The Development of Mathematical Prediction Model to Predict Resilient Modulus for Natural Soil Stabilized by Pofa-Opc Additive for the Use in Unpaved Road Design

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Abstract: Resilient Modulus (Mr) is considered one of the most important parameters in the design of road structure. This paper describes the development of the mathematical model to predict resilient modulus of organic soil stabilized by the mix of Palm Oil Fuel Ash – Ordinary Portland Cement (POFA-OPC) soil stabilization additives. It aims to optimize the use of the use of POFA in soil stabilization. The optimization models enable to eliminate the arbitrary selection and its associated disadvantages in determination of the optimum additive proportion. The model was developed based on Scheffe regression theory. The mix proportions of the samples in the experiment were adopted from similar studies reported in the literature Twenty five samples were designed, prepared and then characterized for each mix proportion based on the MR in 28 days curing. The results are used to develop the mathematical prediction model. The model was statistically analyzed and verified for its adequacy and validity using F-test.

Keywords: Resilient Modulus, Soil Stabilization.

1. Introduction

A number of studies focused on the optimization use of POFA in concrete based on the UCS have been reported in the literature. This particular study aims to optimize the use POFA in soil stabilization of palm oil plantation soil road. The optimization seeks to find the optimum content of POFA in mixture to achieve the properties concerned which in this study is the Resilient Modulus. There are many mathematical methods of optimization which have been developed to simplify the analytical process. Scheffe [15] has developed a method of optimization for experiment with mixtures based on the regression theory. The pursued model in this study shall be used to predict the Mr and the amount of additives required to achieve certain strength of stabilized soil in which so that the stabilized soil can be used in road construction.

Mbadike and Osadeb [11] used Scheffe's simplex theory for the optimization of the compressive strength of lateritic concrete. [11] used modified regression theory to predict of concrete mix ratios with most economical and durable concrete that meet with certain properties as consistency. Okere et al. [6] have also used Scheffe method to optimize the concrete mix cost.

Several effluents are resulted from the production of palm oil. One of the wastes is POFA which is a by-product produced in palm oil mill [16] After palm oil is extracted from the palm oil fruit, both palm oil husk and palm oil shell are burned as fuel in the boiler of palm oil mill. Generally, after combustion about 5% palm oil fuel ashes by weight of solid wastes is produced [14]. Large quantity of ash is produced and creates problems of disposal [17]. POFA produced in Malaysian palm oil mill is dumped as waste without any profitable return

[9].Either in the 20th or 21st century, POFA is still considered as a nuisance to the environment and disposed without being put for any other use as compared to other types of palm oil by-products [10]. Since Malaysia maintains to increase yield of palm oil, more ashes will be brought out and a failure to find any solution in making use of POFA will create various environmental problems. This prompted a number of researchers to initiate studies on engineering characteristics and usability of POFA as cement replacement material. Palm oil waste is an agro-waste material [18].It was reported that, for every 100 tons of fresh fruit bunches processed, approximately 7 tons of fibers, 20 tons of nut shells, and 25 tons of empty bunches discharge from the mill. In 2009 the Malaysian palm oil industry produced around 60 million tons of total solid waste annually finally husk and fiber are used extensively as fuel for the production of steam in the palm oil mills [1].

The physical and chemical studies of POFA have indicated that the POFA is a pozzolanic material and can be used as a binder and has recently been accepted as a pozzolanic material [2, 3].In another review by Tangchirapat, Jaturapitakkul & Kiattikomol [19] it was shown that the ash has great potential in concrete technology advancement, when used in premixing with Ordinary Portland Cement. In terms of soil stabilization the addition of fly ash reduces the plasticity of expansive soils. The liquid limit decreases and the plastic limit increases with an increase in fly ash content. The free swell index of expansive soils can be effectively reduced by the addition of fly ash. Free swell index of expansive soil was reduced by about 50% by adding 20% fly ash [13]. Eldagal, A., & Elmukhtar, [7] have studied the behavior of high strength palm oil fuel ash (POFA) concrete. It was reported that POFA contains silica oxide which can react with calcium hydroxide (Ca(OH)2) generated from the hydration process; and the pozzolanic reactions produce more calcium silicate hydrate (C-S-H) gel compound as well as reducing the amount of calcium hydroxide .

2. Methodology

Fig. 1 shows the flow research work and the phases of the research.

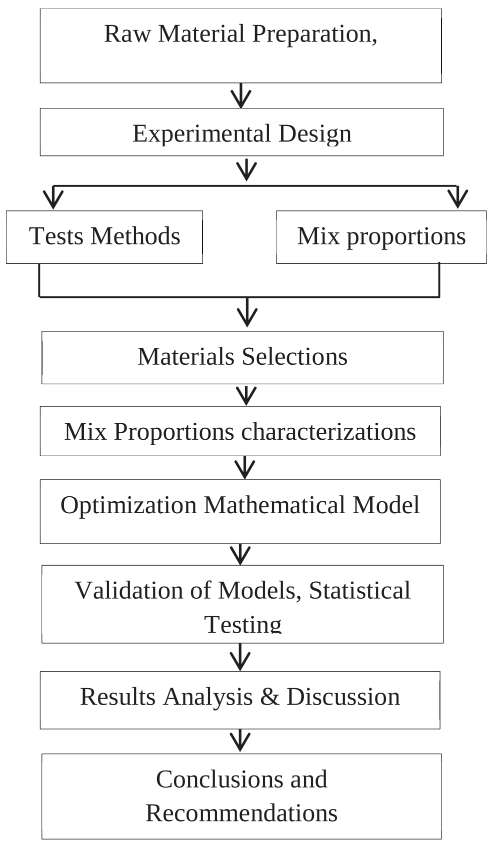

Figure 1. Flow chart of research work

3. Materials

The raw POFA was collected from palm oil mill located in Lot 835, Batu 3, Jalan Batu Pahat, Kluang, Johor Bahru, Malaysia. The laterite gravel was collected from Bukit Naning quarry in Muar, Malaysia. The cement used was ASTM Type I cement Portland cement is commonly used for general purpose.

Natural soil was collected from Ulu Tiram,Johor Bahru, Malaysia. The soil was basically collected from plantation site nearby in which a road is constructed by Probase Company. Disturbed sampling was collected at depth more than 0.4 m. The soil sample obtained was in slightly wet condition, and then the natural moisture content was taken besides the pH was also measured to test the acidity of the natural plantation soil. All of Mr Laboratory experiments were carried out in accordance to the specified standard of AASHTO T 307-99 (2003) [5].

4. Experimental Design

The proposed method of selecting the optimum mix design based on trial mixes having different combinations of five materials i.e.: POFA, water, OPC, natural soil and laterite gravel. The design content for each material is described in order to get a control design proportion that can be used to develop the mix proportions.

To adopt the use of Scheffe's method in optimization model. A simplex lattice is developed to be as structural representation of lines joining the atoms of mixture which is constrained with the theoretical findings. It means the values have to be within the factor space for a trial. Mixtures with proportion of outside the factor space were used for verifications of the developed model. The atoms indicate the constituent components of the mixture (POFA, Soil, Laterite gravel and OPC). It produced a simplex of a mixture with four components. Though, the simplex lattice of these four components mixture is three-dimensional solid equilateral tetrahedron. In a condition of Scheffe's method (1985), the components are subjected to the constraint that the sum of all the components must be equal to "one" which means:

$$x_1 + x_2 + x_3 + x_4 + \cdots \ldots \ldots . . x_q + \tag{1}$$

$$\textstyle\sum_i^q x_i = 1 \tag{2}$$

Where "q" is the number of components of a mixture and Xi is the proportion of the ith component in the mixture, in the mix proportion it is impossible to use the, it is necessary to carry out a transformation from actual to pseudo components actual mix to develop the model since the sum of the mix component must be one. Hence, the actual components represent the proportion of the ingredients based in the theoretical constraints while the pseudo components represent the proportion of the components of the ith components in the mixture i.e. X1, X2, X3, X4. However, it has to be taking into account the four- component mixture tetrahedron simplex lattice, let the vertices of this tetrahedron (principal coordinates) be described by A1, A2, A3, A4.

The following arbitrary mixes are developed based on past practicing manual and literatures which are prescribed for the vertices of the tetrahedron in Fig. 2.The developed mixes is a 4*4 matrix which can be developed to experimentally design the mixes.

Ai :(POFA: Soil: Laterite Gravel: Cement)

A1 :(0.04: 1: 0.1: 0.04)

A2 :(0.024: 1: 0.15: 0.036)

A3 :(0.012: 1: 0.2: 0.028)

A4 :(0.004: 1: 0.3: 0.016)

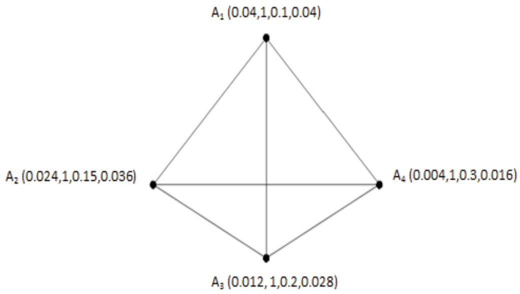

Figure 2. Vertices of Lattice (Represent Actual)

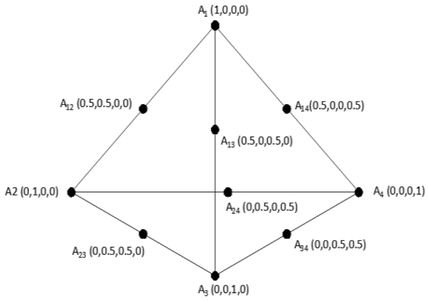

Figure 3. Vertices of Simplex Lattice (Represent Pseudo)

To continue analytical process let X represents the pseudo components and Z represents actual components then, for the processes of transforming pseudo to actual use the following equations:

$$Z = AX \tag{3}$$

In which A is a matrix whose elements are from the arbitrary mix proportions which will be selected when equation (3) is opened and solved mathematically however (A) is the inverse of matrix (X) times (Z) and X is chosen from the structure of pseudo components in Fig. 2 then, by expanding equations (3) the actual components of Z will be determined respectively. Table 1 shows the corresponding values of pseudo and actual.

The MR samples were 70 mm in diameter and 140 mm in height. Twenty five samples were prepared to develop the mathematical model. The compaction energy per unit volume was used to determine the number of blows to produce the samples with identical dry density the same as those achieved on the standard compaction test. After 28 days curing time the samples are tested for the Resilient Modulus.

5. Results and Analysis

Development of Mathematical Model

The use of the following polynomial equation developed by Scheffe [15]:

$$Y = \alpha_1 z_1 + \alpha_2 z_2 + \alpha_3 z_3 + \alpha_4 z_4 + \alpha_{12} z_1 z_2 + \alpha_{13} z_1 z_3 + \alpha_{14} z_1 z_4 + \alpha_{23} z_2 z_3 + \alpha_{24} z_2 z_4 + \alpha_{34} z_3 z_4 \tag{4}$$

Where:

αi and αij are coefficients and Zi are the pseudo components of mixture. The Y function is the response function at any point of observation, zi is the predictor and αi is the coefficient of the optimization model equations.

5.1. Determination of Optimization Model Coefficients

In the optimization model equation (4) the substitution of different values observations then there will be different response and different estimators at constant coefficient then, at nth observation point, Y (n) will correspond with Zi (n). That is,

According to equation (4):

$$Y^{(n)} = \sum \alpha_i z_i^{(n)} + \sum \alpha_{ij} z_i^{(n)} \quad z_j^{(n)} \tag{5}$$

Where $1 \leq I \leq j \leq 4$ and n = 1, 2, 3, …….,10.

Equation (5) can be transformed in the form of matrix

$$\left[Y^{(n)}\right] = \left[Z^{(n)}\right]\{\alpha\} \tag{6}$$

rearranging equation (6) then results the following equation:

$$\{\alpha\} = \left[Z^{(n)}\right]^{-1}\left[Y^{(n)}\right] \tag{7}$$

By Letting the actual mixes in table 3 be represented by Zi and the corresponding fractional portions, Z(i)f are presented in Table 3. The table shows the actual mixes which were developed based on Scheffe simplex lattice and the corresponding fractional values which will be used to continue the mathematical calculations to determine the coefficients in the mathematical optimization model these values of the fractional portions Z(n) were used to develop Z(n) matrix and the inverse of Z(n) matrix. The values of Y(n) matrix are determined from laboratory tests from resilient modulus and unconfined compressive strength. With the values of the matrices Y(n) and Z(n) known, it is easy to determine the values of the constant coefficients of equation (5).

The resilient modulus response defers for each mix ratio hence to develop optimization model the first 10 mixes are chosen to fulfill the condition in Equation (4) The MR responses are:

$$\left[Y^{(n)}\right] = \begin{bmatrix} 15645000 \\ 15505420 \\ 15093100 \\ 15261300 \\ 15122960 \\ 15005460 \\ 14941830 \\ 15228080 \\ 15565240 \\ 15210120 \end{bmatrix}$$

The coefficient values of the model are:

$$\{\boldsymbol{\alpha}\} = \begin{bmatrix} 33934348 \\ 1703582 \\ 16545683 \\ -1.1\text{E}+08 \\ -738592 \\ 376814.7 \\ 7784832 \\ -286502 \\ 843624.9 \\ 2177480 \end{bmatrix}$$

By substituting the values of α into Equation 4 then the optimization model at the response of MR is:

$$Y = 33934348z_1 + 1703582z_2 + 16545683z_3 - (1.1\mathrm{E}+08)z_4 - 738592z_1z_2 + 376814.7z_1z_3 + 7784832z_1z_4 - 286502z_2z_3 + 843624.9z_4 + 2177480z_3z_4 \quad (8)$$

5.2. Statistical Tests For the Adequacy of the Optimization Models

The optimization models are tested to evaluate the validity and adequacy based on the experimental results and to prove the agreement of the optimization models with the actual experimental results. In addition to that, the statistical tests most often used when comparing statistical models that have been fitted to a data set, in order to identify the model that best fits the population from which the data were sampled.

Letting H0 denotes the statistical Null Hypothesis and H1 represents the alternative Hypothesis.

The hypothetical conditions to validate the models are as follow: Null Hypothesis (H0): there are no statistical significant differences among the analytically observed results and the predicted results However, the Alternative Hypothesis (H1): There is a statistically significant difference between the analytical results and the expected results.

The use of control samples is adopted by taking the Y observed from laboratory results and Y predicted from the models by substituting the values of Zi .By the use of Microsoft excel it runs the F-test analysis. The mean is calculated for observed and predicted values and symbolized by (yobs) and (ypred) respectively. Mean is given by the summation of the samples divided by the number of samples.

The variance is given by:

$$v^2 = \left[\frac{1}{n-1}\right]\left[\sum(Y-y)^2\right] \quad (9)$$

$$y = \frac{\sum Y}{n} \quad (10)$$

Where n is the number of responses of the analysis.

According to table 5 the variances are calculated as follows:

$v^2_{pred} = \frac{0.53}{14} = 0.04$

$v^2_{ob} = \frac{2.15}{14} = 0.15$

By substituting the values into equation

$F = \frac{0.04}{0.15} = 0.27$

Referring to the F-distribution tables F0.95(14,14) = 2.48 which is higher than the calculated value 0.27 therefore the null hypothesis is accepted and the mathematical model at resilient modulus is considered adequate.

6. Conclusion

The use of Scheffe method to develop Resilient Modulus predication model for natural plantation soil stabilized by POFA has been prescribed and introduced. The mathematical prediction model can basically be used also to determine the optimum content of POFA required resulting in better stiffness at the same time eliminating the repetitions of testing and the arbitrary selection of additive percentage. For further analysis the mathematical model in Equation (8) can also be used for determining the optimum OPC required in stabilization of plantation soil.

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Appendices

Table 2. Pseudo Component with Their Corresponding Actual Component Values Where: Actual (Zi) and Pseudo (Xi) components

No of mix	X1	X2	X3	X4	Response	Z1	Z2	Z3	Z4
1	1	0	0	0	Y1	0.04	1	0.1	0.04
2	0	1	0	0	Y2	0.024	1	0.15	0.036
3	0	0	1	0	Y3	0.012	1	0.2	0.028
4	0	0	0	1	Y4	0.004	1	0.3	0.016
5	0.5	0.5	0	0	Y12	0.032	1	0.125	0.038
6	0.5	0	0.5	0	Y13	0.026	1	0.15	0.034
7	0.5	0	0	0.5	Y14	0.022	1	0.2	0.028
8	0	0.5	0.5	0	Y23	0.018	1	0.175	0.032
9	0	0.5	0	0.5	Y24	0.014	1	0.225	0.026
10	0	0	0.5	0.5	Y34	0.008	1	0.25	0.022
Control Points Calculated within the factor space									
11	0.5	0.25	0.25	0	C1	0.029	1	0.1375	0.036
12	0.25	0.25	0.25	0.25	C2	0.02	1	0.1875	0.03
13	0	0.25	0.25	0.5	C3	0.011	1	0.2375	0.024
14	0	0.25	0	0.75	C4	0.009	1	0.2625	0.021
15	0.75	0	0.25	0	C5	0.033	1	0.125	0.037
16	0	0.5	0.25	0.25	C6	0.016	1	0.2	0.029
17	0.25	0	0.5	0.25	C7	0.017	1	0.2	0.028
18	0.75	0.25	0	0	C8	0.036	1	0.1125	0.039
19	0	0.75	0.25	0	C9	0.021	1	0.1625	0.034
20	0	0.4	0.4	0.2	C10	0.0152	1	0.2	0.03
Control points Calculated outside the factor space									
21	0.5	0.5	0.5	0.5	C11	0.016	1	0.2	0.027
22	0.25	0	0.25	0.5	C12	0.015	1	0.225	0.025
23	0.5	0	0.5	0	C13	0.026	1	0.15	0.034
24	0.25	0.25	0.25	0	C14	0.019	0.75	0.1125	0.026
25	0	0.5	0.5	0.25	C15	0.019	1.25	0.25	0.036

Table 3. Actual Mix Proportions and Their Corresponding Fractional values

Mix No	Z1	Z2	Z3	Z4	Total of Si	Response	Z1f	Z2f	Z3f	Z4f
1	0.04	1	0.1	0.04	1.18	Y1	3.390	84.746	8.475	3.390
2	0.024	1	0.15	0.036	1.21	Y2	1.983	82.645	12.397	2.975
3	0.012	1	0.2	0.028	1.24	Y3	0.968	80.645	16.129	2.258
4	0.004	1	0.3	0.016	1.32	Y4	0.303	75.758	22.727	1.212
5	0.032	1	0.125	0.038	1.195	Y12	2.678	83.682	10.460	3.180
6	0.026	1	0.15	0.034	1.21	Y13	2.149	82.645	12.397	2.810
7	0.022	1	0.2	0.028	1.250	Y14	1.760	80.00	16.00	2.240
8	0.018	1	0.175	0.032	1.225	Y23	1.469	81.633	14.286	2.612
9	0.014	1	0.225	0.026	1.265	Y24	1.107	79.051	17.787	2.055
10	0.008	1	0.25	0.022	1.280	Y34	0.625	78.125	19.531	1.719
11	0.029	1	0.1373	0.036	1.202	C1	2.413	83.195	11.423	2.995
12	0.02	1	0.1875	0.03	1.238	C2	1.616	80.775	15.145	2.423
13	0.011	1	0.2375	0.024	1.273	C3	0.864	78.555	18.657	1.885
14	0.009	1	0.2625	0.021	1.293	C4	0.696	77.340	20.302	1.624
15	0.033	1	0.125	0.037	1.195	C5	2.762	83.682	10.460	3.096
16	0.016	1	0.2	0.029	1.245	C6	1.285	80.321	16.064	2.329
17	0.017	1	0.2	0.028	1.245	C7	1.364	80.257	16.051	2.247
18	0.036	1	0.1125	0.039	1.188	C8	3.030	84.175	9.470	3.283
19	0.021	1	0.1625	0.034	1.218	C9	1.724	82.102	13.342	2.791
20	0.0152	1	0.2	0.0288	1.244	C10	1.222	80.386	16.077	2.315
21	0.016	1	0.2	0.027	1.24	C11	1.20	80.41	16.10	2.28
22	0.015	1	0.225	0.025	1.265	C12	1.186	79.051	17.787	1.976
23	0.026	1	0.15	0.034	1.210	C13	2.149	82.645	12.397	2.810
24	0.019	0.75	0.1125	0.026	0.908	C14	2.093	82.599	12.390	2.863
25	0.019	1.25	0.25	0.036	1.555	C15	1.222	80.386	16.077	2.315

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Table 4. The determination of Z n matrix values based on table 2

No	Z1	Z2	Z3	Z4	Z1Z2	Z1Z3	Z1Z4	Z2Z3	Z2Z4	Z3Z4	MR response *10^5 (kPa)
1	3.40	84.70	8.50	3.40	288.00	28.90	11.60	720.00	288.00	28.90	156.45
2	2.00	82.60	12.40	3.00	165.20	24.80	6.00	1024.20	247.80	37.20	155.054
3	1.00	80.60	16.10	2.30	80.60	16.10	2.30	1297.70	185.40	37.00	150.931
4	0.30	75.80	22.70	1.20	22.70	6.80	0.40	1720.70	91.00	27.20	152.613
5	2.70	83.70	10.50	3.20	226.00	28.40	8.60	878.90	267.80	33.60	151.2296
6	2.10	82.60	12.40	2.80	173.50	26.00	5.90	1024.20	231.30	34.70	150.0546
7	1.80	80.00	16.00	2.20	144.00	28.80	4.00	1280.00	176.00	35.20	149.4183
8	1.50	81.60	14.30	2.60	122.40	21.50	3.90	1166.90	212.20	37.20	152.2808
9	1.10	79.10	17.80	2.10	87.00	19.60	2.30	1408.00	166.10	37.40	155.6524
10	0.60	78.10	19.50	1.70	46.90	11.70	1.00	1523.00	132.80	33.20	152.1012
11	2.40	83.20	11.40	3.00	199.68	27.36	7.20	948.48	249.60	34.20	150.0034
12	1.70	80.80	15.10	2.40	137.36	25.67	4.08	1220.08	193.92	36.24	156.7584
13	0.86	78.56	18.66	1.89	67.56	16.05	1.63	1465.93	148.48	35.27	152.42
14	0.80	77.30	20.30	1.60	61.84	16.24	1.28	1569.19	123.68	32.48	148.7
15	2.80	83.70	10.50	3.10	234.36	29.40	8.68	878.85	259.47	32.55	155.91
16	1.30	80.30	16.10	2.30	104.39	20.93	2.99	1292.83	184.69	37.03	148.52
17	1.40	80.30	16.10	2.20	112.42	22.54	3.08	1292.83	176.66	35.42	150.261
18	3.00	84.20	9.50	3.30	252.60	28.50	9.90	799.90	277.86	31.35	153.52
19	1.70	82.10	13.30	2.80	139.57	22.61	4.76	1091.93	229.88	37.24	148.69
20	1.20	80.40	16.10	2.30	96.48	19.32	2.76	1294.44	184.92	37.03	152.04
21	1.20	80.41	16.10	2.28	96.49	19.32	2.74	1294.60	183.33	36.71	151.22
22	1.20	79.10	17.80	2.00	94.92	21.36	2.40	1407.98	158.20	35.60	146.36
23	2.10	82.60	12.40	2.80	173.46	26.04	5.88	1024.24	231.28	34.72	146.76
24	2.10	82.60	12.40	2.90	173.46	26.04	6.09	1024.24	239.54	35.96	143.222
25	1.20	80.40	16.10	2.30	96.48	19.32	2.76	1294.44	184.92	37.03	144.03

Table 5. The Determination of First 10 Mixes [Z n]-1 Matrix Inverse Values Based On Table 3

Z1	Z2	Z3	Z4	Z1Z2	Z1Z3	Z1Z4	Z2Z3	Z2Z4	Z3Z4
95.90	100.80	283.60	79.90	-196.20	-39.30	87.70	-136.70	-18.40	-257.30
1.20	0.30	-5.80	-0.60	-2.30	-2.30	0.10	8.10	-1.50	2.90
11.80	0.90	-46.10	-0.90	-24.90	-12.30	-2.60	66.30	-3.80	11.70
-141.40	96.10	266.50	-40.70	172.90	339.70	-62.10	-786.00	103.90	51.30
-1.30	-1.30	-1.60	-0.70	2.60	1.40	-1.10	-0.70	0.80	2.00
-0.20	-0.30	-4.80	-0.90	0.60	-1.50	-0.20	5.10	-1.10	3.30
5.80	1.70	-33.20	-4.60	-7.80	-21.00	3.00	48.50	-11.80	19.50
-0.20	0.00	0.90	0.00	0.40	0.30	0.00	-1.20	0.10	-0.30
1.30	-1.00	-1.50	0.60	-1.40	-3.10	0.60	6.60	-0.80	-1.30
2.00	-0.90	-6.30	-0.30	-2.90	-4.20	0.30	11.90	-1.20	1.60

Table 6. F-Test Analysis Optimization Model

Sample No	Y observed (kPa*106)	Y predicted (kPa*106)	Y(ob) - y(ob) (kPa*106)	Y(pred) - y(pred) (kPa*106)	(Y(obs) -y (obs))2 (kPa*106)	(Y(pred) - y(pre))2 (kPa*106)
C1	15.00	15.08	0.01	-0.05	0.00	0.00
C2	15.68	14.99	0.69	-0.14	0.47	0.02
C3	15.24	15.41	0.25	0.28	0.06	0.08
C4	14.87	15.19	-0.12	0.06	0.01	0.00
C5	15.59	15.01	0.60	-0.12	0.36	0.02
C6	14.85	15.24	-0.14	0.11	0.02	0.01
C7	15.03	14.70	0.04	-0.43	0.00	0.19
C8	15.35	15.38	0.36	0.25	0.13	0.06
C9	14.87	15.31	-0.12	0.18	0.01	0.03
C10	15.20	15.20	0.21	0.07	0.05	0.00
C11	15.12	15.13	0.13	0.00	0.02	0.00
C12	14.64	14.94	-0.35	-0.19	0.13	0.04
C13	14.68	14.91	-0.31	-0.22	0.10	0.05
C14	14.32	15.25	-0.67	0.12	0.45	0.01
C15	14.40	15.20	-0.59	0.07	0.34	0.00
Sum	224.84	226.93			2.15	0.53
Mean	14.99	15.13				