine, MINEBEA-CHA-20KN.

2.2. Test procedure

Before the tearing specimens were installed on the testing machines, some preparations were made to facilitate the crack observation and the test data evaluation. All specimens were carefully made along the yarn directions. Orthogonal grid lines were so strictly drawn on the specimen surface that crack propagation can be identified easily. A specific crack was cut in a specimen according to the crack length and orientation determined by its test purpose.

In the uniaxial test, the variations of initial crack length, initial crack orientation and stretching rate were investigated. Hence three groups of specimens were used to reveal the effects of the three variables, respectively. For each group, three specimens were tested to examine their influence. During the test, a pre-tension of 5 N was first applied to the specimen. Then the load is increased at a specific loading rate as listed in table 1. The details of the crack length and crack orientations were summarized in table 1.

Table 1. Layout of the uniaxial experimental protocol.

Directions	Stretching rate (mm/min)	Initial crack length 2a (mm)	Crack orientation
Warp	3,5,10	10,20,40,50	0°, 45°, 60°
Weft			

3. Results and Discussions

3.1. Influence of Stretching rate

To aid the understanding of failure mode, a specified illustration on the crack initiation and propagation was offered first. Specifically, a tearing specimen was stretched slowly with the coating and the fabric extending together. Meanwhile, neither remarkable elongation nor crack expanding occurred. However, the initial crack then translated from an approximate line to ellipse form with tiny gap emerging on the edge of the initial crack. The yarns behaved as a nonlinear material, and a plastic

region on the crack adjacent area formed gradually. Such phenomenon indicated the second stage was coming. Then, as the elliptic crack increasing, one yarn on an arbitrary slit tip detached suddenly and adjacent yarns began to slip out alternately from the initial crack tips with the first detached yarn extracted. The above mentioned phenomena indicated the initial crack in the slit tip region was propagating. Finally, the extracted yarns appeared to be ruptured successively, and the specimen cannot sustain anymore load, then catastrophic failure came into being suddenly. The typical failure phenomena of specimens subjected to uniaxial tensile loading were shown in figure 2.

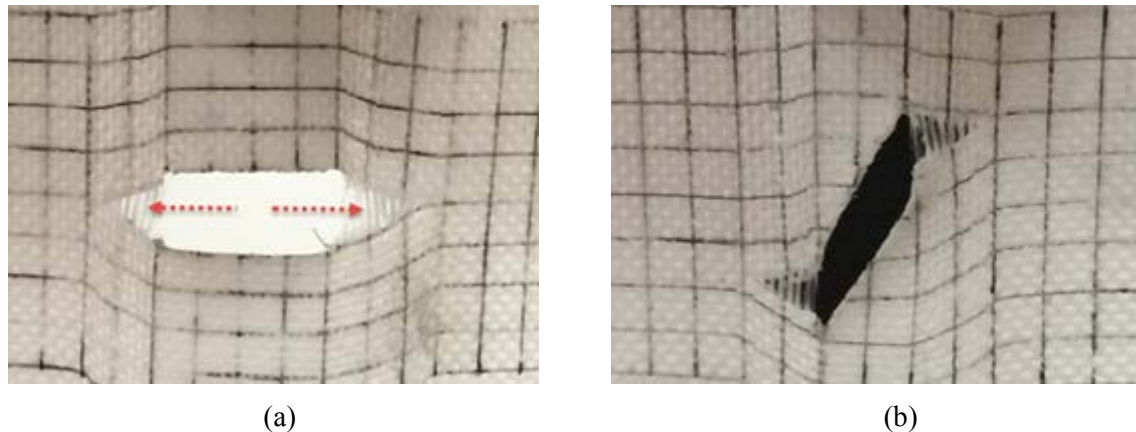


Figure 2. Typical failure modes for uniaxial tearing tensile specimens (20mm). (a) the transverse rupture and (b) the 'Z' shaped rupture.

From figure 2(a) and figure 2(b), it can be found all crack propagation of the uniaxial specimens were vertical to loading direction. Specifically, the specimens with in-axial initial cracks, which are the typical mode I crack, were ruptured without doubt by enlarging initial crack along its orientation. Such failure was defined as the transverse rupture. However, the initial cracks of the off-axial specimens, which were mixed between I and II crack modes, still propagated perpendicular to the loading direction. Such interesting phenomena were not similar to mental materials which could appear oblique crack or branching crack. That was because the application of the plain weave technology in the new textile. Consequently, the 'Z' shaped rupture was defined to the second failure mode.

In the uniaxial tearing tensile tests, three different stretching rates were carried out on the new textile to evaluate its sensitivity to such influence. First, prepare a set of specimens with 20mm initial crack length and oriented at 0° for an example. The corresponding force-elongation curves were revealed in figure 3. Then, to make a vivid illustration, the three maximum tearing tensile forces of specimens oriented at 45° were averaged and summarized in figure 4.

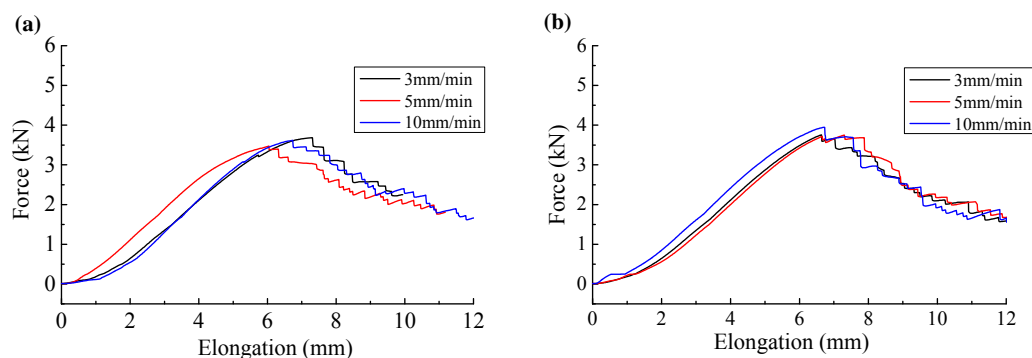


Figure 3. Stretching rate influence on uniaxial specimens. (20mm, 0°) (a) the warp specimens and (b) the weft specimens.

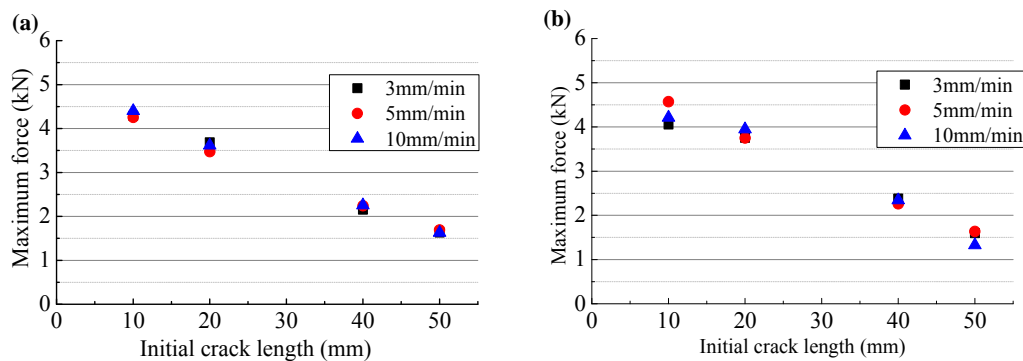


Figure 4. Stretching rate influence on maximum force of uniaxial specimens. (45°) (a) the warp specimens and (b) the weft specimens.

Figure 3 shows the rigid of each specimen was almost the same and increasing moderately. Meanwhile, few fluctuations were observed until the first yarn ruptured. Such phenomenon testifies that the inter-yarn friction of the new woven fabric could perform well on absorbing energy [18]. What is more, it is apparent that the stretching rate has no remarkable effect on the maximum tensile forces of specimens, which can be seen in figure 4. However, an interesting phenomenon is the maximum tensile forces of the warp and weft specimens are nearly identical for the same crack length with almost 20% discount on the weft direction for the intact specimens. That was due to the density of the warp and weft yarns almost the same for GQ-6. Therefore, the friction forces in the warp and weft tearing specimens are nearly identical. Meanwhile, the initial crack can break the weaving process exerting well on the woven fabric.

From the above analysis, consequently, the stretching rate has no remarkable effect on the new coated fabrics under uniaxial tension condition. Therefore, the stretching rate of 10mm/min was chosen arbitrarily at the following researches.

3.2. Influence of Initial crack length

To investigate the influence of the initial crack length on uniaxial specimens, the force-elongation curves corresponding to the warp, weft directions for different initial crack lengths oriented at 0° are shown in figure 5 as examples. All the maximum forces of specimens in warp and weft directions are concluded in figure 6. What should be noticed is the failure mode of uniaxial specimens with various initial crack lengths all ruptured in two modes. Generally, the in-axial specimens ruptured as the transverse mode, and the off-axial specimens ruptured as the “Z” shape mode.

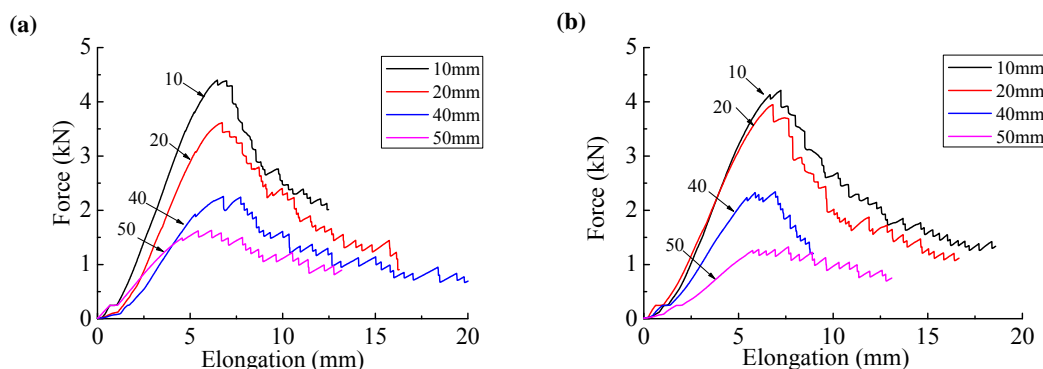


Figure 5. Initial crack length influence on uniaxial specimens (10mm/min, 0°). (a) the warp specimens and (b) the weft specimens.

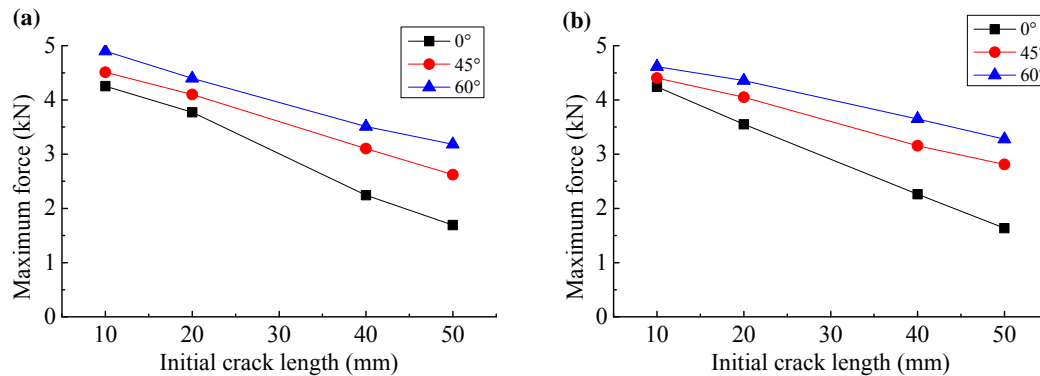


Figure 6. Maximum forces of uniaxial specimens (10mm/min). (a) Warp direction specimens and (b) Weft direction specimens.

From figure 5, it can be observed that the crack length has a significant influence on the new coated fabrics. In general, the maximum tensile force of the specimens decreased considerably with crack length increasing. A similar phenomenon has also been observed experimentally in Zylon airship envelope [11]. Slopes of the force-elongation curves were decreased while getting gentle with the initial crack length getting larger. That was due to fewer yarns left for a specimen with larger initial crack length. As for the maximum strain of such textile, around 4.2% to 5% was observed, which means tiny effect occurred on textile distortion for tearing specimens. From figure 6, the maximum tearing forces reduced rapidly with initial crack length increasing for specimens in various orientations, especially when the crack length was larger than 20mm. Therefore, we can conclude the larger initial crack can lead to less maximum tearing tensile force when subjected to uniaxial tension.

3.3. Influence of Crack orientation

In uniaxial tearing tensile tests, the specimens stretched at 10mm/min with 40mm initial crack length were taken as an example to show the crack orientation influence on mechanical properties of such textile. All failure modes of uniaxial specimens oriented at 0° were the transverse failure. However, the off-axial specimens, i.e., oriented at 45° or 60° were ruptured as “Z” shape failure mode. The force-elongation curves between the warp and the weft specimens are shown in figure 7. Meanwhile, all the maximum forces of specimens in warp and weft directions are plotted in figure 8, respectively.

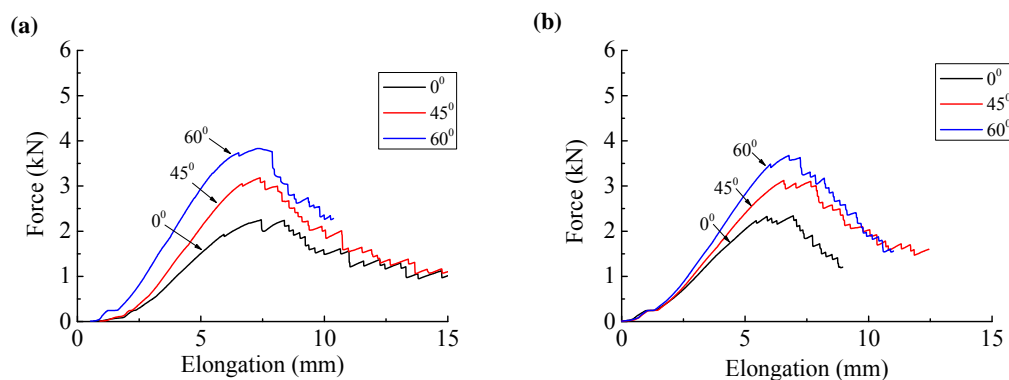


Figure 7. Crack orientation influence on uniaxial specimens (40mm, 10mm/min). (a) the warp specimens and (b) the weft specimens.

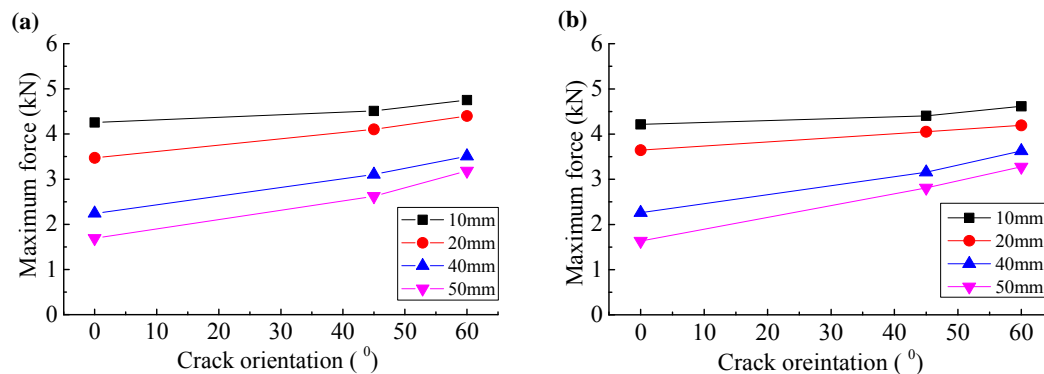


Figure 8. The maximum force of uniaxial specimens. (10mm/min) (a) the warp specimens and (b) the weft specimens.

In figure 7 and figure 8, it is obvious that the maximum tensile forces are increasing with the initial crack orientation getting larger. Theoretically, for a specific crack length, as the orientation getting larger, the number of cutting yarns in the projection region which were perpendicular to the loading direction got to be much smaller, i.e., there were more yarns sustaining stretching load, and the maximum force increased consequently. Meanwhile, it was generally known, for a specific airship, the crack may occur at a random orientation during working condition. Therefore, cracks inclined to the axis of the textile are less critical than those of equal length paralleled to a natural axis.

To cope with the aforementioned circumstances, Rigaud et al.[7] proposed a concept of equivalent initial crack length in their paper. Such length was obtained by projecting the initial crack length in an axis paralleled to the loading direction. For instance, a specimen oriented at 0° with 10mm initial crack length has an identical equivalent initial crack length to a specimen oriented at 60° with 20mm initial crack length. To obtain the relationship between maximum force and the equivalent initial crack length of such new coated fabric, table 2 is concluded.

Table 2. Maximum force of specimens with each equivalent initial crack length.

Specimens	Equivalent initial crack length (mm)	Warp specimens (kN)				Weft specimens (kN)			
		Maximum force		Average force		Maximum force		Average force	
10- 0°	10	4.29	4.26	4.40	4.32	4.06	4.57	4.21	4.28
20- 60°	10	4.44	4.39	4.32	4.38	4.44	4.36	4.17	4.32
20- 0°	20	3.67	3.77	3.62	3.69	3.66	3.55	3.45	3.55
40- 60°	20	3.58	3.51	3.83	3.64	3.57	3.65	3.68	3.63

As can be seen from table 2, the maximum tensile stresses are approximate with each other for the specimens with identical equivalent initial crack length. For instance, the average maximum force of 20- 60° specimen was 4.32kN, only 1.3% higher than that of 10- 0° specimen. Accordingly, the crack orientation and the initial crack length can be combined as one factor -- equivalent initial crack length. To make a preparation for the next theoretical simulation, the critical stresses of all uniaxial specimens with effective critical length are listed in table 3 then.

Table 3. Critical stress of uniaxial tearing tensile specimens.

Specimens	Equivalent initial crack length (mm)	Warp (N/mm)				Weft (N/mm)			
		1	2	3	Average	1	2	3	Average
10-60°	5	58.80	63.36	65.76	62.64	58.03	61.53	61.60	60.39
10-45°	7.08	56.87	60.13	58.33	58.44	59.60	58.72	60.80	59.71
10-0°	10	57.23	56.73	58.69	57.55	54.07	56.55	56.16	55.59
20-60°	10	59.23	58.63	57.63	58.50	59.13	58.09	55.53	57.58
20-45°	14.16	56.60	54.69	55.80	55.70	53.88	54.01	54.58	54.16
20-0°	20	49.16	50.29	48.23	49.23	48.73	47.34	45.96	47.34
40-60°	20	47.70	46.80	51.07	48.52	47.56	48.67	48.42	48.22
50-60°	25	40.94	42.44	43.47	42.28	42.27	43.68	42.16	42.70
40-45°	28.32	43.03	41.38	42.43	42.28	42.54	42.08	41.67	42.10
50-45°	35.4	36.23	34.96	36.56	35.92	35.89	37.47	37.36	36.91
40-0°	40	28.73	29.89	30.07	29.56	31.73	30.13	31.23	31.03
50-0°	50	21.47	22.56	21.73	21.92	21.29	21.80	17.67	20.25

4. Theoretical Simulation

It is well known that an airship will undergo various pressures in different working conditions. The relationship between the threshold values of initial crack length and various critical stresses of envelope is very important. However, it is almost impossible to do experimental researches and unnecessary to study all the operating conditions. Therefore, precise descriptions on mechanical properties of membrane material by the numerical simulations are unquestionable and proper methods. Hence, three theoretical simulations, that is to say, the Griffith theory, the Stress intensity factor theory and the Thiele's empirical theory were made then.

4.1. Three theoretical simulations

The Griffith theory is based on the energy balance method according to Griffith's studies on the glass fracture. The fundamental of fracture mechanics is the definition of a function known as the stress intensity factor. Such function can describe the stress field concentration around the slit tip regarding

the overall stress and crack length. Irwin [20] introduced the plastic work to modify the fracture mechanics theory. Further, he proposed a simplified model to analyze the stress redistribution in the plastic region. Isida et al. [21, 22] and Tada [23] studied the semi-empirical solutions according to a large amount of experimental data. Topping [10] researched the relationship between critical crack length and the stress with a pressurized cylinder. Thiele performed tests on two full-size airship envelopes to further Topping's investigation. He deduced an equation called Thiele's experimental formula to correlate cut slit tear stress with critical slit length.

The critical tearing tensile stress based on the Griffith theory is shown as follows:

$$\sigma_c = \left(\frac{G_{IC} E}{\pi a} \right)^{\frac{1}{2}} \quad (1)$$

Where E is the elastic modulus, and a is half of the center crack length, σ_c is tearing tensile stress, G_{IC} is the critical energy release rate.

The form of stress intensity factor is applied as:

$$K_I = F_T \sigma (\pi a)^{\frac{1}{2}} \quad (2)$$

Where σ is maximum tensile stress, a is half of the center crack length, and F_T stands for geometrical factor considering specimen dimension. The units of the above mentioned variables were N/mm, mm, mm, respectively. Based on the research output of Isida [22], when the ratio of length to width for specimen is 0.75, the fitting function of is written as

$$F_T = 2.08 \left(\frac{a}{b} \right)^2 + 0.078 \left(\frac{a}{b} \right) + 1 \quad (3)$$

Therefore, the stress intensity factor, K_I , is deduced to be $276.74 \text{ N/mm}\sqrt{\text{mm}}$ and $274.17 \text{ N/mm}\sqrt{\text{mm}}$ by warp and weft experimental data, respectively.

Thiele's experimental formula to correlate cut slit tear stress with critical slit length is written as follows:

$$\sigma_c = Pr = \frac{C_I C_s}{(2a)^n \left(1 + \frac{2a}{r} \right)} \quad (4)$$

Where σ_c is the tearing stress, P is the filling pressure, r is the diameter of cylinder, C_I is the tearing stress in accordance with the FAA-P-8110-2[6], and n is constant and can be deduced by the least square method.

4.2. Comparison of the stresses on the experimental data and theoretical simulations

The relationship between the critical stress and the equivalent initial crack length were simulated by the above mentioned theories. Then, evaluated by observing how well they correlate with the experimental results, we get the calculated results depicted in figure 9 for the uniaxial tests.

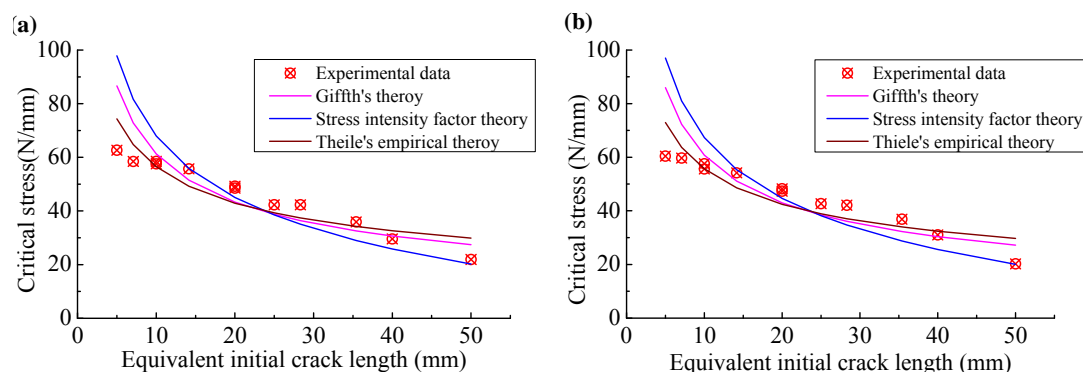


Figure 9. Comparison between experimental data and the theoretical values for uniaxial specimens (a) the warp specimens and (b) the weft specimens.

As for the uniaxial tearing tensile tests, theoretical data was higher than the experimental data when the equivalent initial crack length was less than 10mm. It should be noticed that theoretical simulations show unreasonable high values approaching infinity as equivalent initial crack length near zero. Such phenomenon has also been observed by other previous researchers. However, the theoretical resolutions were approximate to the experimental data during the equivalent initial crack length in the range of 10mm to 40mm. However, variation emerged when crack length was getting larger, as in figure 9. Among the three theory resolutions, the Stress intensity factor theory is the closest to the experimental data, whereas resolution of the Griffith theory and Thiele's empirical theory are higher than the experimental data. Theoretically, Griffith theory is based on the infinite plank and the Thiele's empirical theory based on the pneumatic cylinder tearing tensile tests in which irrespective of the dimension influence. Therefore, deviation comes to be higher during larger initial crack length. As for the stress intensity factor theory, the specimen's dimension is considered, hence such theory is the most suitable one for the uniaxial tearing tensile tests.

4.3. The allowable crack length

According to the standard of FAA-P-8110-2[6], there is no permission on the crack tear propagation under working condition for an airship envelope material. The limit stress of the envelope fabric is assumed to be one forth (4 is the safety factor specified in the FAA) of the ultimate tensile stress of the intact specimens. Consequently, the relevant minimum slit size can be calculated by the Stress intensity factor theory, as shown in figure 10. That is to say, the technology demonstrator can withstand up to around 48mm initial effective crack during the operation condition. Such dimension is higher than that of the Zylon envelope material Z2929T-AB whose technology demonstrator is 40mm slit[11].

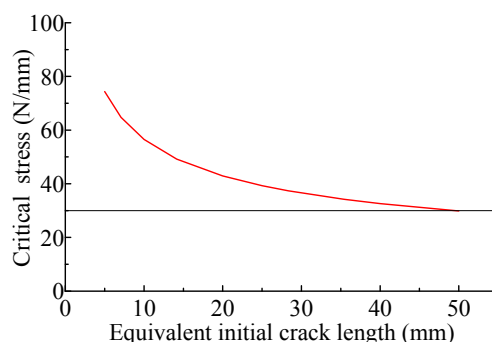


Figure 10. The allowable crack length of the technology demonstrator.

5. Conclusions

In the present work, the tear propagation characteristics of a new envelope fabric for airships, GQ-6, have been successfully identified by uniaxial tearing tensile tests. It is observed that the woven fabric has significant sensitivity to the crack orientation and initial crack length, whereas the stretching rate factor shows tiny sensitivity to specimens subjected to uniaxial tensile condition. After thoroughly researching the uniaxial tensile experimental data, we conclude that the initial crack length and the initial crack orientation can be combined with one factor of equivalent initial crack length. Then, by the experimental data, three theoretical simulations (Griffith energy theory, Stress intensity factor theory, Thiele's empirical theory) are deduced, and comparisons between the theoretical solutions and the experimental data are made. The stress intensity factor theory is selected to be the optimal one while analyzing the tearing mechanical properties of the GQ-6 textile. Finally, an equivalent initial crack length, i.e., 48mm, is testified to be safe during the operation condition.

The above conclusions can be directly applied in the preliminary design in practical engineering. Meanwhile, these work should be extended further to biaxial and multi-axial tearing tensile tests simulating stable crack growth by FEM analysis for arbitrary cracks.

Acknowledgments

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