Numerical solution of optimal departure frequency of Taipei TMS

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Abstract. Route Number 5 (Bannan Line) of Taipei Mass Rapid Transit (MRT) is the most popular line in the Taipei Metro System especially during rush hours periods. It has been estimated there are more than 8,000 passengers on the ticket platform during 18:00~19:00 at Taipei main station. The purpose of this research is to predict a specific departure frequency of passengers per train. Monte Carlo Simulation will be used to optimize departure frequency according to the passenger information provided by 22 stations, i.e., 22 random variables of route number 5. It is worth mentioning that we used 30,000 iterations to get the different samples of the optimization departure frequency, i.e., 10 trains/hr which matches the practical situation.

1. Introduction

The mass rapid transit (MRT) system is the most important, efficient, convenient transportation between modern metropolitan and urban areas. Because the rail is always underground or elevated, MRT offers the fastest way from downtown to any other place [1]. MRT is therefore the most important mode of modern city transportation. It can carry more than 1,000 passengers on each train to reach their destination get elsewhere in even less than 10 minutes. However, during the 18:00~19:00 rush hour more than 8,000 passengers crowd into the platform of Route Number 5 (Bannan Line) of Taipei main station [2]. Maintaining optimization, i.e., get the optimization departure frequency to have these passengers on their way quickly and of course maintain the most benefit of the Taipei Metro System is the main goal of this research [3].

2. Monte Carlo Simulation

Due to the reasons of nonparametric statistics, full valuation, probabilistic iteration and easy operating, the Monte Carlo Simulation is chosen to model the complex situation of the train Taipei MRT system during the rush hours. Monte Carlo

Simulation is a very popular method of estimating a large amount of calculation. It is used in many research fields, such as statistical engineering, bio-medicine, psychology, atmospheric science, and management, but this application is specific to mass rapid transit transfer system. A lot of papers outline the risk estimate by MCS such as [4-7]. Probability estimate is based on the assumption of normal distribution. Gross [8], estimate the probability by MCS. Remaining life assessment of low pressure turbine rotor has been recommended based on MCS [9]. With the advent of computers, MCS has become a very common numerical method. Mark Demaria [10] used MCS to estimate Tropical Cyclone Wind Speed Probabilities. It is a random event process, similar to the scientific experimental process [11]. It is also used to model error estimation in order to estimate social risk [12].

Table 1 Numbers of Departure passenger of each station of Bannan Line during 18:00~19:00 per weekday.

	Station	**Departure Passengers**
a	Yongning	1607
b	Tucheng	860
c	Haishian	2951
d	Far East Hospital	2080
e	Futhong	4271
f	Banqiao	4956
g	Xinpu	6074
h	Jiang zihui	3863
i	Longshan Temple	3244
j	Ximen	6858
k	Taipei Main Station	8746
l	Shandao Temple	2541
m	Zhongxiao Xinsheng	2377
n	Fuxing	4784
o	Zhongxiao Dunhua	4701
p	S.Y.S. Memorial Hall	2603
q	Taipei City Hall	5015
r	Yongchun	2449
s	Houshanpi	2537
t	Kunyang	1502
u	Nangang	1103
v	Taipei Nangang Exhibition Center	1977

To analyze the optimization frequency of the Bannan Line of Taipei MTR from inputting each of the 22 independent parameters from each correspondence station the final income of each train plays a important role of this research. Table 1 shows the arrival of passengers for 22 stations of the Bannan Line during the time 18:00~19:00 of one weekday. In this paper, the Random Number Generator is made by Linear Congruential Generator (LCG) the equation is

$$Y_{i+1} = [AY_i + C] \quad MOD \quad M \qquad \text{i} = 0,1,2,..... \tag{1}$$

where Y_0 is the seed value, A is the constant multiplier, C is the increment, and M is the modulus. The value of Y_0 is preset. The sequence of all other Y_i's will be obtained according to eq. (1) and the normalized value (Pseudo-Random Number) R_0, R_1, R_2, …., R_i will be assigned from eq. (2)

$$R_i = \frac{Y_i}{M} \quad . \tag{2}$$

Performing each Monte Carlo iteration, a random number is generated for each random variable of those 22 stations as described above. The 22 simulations are processed deterministically for a few chosen standard deviations of the above mentioned 22 random variables. It is worth mentioning we repeated the iteration 30,000 times to get the sample of the profile of the Bannan Line with different departure frequencies.

3. Simulation the income of each train

To simulate the income of each train we have to set up a model to get the beginning and the end point of each passenger journey. It is reasonable to assume the alighting rate can be obtained as per the equations below:

$$GOR_s = \left(\frac{DP_s - \frac{DP_s}{n}}{\sum_{s=1}^{22} DP_s}\right)100\% \qquad s = 1,2,3,.....,11 \quad \text{(before station k)} \tag{3}$$

$$GOR_s = \left(\frac{\frac{DP_s}{n}}{\sum_{s=1}^{22} DP_s}\right)100\% \qquad s = 12,13,14,.....,22 \quad \text{(after station k)} \tag{4}$$

where GOR_s is the alighting rate of each station,

DP_s is the number of the departing passengers from of each station,

n is a specific number corresponding to each station and described in Table 2.

Table 2 Values of *n* at each station where the alphabet corresponds to Table 1.

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*	*k*	*l*	*m*	*n*	*o*	*p*	*q*	*r*	*s*	*t*	*u*	v
n	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2	1.9	1.8	1.7	1.6	1.5	1.4	1.3	1.2	1.1	1.1	1

It is worthy of mention that the value of *n* for each station is assumed to have linearly increase from station *a* to *k* and then to decrease linearly from station *k* to *v*. It is reasonable that $n = 2$ at station *k* because station *k* is the central station of the line, i.e., the number of passenger for each journey is equal.

Table 3 Ticket Price of Bannan Line where the alphabet corresponds to Table 1.

Station	***a***	***b***	***c***	***d***	***e***	***f***	***g***	***h***	***i***	***j***	***k***	***l***	***m***	***n***	***o***	***p***	***q***	***r***	***s***	***t***	***u***	***v***
a	0																					
b	20																					
c	20	20																				
d	20	20	20																			
e	25	20	20	20																		
f	25	25	20	20	20																	
g	25	25	20	20	20	20																
h	30	25	25	20	20	20	20															
i	35	30	30	25	25	25	20	20														
j	35	35	30	30	25	25	25	20	20													
k	35	35	35	30	30	25	25	25	20	20												
l	40	35	35	30	30	30	25	25	20	20	20											
m	40	40	35	35	30	30	30	25	20	20	20	20										
n	40	40	40	35	35	30	30	30	25	20	20	20	20									
o	45	40	40	35	35	35	30	30	25	20	20	20	20	20								
p	45	45	40	35	35	35	30	30	25	25	20	20	20	20	20							
q	45	45	40	40	35	35	35	30	25	25	20	20	20	20	20	20						
r	50	45	45	40	40	35	35	35	30	25	25	25	20	20	20	20	20					
s	50	45	45	40	40	40	35	35	30	30	25	25	25	20	20	20	20	20				
t	50	50	45	45	40	40	40	35	30	30	25	25	25	25	20	20	20	20	20			
u	55	50	50	45	45	45	40	40	35	30	30	30	25	25	25	25	20	20	20	20		
v	55	55	50	50	45	45	45	40	35	35	30	30	30	25	25	25	25	20	20	20	20	0

After we have GOR_s of each station and consult the transportation fee listed in, we can than calculate the total income of each train by summation all incomes of each 22 stations. The ticket price of Bannan Line is shown on Table 3.

4. Results

The results of five different departure frequencies, i.e., 1, 2, 3, 10 and 20 trains each hour are shown in Tables 4 to 8. It is obvious that the income of each train can be calculated by the equations (5) and (6) below.

$$\mathrm{I} = \sum_{\mathrm{i}=1}^{21} (\mathrm{Inc}) \tag{5}$$

$$\mathrm{Inc} = \sum_{1}^{\mathrm{Num}} \mathrm{Pri} \tag{6}$$

I: Total income of the train,

Inc: Income of each station,

Num: Number of passengers who board the train at on a specific station,

Pri: Ticket price that of each passenger needs to pay.

It is obvious from Table 4 where the departure frequency is 1 train/hr that some passengers cannot board the train at stations f, j, k, n, o, and q for the reason that the train's upper loading limit is 1,936 passengers (highlighted with yellow background). Therefore, the total income is NTD$584,972 which is less than others. However, if we double the frequency, it can be seen from Table 5 that only if the passengers board the train at stations other than station k, then the total income will increase to NTD$677,840 ($338920 \times 2$). If the frequency increase three times, as the original Table 6 shows, the passengers of every station can get on the train and the total income increases to NTD$691,308 ($230436 \times 3$). Even if everyone can board the train, it will likely be crowded, especially at stations j and k. For the sake of passenger comfort level, we increase the frequency ten times from the original. It is very clear that even if the income is NTD$691,330 ($69133 \times 10$) which maintains the same income as Table 7, the comfort level is increased along with the willingness of passengers to take the line. Table 8 shows the result of 20 trains/hr with the income NTD$681,040 ($34052 \times 20$).

Table 4 Passengers of each station and total income of each train with departure frequency 1 train/hr where the alphabet corresponds to Table 1.

1 train/hr, passangers and income

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*	*k*	*l*	*m*	*n*	*o*	*p*	*q*	*r*	*s*	*t*	*u*	*v*	Rev
GOR	0	0.2	1.3	1.2	3.1	4.2	5.8	4	3.7	9.2	11.1	3.4	3.4	7.2	7.5	4.4	9.1	4.7	0.1	3.5	2.6	0	
Ticket Price	0	20	20	20	25	25	25	30	35	35	35	40	40	40	45	45	45	50	50	50	55	55	
Passangers	1620	410	930	1020	1260	1936	1860	1010	1230	1936	1936	1910	1780	1936	1936	1180	1936	570	330	210	120	0	
a	65	421	389	1256	1701	2349	1944	2098	5216	6294	2203	2203	4666	5468	3208	6634	3807	81	2835	2317	0	0	55153
b		107	98	318	431	595	492	531	1320	1593	558	558	1181	1384	812	1679	964	21	718	586	0	0	13942
c			223	721	977	1349	1116	1204	2995	3613	1265	1265	2678	3139	1841	3808	2186	47	1628	1330	0	0	31383
d				791	1071	1479	1224	1321	3284	3963	1387	1387	2938	3443	2020	4177	2397	51	1785	1459	0	0	34175
e					1323	1827	1512	1632	4057	4895	1714	1714	3629	4253	2495	5160	2961	63	2205	1802	0	0	41240
f						2807	2323	2507	6234	7521	2633	2633	5576	6534	3833	7928	4550	97	3388	2768	0	0	61332
g							2232	2409	5989	7226	2530	2530	5357	6278	3683	7617	4371	93	3255	2660	0	0	56228
h								1308	3252	3924	1374	1374	2909	3409	2000	4136	2374	51	1768	1444	0	0	29320
i									3961	4779	1673	1673	3542	4151	2435	5037	2891	62	2153	1759	0	0	34114
j										7521	2633	2633	5576	6534	3833	7928	4550	97	3388	2768	0	0	47461
k											2633	2633	5576	6534	3833	7928	4550	97	3388	2768	0	0	39940
l												2598	5501	6446	3782	7821	4489	96	3343	2731	0	0	36806
m													5126	6008	3524	7289	4183	89	3115	2545	0	0	31880
n														6534	3833	7928	4550	97	3388	2768	0	0	29098
o															3833	7928	4550	97	3388	2768	0	0	22564
p																4832	931	20	693	566	0	0	7042
q																	4550	97	3388	2768	0	0	10803
r																		29	998	815	0	0	1841
s																			193	157	0	0	350
t																				300	0	0	300
u																					0	0	0
v																							

Total Income= 584972

Note: 1936 is the upper limit of passangers each train

Table 5 Passengers of each station and total income of each train with departure frequency 2 trains/hr where the alphabet corresponds to Table 1.

2 trains/hr, passangers and income

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*	*k*	*l*	*m*	*n*	*o*	*p*	*q*	*r*	*s*	*t*	*u*	*v*	Rev
GOR	0	0.2	1.3	1.2	3.1	4.2	5.8	4	3.7	9.2	11.1	3.4	3.4	7.2	7.5	4.4	9.1	4.7	0.1	3.5	2.6	0	
Ticket Price	0	20	20	20	25	25	25	30	35	35	35	40	40	40	45	45	45	50	50	50	55	55	
Passangers	810	205	465	510	630	1020	930	505	615	1770	1936	955	890	1030	1100	590	1220	285	165	105	60	0	
a	32	211	194	628	851	1175	972	1049	2608	3147	1102	1102	2333	2734	1604	3317	1904	41	1418	1158	0	0	27576
b		53	49	159	215	297	246	265	660	796	279	279	590	692	406	839	482	10	359	293	0	0	6971
c			112	360	488	674	558	602	1497	1807	632	632	1339	1569	921	1904	1093	23	814	665	0	0	15691
d				395	536	740	612	660	1642	1981	694	694	1469	1721	1010	2088	1199	26	893	729	0	0	17088
e					662	914	756	816	2029	2448	857	857	1814	2126	1247	2580	1481	32	1103	901	0	0	20620
f						1479	1224	1321	3284	3963	1387	1387	2938	3443	2020	4177	2397	51	1785	1459	0	0	32314
g							1116	1204	2995	3613	1265	1265	2678	3139	1841	3808	2186	47	1628	1330	0	0	28114
h								654	1626	1962	687	687	1454	1704	1000	2068	1187	25	884	722	0	0	14660
i									1980	2389	836	836	1771	2076	1218	2518	1445	31	1076	879	0	0	17057
j										6876	2407	2407	5098	5974	3505	7248	4160	89	3098	2531	0	0	43392
k											2633	2633	5576	6534	3833	7928	4550	97	3388	2768	0	0	39940
l												1299	2750	3223	1891	3911	2244	48	1671	1366	0	0	18403
m													2563	3004	1762	3645	2092	45	1558	1273	0	0	15940
n														3476	2039	4218	2421	52	1803	1473	0	0	15481
o															2178	4505	2585	55	1925	1573	0	0	12821
p																2416	931	20	693	566	0	0	4626
q																	2867	61	2135	1745	0	0	6808
r																		14	499	408	0	0	921
s																			193	157	0	0	350
t																				150	0	0	150
u																					0	0	0
v																							

Total Income= 338920

Note: 1936 is the upper limit of passangers each train

Table 6 Passengers of each station and total income of each train with departure frequency 3 trains/hr where the alphabet corresponds to Table 1.

3 trains/hr, passangers and income

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*	*k*	*l*	*m*	*n*	*o*	*p*	*q*	*r*	*s*	*t*	*u*	*v*	Rev
GOR	0	0.2	1.3	1.2	3.1	4.2	5.8	4	3.7	9.2	11.1	3.4	3.4	7.2	7.5	4.4	9.1	4.7	0.1	3.5	2.6	0	
Ticket Pric	0	20	20	20	25	25	25	30	35	35	35	40	40	40	45	45	45	50	50	50	55	55	
Passangers	538	136	318	336	421	680	628	346	409	1189	1426	637	595	689	730	396	815	199	110	55	37		
a	22	140	129	417	565	780	646	697	1732	2090	732	732	1549	1816	1065	2203	1264	27	942	769	0	0	18316
b		35	33	105	143	197	163	176	438	528	185	185	392	459	269	557	320	7	238	194	0	0	4625
c			76	246	334	461	382	412	1024	1235	432	432	916	1073	630	1302	747	16	557	455	0	0	10731
d				260	353	487	403	435	1082	1305	457	457	968	1134	665	1376	790	17	588	480	0	0	11258
e					442	610	505	545	1356	1636	573	573	1212	1421	834	1724	989	21	737	602	0	0	13779
f						986	816	881	2190	2642	925	925	1958	2295	1346	2785	1598	34	1190	972	0	0	21542
g							754	813	2022	2440	854	854	1809	2120	1243	2572	1476	31	1099	898	0	0	18984
h								448	1114	1344	471	471	996	1168	685	1417	813	17	606	495	0	0	10044
i									1317	1589	556	556	1178	1380	810	1675	961	20	716	585	0	0	11344
j										4619	1617	1617	3424	4013	2354	4869	2794	59	2081	1700	0	0	29148
k											1939	1939	4107	4813	2823	5839	3351	71	2496	2039	0	0	29418
l												866	1835	2150	1261	2609	1497	32	1115	911	0	0	12275
m													1714	2008	1178	2437	1398	30	1041	851	0	0	10656
n														2325	1364	2821	1619	34	1206	985	0	0	10356
o															1445	2989	1716	37	1278	1044	0	0	8508
p																1622	931	20	693	566	0	0	3831
q																	1915	41	1426	1165	0	0	4548
r																		10	348	285	0	0	643
s																			193	157	0	0	350
t																				79	0	0	79
u																					0	0	0
v																							

Tota Income = 230436

Table 7 Passengers of each station and total income of each train with departure frequency 10 trains/hr where the alphabet corresponds to Table 1.

10 trains/h, passangers and income

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*	*k*	*l*	*m*	*n*	*o*	*p*	*q*	*r*	*s*	*t*	*u*	*v*	Rev
GOR	0	0.2	1.3	1.2	3.1	4.2	5.8	4	3.7	9.2	11	3.4	3.4	7.2	7.5	4.4	9.1	5	0.1	3.5	2.6	0	
Ticket Pric	0	20	20	20	25	25	25	30	35	35	35	40	40	40	45	45	45	50	50	50	55	55	
Passangers	162	41	93	102	126	204	186	101	123	354	427	191	178	206	220	118	244	57	33	21	12	0	
a	6	42	39	126	170	235	194	210	522	629	220	220	467	547	321	663	381	8	284	232	0	0	5515
b		11	10	32	43	59	49	53	132	159	56	56	118	138	81	168	96	2	72	59	0	0	1394
c			22	72	98	135	112	120	299	361	126	126	268	314	184	381	219	5	163	133	0	0	3138
d				79	107	148	122	132	328	396	139	139	294	344	202	418	240	5	179	146	0	0	3418
e					132	183	151	163	406	490	171	171	363	425	249	516	516	6	221	180	0	0	4344
f						296	245	264	657	793	277	277	588	689	404	835	479	10	357	292	0	0	6463
g							223	241	599	723	253	253	536	628	368	762	437	9	326	266	0	0	5623
h								131	325	392	137	137	291	341	200	414	237	5	177	144	0	0	2932
i									396	478	167	167	354	415	244	504	289	6	215	176	0	0	3411
j										1375	481	481	1020	1195	701	1450	832	18	620	506	0	0	8678
k											581	581	1230	1441	845	1749	1003	21	747	611	0	0	8809
l												260	550	645	378	782	449	10	334	273	0	0	3681
m													513	601	352	729	418	9	312	312	0	0	3245
n														695	408	844	484	10	361	295	0	0	3096
o															436	901	517	11	385	315	0	0	2564
p																483	277	6	207	169	0	0	1142
q																	573	12	427	349	0	0	1362
r																		3	100	82	0	0	184
s																			58	47	0	0	105
t																				30	0	0	30
u																					0	0	0
v																						0	0

Total Income= 69133

Table 8 Passengers of each station and total income of each train with departure frequency 20 trains/hr where the alphabet corresponds to Table 1.

20 trains/hr, passangers and income

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*	*k*	*l*	*m*	*n*	*o*	*p*	*q*	*r*	*s*	*t*	*u*	*v*	Rev
GOR	0	0	1	1	3	4.2	5.8	4	3.7	9.2	11.1	3.4	3.4	7.2	7.5	4.4	9.1	5	0.1	3.5	3	0	
Ticket Pric	0	20	20	20	25	25	25	30	35	35	35	40	40	40	45	45	45	50	50	50	55	55	
Passangers	81	20	46	51	63	102	93	50	61	177	213	95	89	103	110	29	122	28	16	10	6	0	
a	3	21	19	63	85	117	97	105	261	315	110	110	233	273	160	332	190	4	142	116	0	0	2758
b		5	5	16	21	29	24	26	64	78	27	27	58	68	40	82	47	1	35	29	0	0	680
c			11	36	48	67	55	60	148	179	63	63	132	155	91	188	108	2	81	66	0	0	1552
d				40	54	74	61	66	164	198	69	69	147	172	101	209	120	3	89	73	0	0	1709
e					66	91	76	82	203	245	86	86	181	213	125	258	148	3	110	90	0	0	2062
f						148	122	132	328	396	139	139	294	344	202	418	240	5	179	146	0	0	3231
g							112	120	299	361	126	126	268	314	184	381	219	5	163	133	0	0	2811
h								65	161	194	68	68	144	169	99	205	118	3	88	72	0	0	1452
i									196	237	83	83	176	206	121	250	143	3	107	87	0	0	1692
j										688	241	241	510	597	350	725	416	9	310	253	0	0	4339
k											290	290	613	719	422	872	501	11	373	305	0	0	4394
l												129	274	321	188	389	223	5	166	136	0	0	1831
m													256	300	176	364	209	4	156	127	0	0	1594
n														348	204	422	242	5	180	147	0	0	1548
o															218	450	259	6	193	157	0	0	1282
p																119	68	1	51	41	0	0	281
q																	287	6	214	174	0	0	681
r																		1	49	40	0	0	90
s																			28	23	0	0	51
t																				14	0	0	14
u																					0	0	0
v																							0

Tota Income = 34052

It can be seen from Fig. 1 that the total income almost the same after the frequency increases up to 3 trains/hr and there is a maximum income value at the frequency of 10 trains/hr.

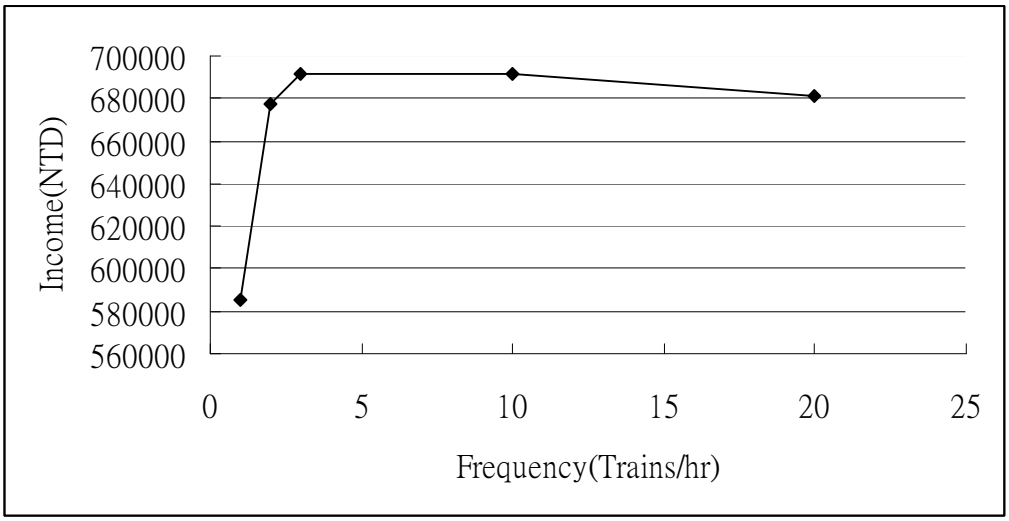

Figure 1 Income of different departing frequencies.

Figs 2-6 show the distribution and statistical data of 30,000 iterations of each different frequency. Each of them is normal standard distribution. Table 9 gives statistical data of the different departure frequency of each train.

According to the results shown above, it is wise to choose the departure frequency 10 trains/hr. The reason why we did not didn't pick up 20 trains/hr is that there is still an electricity cost of NTD$50,000 for each train. After consulting

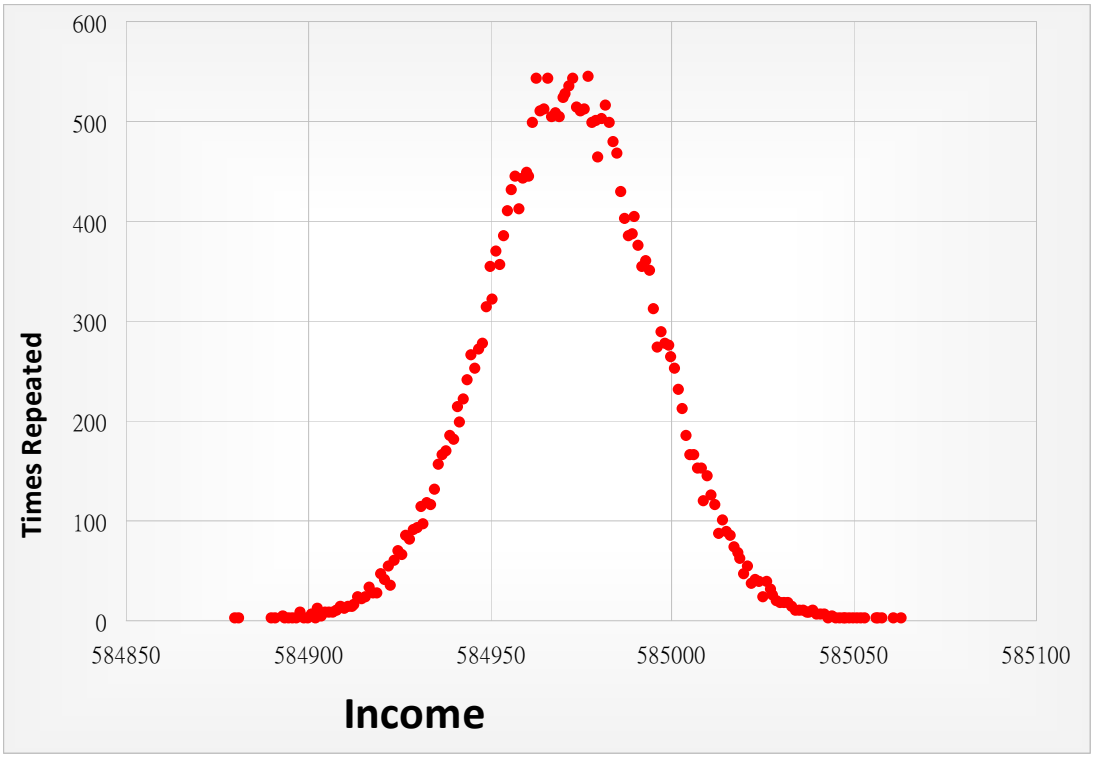

Figure 2 Distribution of 30,000 iterations of departing frequency 1 train/hr.

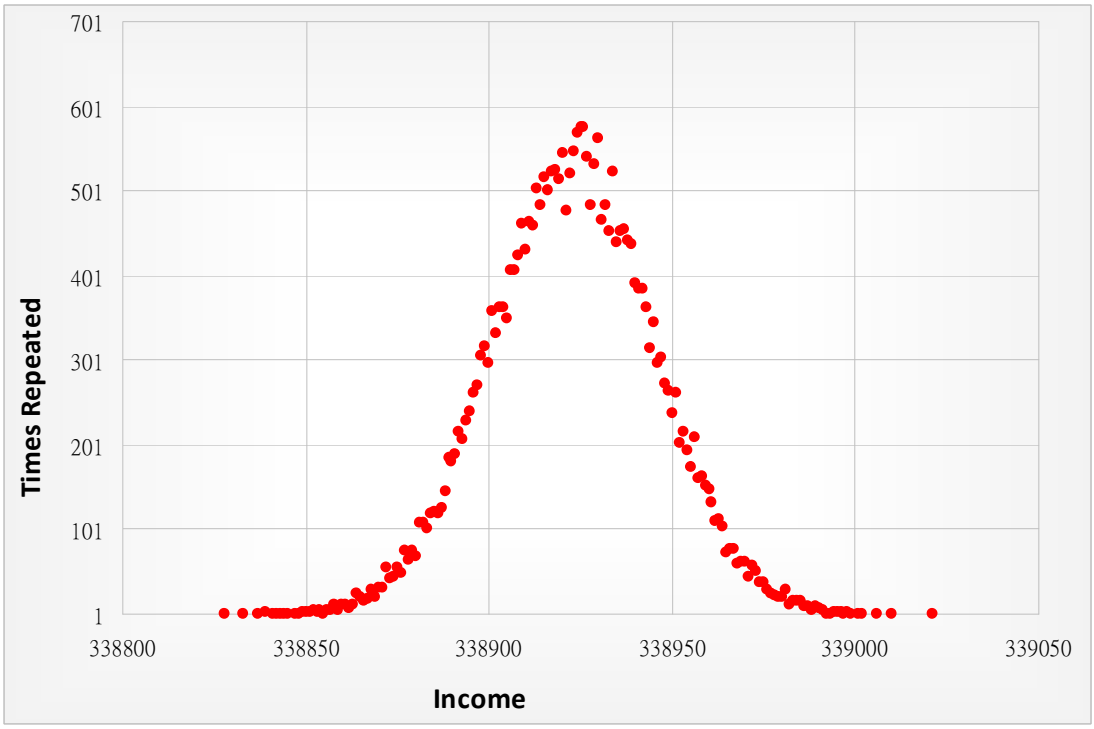

Figure 3 Distribution of 30,000 iterations of departing frequency 2 trains/hr.

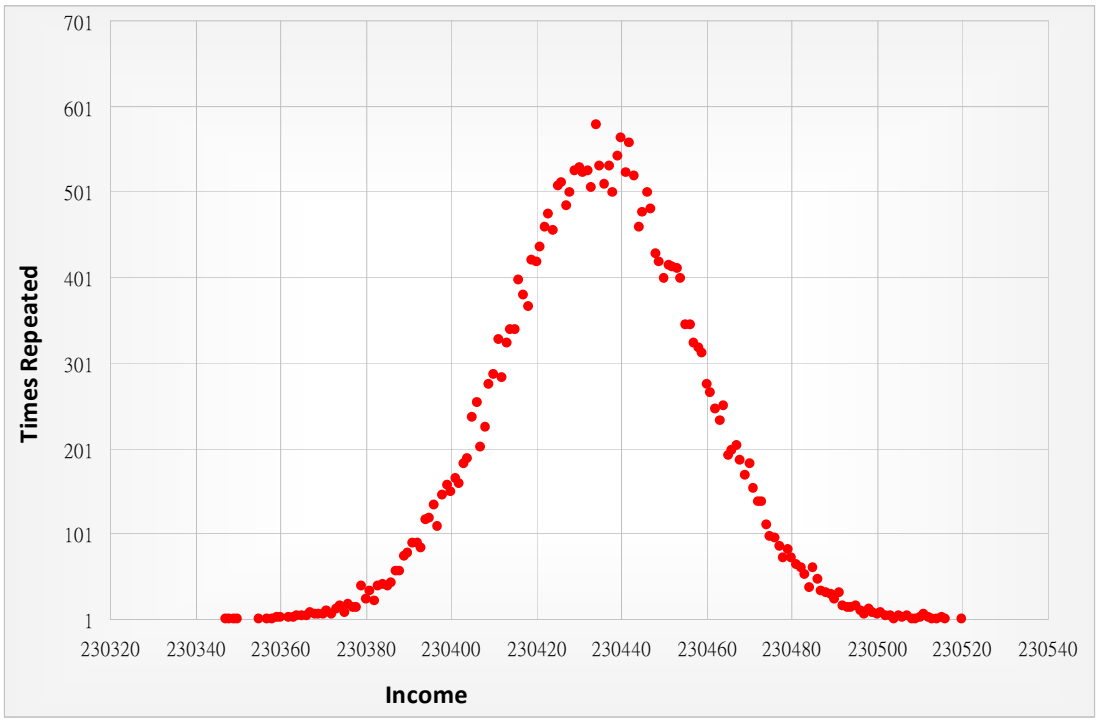

Figure 4 Distribution of 30,000 iterations of departing frequency 3 trains/hr.

the departure time table for the Bannan Line we found that the departure frequency is 11 trains/hr which matches the results of our research.

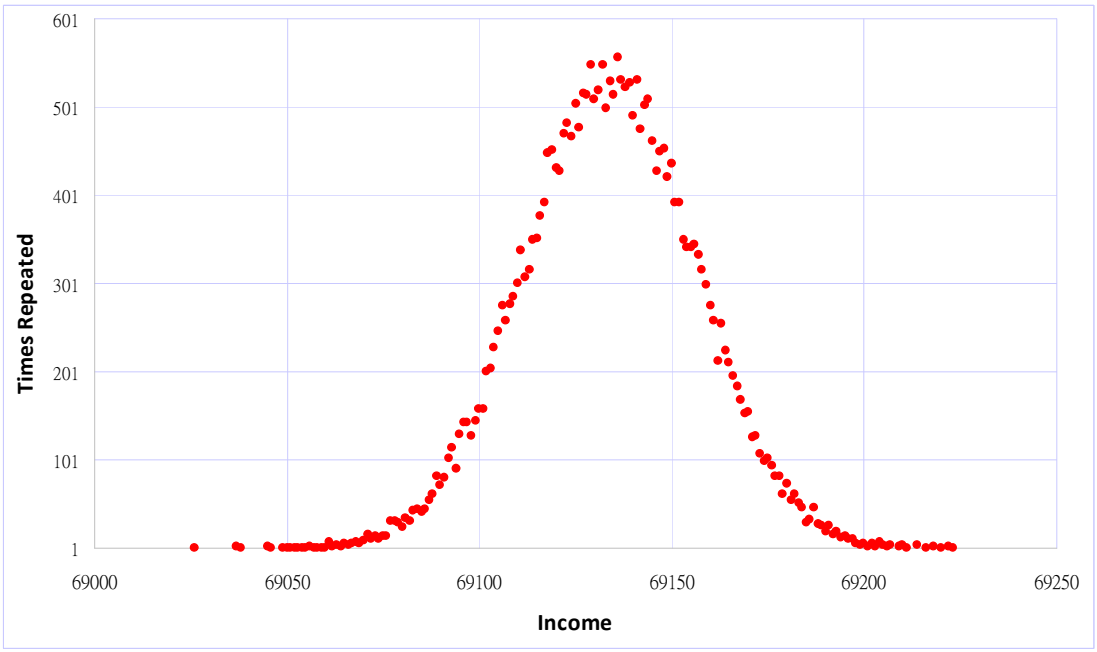

Figure 5 Distribution of 30,000 iterations of departing frequency 10 trains/hr.

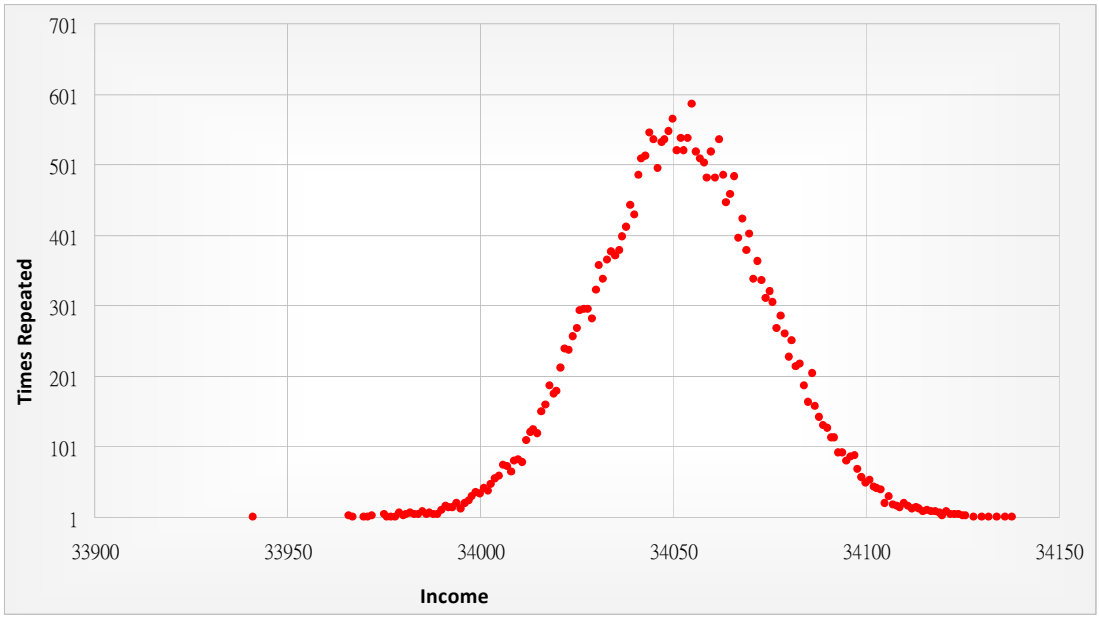

Figure 6 Distribution of 30,000 iterations of departing frequency 20 trains/hr.

Table 9: Statistical data of different departure frequencies.

Statistical Data					
departure frequency	1	2	3	4	5
mean	584972	338923	230435	69134	34052
standard error	0.13	0.129	0.13	0.13	0.13
median	584972	338923	230435	69134	34052
mode	584977	338926	230434	69133	34017
standard deviation	22.58	22.32	22	22.51	22
variance	509.8	498.34	500	507	496
kurtosis	0.018	0.0411	-0.02	0.05	0.001
skewness	-0.015	-0.013	-0.01	-0.02	-0.011
range	183	193	173	197	197
minimum	584880	338828	230347	69026	33941
maximum	585063	339021	230520	69223	34138
numbers of iteration	30000	30000	30000	30000	30000

5. Conclusions

This research focuses on the MRT of the Bannan line, which is the most popular line of the 5 lines of the MRT system. Based on the total entrance of passengers into of each station during the time of 18:00~19:00 on one weekday, we set up a procedure to obtain the optimization of departure frequency of trains by using Monte Carlo Simulation for 60,000 iterations. The frequency is 10 trains/hr which is almost the same as the real situation, i.e., 11 trains/hr. We can apply this research to other lines of the MRT and even other cities. However, there are still a few points that need to be discussed.

a. The given data varies according to the number assigned to the variable n. It is just an assumption for the specific number of each station *n* to calculate the number of passenger of different journeys. The results will be varied according to the different ways this number.
b. The number of passenger alighting the train is another assumption in this research. It is based on the GOR_s number which is the alighting rate calculated from the number of the departing passengers from each station. The results will be different if this assumption is changed.
c. Cost is an important factor of the total profit of the Bannan Line. We made no mention of any cost of this research owing to the reason that the comfort level is another factor of passengers' willingness to take the MRT.

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