

# Pallet Optimization of the Heavy Rotary Table Load Carrying System

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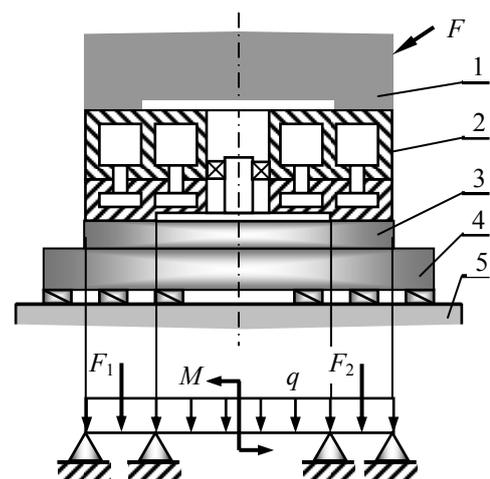
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**Abstract.** The pallet optimization of the heavy rotary table load-carrying system, which is a part of the multi-purpose machine, is considered in terms of the deterministic and probabilistic models. As a result of optimum design in case of the deterministic model the mass of the pallet is reduced by 35.5 % in comparison with a serial model. The evaluation of the influence of optimization problem limitations on design variables confirms the importance of rigidity criterion in relation to other criteria. Calculation for probabilistic model allows reducing the mass of the construction by 27 % in comparison with the deterministic model. Considering a work piece rigidity on the basis of a conventional work piece of the minimum rigidity (without stiffening ribs etc.) leads to reducing of the pallet mass by 22.3 % in comparison with the deterministic model.

## 1. Introduction

Over the past decades, column-and-knee-type machines were widely used due to their ability to process the work piece from all sides. On the first modifications of the machines, the work piece rotation was carried out with the help of crane, the procedure taking from 0.5 to 3 hours. In most cases, the processing of the work piece surface on one side takes less than a working shift duration, which leads to intensive crane operation and working efficiency reduction. The application of rotary tables eliminates the mentioned problems. The work piece is placed onto the rotating part of the table (pallet) and the desired position precision in relation to the machine is achieved by the movement of the rotating part.

Heavy column-and-knee-type machines [1] are often used for the processing of the mechanisms basic parts. The outer dimensions of these parts have greatly changed with the advances in technology. Originally, the dimensions of the work pieces were no bigger than a flatcar and the geometrical precision of the surfaces and their relationship was achieved at the final assembly stage



**Figure 1.** Pallet calculation scheme:  
1 – work piece, 2 – pallet, 3 – rotating table, 4 – bed, 5 – foundation

by the manual fitting. This imposed certain limits to the acceptable table body deformation; e.g. multisupport beam (Figure 1) was used as an analytical model for the body of the rotating part (pallet) with strength of materials methods for the rigidity calculation.

With the growth of the mechanisms operating parameters, such as capacity, pressure, temperature and load, there was an increase in outer dimensions and basic parts production precision, which in turn leads to the enlargement of the rotating tables clamping surface and work piece carrying capacity reducing at the same time the acceptable deformation of the table body caused by the weight of the processed work pieces. Under these conditions the necessity and importance of rotating tables strength, rigidity and other performance criteria validity is increasing. The updated requirements to the rotating table construction can be partially satisfied by the designer decisions aimed at the symmetrical load distribution provision, supporting plane dimensional growth, unsupported length reduction, but mostly at the expense of manifold increase in the cross-section moment of inertia, i.e. outer dimensions and mass.

## 2. The statement of the problem

Spatial repositioning of a spot on the processed work piece, which is placed on the rotating table, in the process of its reorientation within the operating zone depends on the table rigidity, work piece rigidity, the mass of the processed work piece and its center of gravity position in relation to the table pivot axis as well as on the magnitude and direction of the cutting force. The mentioned parameters with the exception of the table rigidity are the initial data to be considered while designing. They are in part defined by the work pieces nomenclature, in part by the specifications of the machine and cutting tools. The table rigidity conditioned by the pallet rigidity, bed and foundation bodies and joints contact rigidity is to be determined during machine designing.

The implemented designing decisions in the table load carrying arrangement allow the bed and the foundation to experience mainly the compressive deformation under the external load. Therefore, the height of the bed and foundation is set to minimal allowed value for the design and technology reasons. The pallet mostly undergoes the bending deformation. Since the bending displacement is considerably greater than the compressive one and considering the fact that the load carrying system rigidity is primarily conditioned by the pallet rigidity, the prior attention will be given to the design of the pallet as the most deformable structural element.

The serial pallet (Figure 2) is a three-dimensional thin-walled rectangular formation of cellular structure with the dimensions  $L = 5.6$  m,  $B = 3.6$  m,  $H = 0.8$  m [2]. There are lengthwise and crosswise stiffening ribs of rectangular cross section along the pallet lower edge. The pallet body rests on the bed guide track of ring cross section (outer diameter 3.6 m).

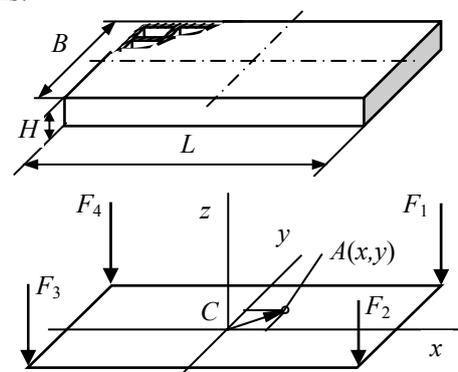
The pallet analytical model is based on the following propositions:

1. The pallets body is modeled with the help of plate-like rectangular finite element and rod finite element (ribs).

2. The pallet rests on rigid circular guide track of the table bed.

3. The calculated load is the pallet weight and the processed work piece weight (2 MN). The cutting forces due to their relative insignificance in comparison with the mentioned loads are not taken into account because during the finishing front milling work the maximum component of cutting force is 3.0 kN.

4. The processed work piece is supposed to be placed on the technological base, which coincides with the pallet corner zones. The external load  $F$  from the weights of the work piece and pallet is in extreme event characterized by forces  $F_i$  ( $i = 1, \dots, 4$ ), applied at the pallet corner points. The distribution of the load from the work piece weight in the pallet corner points is calculated by the strength of materials methods:



**Figure 2.** Pallet calculation scheme

$$F_i = (1/4)F \left[ 1 \pm x/(L/2) \pm y/(B/2) \right] \quad (1)$$

5. In general case the work piece center of gravity  $A$  (Figure 2) is moved in the plane  $xy$  relative to the table pivot axis by  $1/20$  of the length and  $1/30$  of the breadth of the pallet; this is the maximum eccentricity value found out by the analysis of the occurring large scale work pieces geometry.

### 3. Results and Discussion

#### a. Deterministic model

The coordinates of point  $A$  to apply the resultant load:

$$x = L/20 = 5.6/20 = 0.28 \text{ m}, y = B/30 = 3.6/30 = 0.12 \text{ m}.$$

The forces  $F_1, \dots, F_4$  applied at the pallet corner points are defined with the help of the formula (1).

The problem of the pallet optimal design is stated as follows:

$$\text{to minimize} \quad \Psi_0 = \rho \left( \sum_{i=1}^k V_i + \sum_{j=1}^m V_j \right) \quad (2)$$

limited by:

$$\begin{aligned} \text{displacement} & \quad \psi_1 = 1 - \delta / [\delta] \geq 0, \\ \text{stress} & \quad \psi_2 = 1 - \sigma_{\text{eqv}} / \sigma_{\text{allow}} \geq 0, \\ \text{stability} & \quad \psi_3 = 1 - n\sigma / \sigma_{\text{cr}} \geq 0, \\ \text{frequency} & \quad \psi_4 = p_1 / [p_1] - 1 \geq 0 \\ \text{design variables} & \quad \psi_5 = V_i \geq 0, \quad i = 1, \dots, k, \\ & \quad \psi_6 = V_j \geq 0, \quad j = 1, \dots, m, \end{aligned}$$

with  $k, m$  – being the number of plate-like rectangular and rod finite elements (FE);  $\rho$  – density of the material;  $V$  – the volume of the finite element;  $\delta, [\delta]$  – calculated and permissible relative deformation, rated in the direction perpendicular to the pallet plane;  $\sigma_{\text{eqv}}, \sigma_{\text{allow}}$  – equivalent stress and allowable stress;  $n = 2$  – stability factor;  $\sigma, \sigma_{\text{cr}}$  – compressive stress and the critical compressive stress;  $p_1, [p_1] = 12 \text{ Hz}$  – calculated value and the lower limit (defined by the spindle rotating speed  $500 \text{ min}^{-1}$  with 30% resonance offset) of the first natural frequency.

The design variable are the thickness of the body wall –  $t_c$  and the thickness of the rib –  $t_p$  (rib breadth is constant). The pallet outer dimensions (length, breadth and height) are defined by the preliminary specifications and do not vary.

The main criterion to characterize the pallet rigidity is the pallet tilting angle because it directly influences the performance of hydrostatic guide track. The calculated pallet rigidity standard was introduced on the basis of this criterion, i.e. the relative vertical deformation  $[\delta] = 2 \cdot 10^{-5}$  with guide track breadth being 1 m and oil lubricant layer thickness  $4 \cdot 10^{-5} \text{ m}$ .

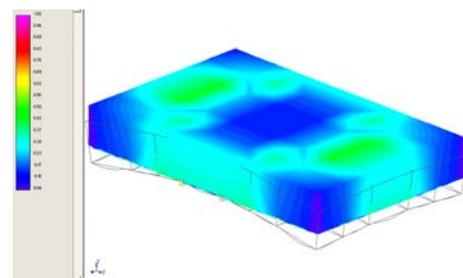
The objective function of the problem (2) is the mass of the formation as firstly, the formation mass of several dozen tons is to be calculated, and secondly, certain criteria such as displacement rigidity and stress stability and some others can be assigned within permissible limits.

The problem (2) is solved by the penalty function method [3, 4] in the form

$$\varphi = \Psi_0 / \Psi_0^H + r \sum_{i=1}^4 (1/\psi_i), \quad (3)$$

with  $\Psi_0^H$  – the serial pallet original mass prior to optimization;  $r$  – small positive parameter. The problem is solved by the function (3) unconstrained minimization for the decreasing sequence of parameter  $r$  values with the help of Davidon-Fletcher-Powell method [4].

Figure 3 demonstrates the deformation of the pallet, Table 1 presents the calculation results based only on Finite Elements Method (FEM) by the limited enumeration of possibilities as well as on FEM and optimization methods jointly. As a result of optimization designing the pallets mass



**Figure 3.** Deformed pallet state

was reduced by 35.5% in comparison with the serial version, which virtually coincides with the results achieved by FEM only. The difference lies in the elements dimensions, which is likely to be connected with different design variables response during the optimal search. The maximum stress for the optimal pallet is 13.4 MPa, the discrepancy in rigidity criterion is 0.65 %. The values of the three first natural frequencies are presented in Table 2. As the table indicates, the lowest natural frequency of the pallet is nearly 9 times as high as the frequency of the constrained oscillations, which additionally proves the rightful limitation by the first natural frequency.

**Table 1.** The Results of Pallet Optimization

Pallet Design	Thickness, mm				Maximum vertical displacement, mm	Mass, tons
	Upper board	Side wall	Inside wall	Rib		
Serial	60.0	60.0	50.0	60.0	0.249	38.12
FEM	60.0	30.0	20.0	60.0	0.427	24.40
Original to be optimized	70.0	40.0	40.0	70.0	-	-
Optimal	29.0	36.3	36.3	69.5	0.452	24.59

**Table 2.** The Range of Optimal Pallet Natural Frequencies

The form of oscillations	Frequency, Hz		
	axis <i>x</i>	axis <i>y</i>	axis <i>z</i>
1	269.2	272.2	88.6
2	283.2	294.5	192.0
3	306.8	526.1	268.2

*Sensitivity analysis* [5]. To assess the impact of problem (2) limitations onto the design variables we investigated the behavior design variables variations around the optimal point decision. To do so we fix all variables but one and investigate the change in displacement, stress and frequency. The parameter change span is set to be  $\pm 25\%$  in order to round the results for the practical use. The sequence of the design variables changes is given in Table 3. The change of limitation was defined in relation to the least value of the corresponding limitation while the design variable was changing from  $-25\%$  to  $+25\%$ , i.e.

$$[(\Psi_{+25\%} - \Psi_{-25\%}) / \Psi_{\min}] 100 \%,$$

with

$$\Psi_{\min} = \begin{cases} \Psi_{+25\%}, & \text{if } \Psi_{+25\%} < \Psi_{-25\%} \\ \Psi_{-25\%}, & \text{if } \Psi_{+25\%} > \Psi_{-25\%} \end{cases}$$

**Table 3.** The Results of Response Analysis

Design Variables	Response Limitation, %			
	Displacement	Stress	Stability	Frequency
The thickness of the side and inside walls (0.0363 m)	73.4	54.6	51.5	5.4
The thickness of the upper board (0.0290 m)	16.7	4.9	8.2	85.8
The rib thickness (0.0695 m)	15.2	10.4	25.1	0.2

The response analysis shows that if the pallet design was to be improved in terms of natural frequency, the best results could be achieved by the varying the upper board thickness. In other cases, best results are achieved by varying the thickness of side and inside walls. Using the response analysis information the designer can carry out a systematic analysis of the design and thus refine it. The received data supports the rigidity dominance over other criteria in the process of load-carrying

structures design. It allows limiting the research efforts to one criterion, others being checked for performance consistency on the final stages of the optimal search.

### b. Probabilistic Model

Deterministic mathematical models are widely used in calculation and design. However, spatial repositioning of a spot on the processed work piece, which is placed on the rotating table, depends on certain factors, namely on the center of gravity position in relation to the table pivot axis. Due to possible operational errors (e.g. work piece misplacement on the table or equipment misuse), the misalignment of the work piece center of gravity with the table pivot axis is of statistical character. The resulting eccentricity can be treated as a normally distributed variable with mathematical expectation  $m = 0$ .

The coordinates  $(x,y)$  of the point  $A$  (Figure 2) factual load application resulting from the work piece load form a system of random variables, normal density for which is expressed by the formula [6]:

$$f(x, y) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-r^2}} \times \exp\left\{-\frac{1}{2(1-r^2)}\left[\frac{(x-m_x)^2}{\sigma_x^2} - \frac{2r(x-m_x)(y-m_y)}{\sigma_x\sigma_y} + \frac{(y-m_y)^2}{\sigma_y^2}\right]\right\} \quad (4)$$

Rectangular areas are characterized by elliptic eccentricity distribution, i.e. there is a dispersion ellipsis with a major  $a$  semiaxis and a minor  $b$  semiaxis. Assuming that semiaxes coincide with the coordinate axes, the origin of coordinates point  $C$  coincides with the dispersion center and random variables  $x,y$  are independent, the formula (4) reads:

$$f(x, y) = \frac{1}{2\pi\sigma_x\sigma_y} \exp\left[-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2}\right].$$

The full dispersion ellipsis is described by the equation [6]

$$\frac{x^2}{(4E_x)^2} + \frac{y^2}{(4E_y)^2} = 1, \quad (5)$$

with  $E_x \approx 0.675\sigma_x$ ,  $E_y \approx 0.675\sigma_y$  being major probable deviations. For the variables normally distributed within the limits of  $[-a; a]$  и  $[-b; b]$  the values are  $a/\sigma_x \approx 3$ ,  $b/\sigma_y \approx 3$ . Considering that  $a = L/20$ ,  $b = B/30$  the equation (5) takes on the following form:

$$\frac{x^2}{0.002L^2} + \frac{y^2}{0.0009B^2} = 1.$$

In order to identify the point  $A$  coordinates we formulate the optimization problem:

$$\begin{aligned} &\text{to maximize} && F_i \\ &\text{limited by} && \frac{x^2}{0.002L^2} + \frac{y^2}{0.0009B^2} = 1. \end{aligned} \quad (6)$$

The problem (6) is solved for the accepted dimensions  $L$  and  $B$  by Lagrange's method of multipliers [3], the resulting coordinates and eccentricity are:

$$x = 0.21 \text{ m}, y = 0.06 \text{ m}, e = 0.218 \text{ m}.$$

The problem of the pallet optimal design in this case is stated similarly to the problem (2). Two occurrences are taken into account while calculating the probabilistic model of pallet loading:

- 1) the values  $t_c$ ,  $t_p$  are limited by non-negativity, i.e.  $t_c = t_p \geq 0$ ;
- 2) the values  $t_c$ ,  $t_p$  are limited by the casting conditions in accordance with the formula [7]

$$t_{\min} = 10 \sqrt{(2L + B + H)/3}, \text{ mm}, \quad (7)$$

with  $L, B, H$  being formation outer dimensions in meters. In our case:  $t_c = t_p = t_{\min} = 23$  mm.

Table 4 shows the calculation results of the pallet probabilistic model in comparison with the previously investigated models.

**Table 4.** The calculation results of the three models

Pallet Model	Thickness, mm				Rigidity Discrepancy, %	Mass, tons
	Upper board	Side wall	Inside wall	Rib		
Serial	60.0	60.0	50.0	60.0	40.0	36.80
Deterministic	29.0	36.3	36.3	69.5	0.65	24.59
Probabilistic:						
$t \geq 0$	8.6	17.2	17.2	63.4	0.54	14.22
$t \geq 23$ mm	23.1	23.2	23.2	38.9	27.0	15.80

The acquired results demonstrate that in case the formation is exposed to ununiform load the probabilistic model calculation allows additional reduction of the formation mass without the loss of performance in comparison with deterministic model. The probabilistic model calculations require considering technological limitations, such as casting conditions, which determine the minimum wall thickness. The resulting formation rigidity reserve (27%) indicates the probability to search for additional designer arrangement improvement.

### c. Considering the processed work piece rigidity

We have analyzed the deterministic and probabilistic approaches to the rotating table pallet calculation. While calculating models we used only those pallet load models, which considered solely the weight of the processed work piece, not its rigidity. However, the spatial repositioning of a spot on the processed work piece, which is placed on the rotating table, depends not only on the table rigidity, but also on the work piece rigidity.

Calculating the work piece rigidity assumes that the processed work piece is firmly fixed in pallet corner zones at three points (Figure 4, the application of force points), which provides for the fact that the two load schemes (considering and not considering the work piece rigidity) become equivalent.

To assess the combined action of the system 'pallet – work piece' we suggest using a conventional basic work piece of minimum rigidity (without bridges, ribs, internal closed contours etc.) with calculated weight of 2000 kN and the cross section providing for the specified center of gravity eccentricity  $A(x,y)$  with coordinates (Figure5):

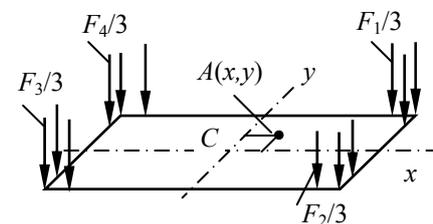
$$x = L/20 = 5.6/20 = 0.28 \text{ m}, y = B/30 = 3.6/30 = 0.12 \text{ m}.$$

All the calculations took into account the limitation (7) on the walls and ribs thickness due to casting conditions. For the specified pallet the value is  $t_{\min} = 23$  mm.

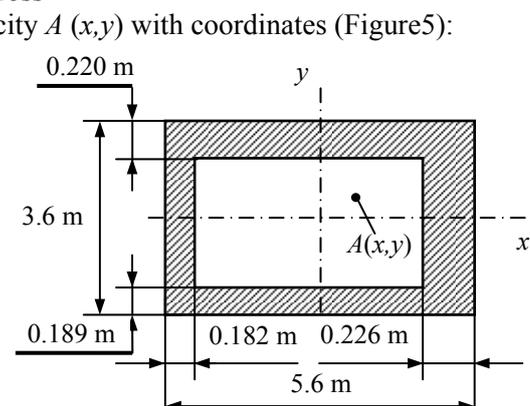
The calculation is executed with the help of bundled software APM WinMachine (version 7.0) by the finite elements method:

- 1) for the pallet with optimal dimensions,
- 2) for the pallet with body wall thickness of 23 mm.

Figure 6 displays the pallet deformed state caused by the load of a conventional work piece. The calculation results given in Table 5 prove that the rigidity of the processed work piece considerably influence the rigidity of the pallet and, consequently, the load carrying formation of the table in general, making it possible to reduce pallet mass by 22.3 %. The minimum value of 23 mm thick pallet natural frequency is 88.18 Hz. It is significantly higher than permissible natural frequency of 10.8 Hz (defined by the spindle highest



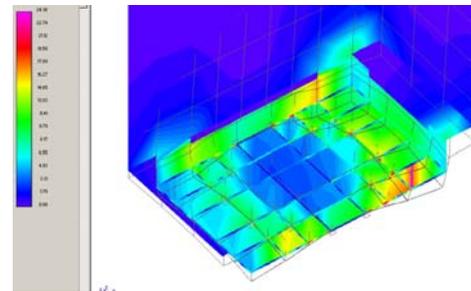
**Figure 4.** Calculation



**Figure 5.** Cross-section of a conventional work piece

frequency  $500 \text{ min}^{-1}$  with 30% resonance offset). Therefore, when the pallet mass is reduced there is no possibility for resonance to appear during the machining.

The real processed work pieces do have various elements, which magnify their rigidity (e.g. walls, ribs, inside contours etc.), thus magnifying the rigidity of the 'pallet-work piece' system. However, due to the variety of possible work pieces arrangements, and, accordingly, various cross section rigidity it proves practical to calculate the design with the help of simplified conventional work piece with minimum rigidity for certain work piece nomenclature in order to obtain the rational table formation structure. The excessive rigidity of the real work pieces in comparison with the conventional work piece becomes the rigidity reserve of the table load-carrying system.



**Figure 6.** The deformed state of the pallet with a work piece

**Table 5.** Calculation results considering the processed work piece rigidity

Loading model	Pallet elements thickness			Pallet mass tons
	Upper board	Walls	Ribs	
	mm			
Not considering the processed work piece rigidity, optimal design	29.0	36.3	69.5	24.59
Considering the processed work piece rigidity	23.0	23.0	69.5	19.11

#### 4. Conclusions

The analyzed calculation models of the heavy rotating table pallet demonstrate a wide range of rational design decisions in improving the load carrying systems of multi-purpose machines. Finite elements method in combination with optimization methods and considering the work piece rigidity on the grounds of a conventional basic work piece of minimum rigidity allow to design load-carrying machine structures without excessive properties.

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