

Mathematical modeling of jet flow around of bodies when applying polymer powder coatings

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Abstract. Mathematical modeling of processes occurring during the spraying of polymer powder compositions on the surface of parts and products is considered. Relations describing the flow around processed bodies, movement of the powder particles in a stream of the carrying medium, dynamics of their deposition on the body surface are presented.

1. Introduction

At the present time, coatings based on the polymer powder compositions (PPC) [1] are becoming more common in many industries, construction and transport. Promising methods of PPC spraying are jet methods such as flameless spraying on a heated surface of parts and products, hot spraying, spraying under the influence of centrifugal forces in an electrostatic field. Internal gas-dynamic, thermal problems in the cavities of spraying guns [2], other spraying devices and external problems when considering jet nonisothermal leakage of the carrying medium (air, combustion products) containing the powder particles on the surface of parts and products are distinguished at mathematical modeling of processes occurring in the implementation of the named methods. It should be noted that in these problems apart from the dynamics of gas and particles the heat exchange of powder particles with the carrying medium, thermal behavior of the powder coating material layer and in some cases the heat exchange of a jet with the environment are of great importance. Similar problems are encountered at jet painting, abrasive machining of parts and products, work of ventilation systems of industrial and residential premises, reversing devices, when studying the processes occurring during the takeoff of the aerospace objects and in other cases [3-6].

2. Relations describing the flow around processed bodies, movement of the powder particles, dynamics of their deposition on the body surface

Models of an ideal either viscous compressible or incompressible fluid can be used for a mathematical description of the medium movement when spraying the polymer powder compositions on parts and products according to the classification of jet streams [7]. In this case, the medium can be considered as a homogeneous fluid. However, in the considered processes it definitely represents a multiphase gas suspension or depending on the particles sizes the aerosol. Wherein disperse particles



may or may not have an electric charge. Furthermore, as mentioned above, it is necessary to take into consideration the interaction of the jet with the environment. A jet can spread in a stationary or moving medium in the absence or presence of limitations (strained jets). When the jet flows around parts and products the latter also represent limitations moreover they can be of very different shapes, sizes, they can move, rotate relatively to the source of the jet or be stationary. Finally, plane, axisymmetric, circular, fan, nonswirled or swirled jet can be identified by the shape and nature of medium movement in the jet.

In this work it is assumed that the jet fluid is ideal, incompressible and contains small sized uncharged particles of a polymer powder. There are no limitations of a jet flow with the exception of a streamlined body. Thermal interaction of a jet with environment and particles with the carrying medium is disregarded. Since in practice mainly circular or slotted nozzles are used when spraying only plane and axisymmetric gas powder jets are considered below.

For a plane-parallel fluid flow solutions are obtained as a rule by the conformal mapping method, for example, the problems of jet inleakage on the plate of finite length, semi-infinite and infinitely stretched plate [8-10 and others]. Peculiarities of the jet flow around the bodies of arbitrary cross-section, circular cylinder, in particular, rotating one are discussed in [11]. Free jet, zone of flow reversal and jets spreading over the surface, probably, separating from it can be marked out in the flow region when a fluid or gas jet flows at a moderate speed on the body. Specificity of studies of jet dynamics, movement of powder particles in the flow when spraying is that from the practical point of view the greatest interest represents the zone of flow reversal in the vicinity of the body front point where particles deposit on the surface.

In the case when the width of the jet cross-section near the body is greater than its transverse size it can be expected that the jet flow in the reversal zone differs only slightly from the flow in the same region when unbounded stream flows around a body. Here, solutions to the problem of an unbounded stream flowing around a body can be used in the said zone to assess the behavior of the powder particles. Thus, for flow around a stationary circular cylinder of radius a velocity vector components of a fluid in the cylindrical coordinate system (z, r, φ) are:

$$v_r = v_0 \cos \varphi \left(1 - a^2/r^2\right), \quad v_\varphi = -v_0 \sin \varphi \left(1 + a^2/r^2\right). \quad (1)$$

Here v_0 – fluid velocity at a distance from the body directed along the r -axis.

If the cylinder rotates around the Oz -axis counterclockwise with an angular velocity ω then flow function is:

$$\psi = v_0 r \sin \varphi \left(1 - a^2/r^2\right) - \omega a^2 \ln r, \quad (2)$$

velocities $v_r = \partial \psi / (r \partial \varphi)$, $v_\varphi = \partial \psi / \partial r$.

Further let us consider the spatial flow around a body. If the body is a sphere of radius a , the flow is axisymmetric and the symmetry axis coincides with the Oz -axis of cylindrical coordinate system (z, r, φ) with the origin at the center of the sphere then according to [12] flow function is:

$$\psi = -0.5 v_0 r^2 \left(1 - \left(a^2 / \sqrt{r^2 + z^2}\right)^3\right), \quad (3)$$

velocities $v_r = \partial \psi / (r \partial z)$, $v_z = -\partial \psi / (r \partial r)$, $v_\varphi = 0$.

In the case of axisymmetric flow around a semi-infinite body of rotation the flow function is

$$\psi = -0.5 v_0 \left(r^2 - 0.5 r_\infty^2 \left(1 + z / \sqrt{r^2 + z^2}\right)\right), \quad (4)$$

where r_∞ – radius of the body cross-section removed far from the front point located at a distance of $0.5r_\infty$ from the origin.

In addition to the mentioned above cylindrical coordinates we introduce Cartesian coordinates (x, y, z) with the origin O . The Ox -axis is directed along the velocity vector of the oncoming flow. At a distance from the surface of the subject body, upstream, in the transition region from the zone of a free jet to the zone of flow reversal we single out section S_0 perpendicular to the velocity vector of the oncoming flow. We assume that in this section polymer powder particles are thrown into the flow. Confining ourselves to the case of two-dimensional flow of the medium (plane-parallel or axisymmetric flow) the initial coordinates of the particles are denoted as x_{p0} , y_{p0} , and velocity vector components as u_{p0} , v_{p0} . For simplicity we assume that all powder particles are spherical, over time their radius r_p does not change. Possible rotation of the particles around its own axis is neglected, particles do not stick together and do not break up.

Following the Lagrangian continual approach, neglecting the forces of gravity, added masses and Bosse, to describe the motion of powder particles in a fluid we use the following relations:

$$\frac{du_p}{d\tau} = R_\mu (u - u_p), \quad \frac{dv_p}{d\tau} = R_\mu (v - v_p). \quad (5)$$

Here τ – time, $u_p = dx_p(\tau)/d\tau$, $v_p = dy_p(\tau)/d\tau$ – velocity components, $x_p = x_p(\tau) = x_{p0} + f_p(\tau)$, $y_p = y_p(\tau) = y_{p0} + g_p(\tau)$ – law of the particles motion; $R_\mu = k_c r_p \mu / m_p$, m_p – particle mass, μ – coefficient of fluid viscosity. We assume that the hydrodynamic forces acting on the particles are proportional to the difference of velocities of the fluid and particles, k_c – proportionality coefficient ($k_c = 19 \div 25$).

Considering that in equations (5) velocity vector components of a fluid which, in particular, can be determined by the formulas (1-4) depend on time $u = u(x_p(\tau), y_p(\tau)) = U(\tau)$, $v = v(x_p(\tau), y_p(\tau)) = V(\tau)$ we present them in the form: $U = a_0 + a_1 \tau$, $V = b_0 + b_1 \tau$. Substituting U, V in (5) and integrating these equations we find

$$u_p = u_{p0} \exp(-R_\mu \tau) + (a_0 - a_1/R_\mu) + a_1 \tau; \quad (6)$$

$$v_p = v_{p0} \exp(-R_\mu \tau) + (b_0 - b_1/R_\mu) + b_1 \tau.$$

Hence the particles coordinates

$$\begin{aligned} x_p &= x_{p0} - (u_{p0}/R_\mu) \exp(-R_\mu \tau) + (a_0 - a_1/R_\mu) \tau + 0.5a_1 \tau^2; \\ y_p &= y_{p0} - (v_{p0}/R_\mu) \exp(-R_\mu \tau) + (b_0 - b_1/R_\mu) \tau + 0.5b_1 \tau^2. \end{aligned} \quad (7)$$

Under condition that $R_\mu \tau < 1$ these dependencies can be simplified:

$$x_p \approx A_0 + A_1 \tau + A_2 \tau^2; \quad y_p \approx B_0 + B_1 \tau + B_2 \tau^2, \quad (8)$$

where coefficients $A_0 = x_{p0} - u_{p0}/R_\mu$, $A_1 = u_{p0} + a_0 - a_1/R_\mu$, $A_2 = 0.5a_1$; $B_0 = y_{p0} - v_{p0}/R_\mu$, $B_1 = v_{p0} + b_0 - b_1/R_\mu$, $B_2 = 0.5b_1$.

Relations (7, 8) are equations of particles trajectories in the parametric form. Using them it is possible to find the intersection points of these trajectories with the streamlined body surface, estimate the mass distribution of the powder on its surface.

In this paper we propose a slightly different approach. It is assumed that the processed part of the surface of a detail, product is convex there are points M_1, M_2 on it at a maximum distance from the Ox -axis their coordinates are $(x_{m1}, y_{m1}), (x_{m2}, y_{m2})$, respectively. Substituting the value x_{m1} instead of x_p in the first of equations (8) we find the time $\tau = \tau_{m1}$, during which the particle with coordinates $(x_{p0}, y_{m1} = y_p(\tau_{m1}))$ in section S_0 at the initial time reaches the point M_1 . Similarly, the initial position of the particle falling into the point M_2 on the body surface is determined. It is obvious that all particles in section S_0 with coordinates $(x_{p0}, y_{m2} < y_p < y_{m1})$ can reach the body surface and stay on it.

Further, it is convenient to break the arc part of the body surface between M_1 and M_2 by points K_i on n equal parts of length l_0 ($i = 1, n-1$), find the coordinates y_{p0i} of the initial position of the powder particles which have fallen into points K_i .

It is easy to make sure that mass flow rate of the particles material passing through the part $(\Delta_{pi} \cdot 1)$ of the section S_0 can be estimated by the dependency:

$$G_i \approx \rho \bar{\varepsilon}_{pi} \bar{u}_i (\Delta_{pi} \cdot 1). \quad (9)$$

Here, ρ – material density of the polymer powder particles, $\bar{u}_i$ – normal to the section S_0 average velocity of the particles which are in the part $\Delta_{pi} = y_{p0(i+1)} - y_{p0i}$ at the initial time, $\bar{\varepsilon}_{pi}$ – average volume concentration of particles of a gas suspension flowing through this part.

For the time of spraying $\tau_H > (\tau_{m1}, \tau_{m2})$ mass of the powder that has entered the part of a body surface between the points K_i and K_{i+1} is $\tau_H G_i$. Accordingly, under the condition of its uniform distribution over the said area thickness of the sprayed layer is

$$h_i \approx k \bar{\varepsilon}_{pi} \bar{u}_i \tau_H (\rho \Delta_{pi} / (\rho_i l_0)), \quad (10)$$

where ρ_i – medium density of the powder layer which depends on the falling speed of particles on the surface, their size, degree of fusion and other factors; k – correction coefficient that can be used to take into account the powder movement along the body surface, its blowing.

3. Conclusions

In conclusion we note that peculiarities of flow around bodies by carrying medium, influence of such parameters as speed of oncoming flow v_0 , angular velocity ω when the body rotates, characteristic dimension a , body shape (see (1)-(4)) as well as the value R_μ depending on the particles radius r_p , their mass m_p , medium viscosity μ , finally, distance of the initial section S_0 from the body, position of the reference part K_i, K_{i+1} on its surface are taken into account in the expression for determining the thickness of the sprayed layer h_i (10) indirectly through the average speed $\bar{u}_i$.

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