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Dynamic Models of Robots with Elastic Hinges

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Abstract. Two dynamic models of robots with elastic hinges are considered. Dynamic models are the implementation of the method based on the Lagrange equation using the transformation matrices of elastic coordinates. Dynamic models make it possible to determine the elastic deviations from programmed motion trajectories caused by elastic deformations in hinges, which are taken into account in directions of change of the corresponding generalized coordinates. One model is the exact implementation of the Lagrange method and makes it possible to determine the total elastic deviation of the robot from the programmed motion trajectory. Another dynamic model is approximated and makes it possible to determine small elastic quasi-static deviations and elastic vibrations. The results of modeling the dynamics by two models are compared to the example of a two-link manipulator system. The considered models can be used when performing investigations of the mathematical accuracy of the robots.

1. Introduction

Let us consider that in robots the elastic elements are conventionally accumulated in the joint nodes of links – hinges. The hinge rigidity can be determined from rigidities of actual elastic elements of the drive established in this hinge by means of its reduction to one of the methods known in the applied theory of elastic vibrations.

Elastic elements are deformed under the effect of static and dynamic loads, which results in an actual motion law that will differ from the programmed one. We denote the magnitude of the deviation of the vector of generalized coordinates from the programmed motion as $\{\Delta q\} = \{\Delta q(t)\}$.

The equation of motion of the manipulator systems of robots allowing for the elastic compliance, which is taken into account in varying the direction of the generalized coordinate corresponding to this hinge, can be derived from [1] by the force of substitution of vector $\{q(t)\}$ by vector $\{q(t) + \Delta q(t)\}$.

$$[M_i(q+\Delta q)]\{\ddot{q}+\Delta\ddot{q}\}+\{\dot{q}+\Delta\dot{q}\}^T[C_i(q+\Delta q)]\{\dot{q}+\Delta\dot{q}\}=Q_{Di}(q+\Delta q)+Q_{Gi}(q+\Delta q)+Q_{Fi}(q+\Delta q),\ i=(1,\dots,n). \quad (1)$$

In this expression Q_{Di} is the generalized force, which corresponds to the forces developed by an i-drive; Q_{Gi} and Q_{Fi} *are* the generalized forces, which correspond to the force of gravity and external forces.

Equation (1) should be supplemented by the equation associating the forces developed by drives Q_{Di} with elasticity forces, which appear in transmission mechanisms of these drives. If the mass of elastic elements can be neglected and the elastic force linearly depends on Δq, then we have the expression

$$Q_{Di} = w_i \Delta q(q,t),\ i = (1,\dots,n), \tag{2}$$

where w_i is a stiffness coefficient of an *i*-hinge.

Equation (2) adds Equation (1) to the set of equations resolvable relatively all generalized coordinates $\{q\}$ and $\{\Delta q\}$. Equations (1) and (2) jointly constitute the mathematical model for modeling the dynamics of manipulator systems of robots with elastic hinges by the composite variables, which are the sum of rigid $\{q\}$ and elastic $\{\Delta q\}$ generalized coordinates.

If rigidities of elastic hinges are sufficiently large, then elastic deformations that appear in them and determine elastic coordinates $\{\Delta q\}$ can be considered as small quantities. In this case, when calculating each composite variable, we are forced to sum the quantities of different orders of magnitude in Equation (1). We can consider that another disadvantage of Equation (1) is the fact that this equation contains slow and rapid variables. The variables, which determine the programmed motion of the manipulator system, belong to a slow group of variables; while the variables, which reflect small elastic vibrations, can be referred to a rapid group. This leads to the fact that when numerically integrating differential Equation (1), we are forced to calculate high-frequency components at long time intervals with a small step.

2. Creating a dynamic model

Based on the assumption of smallness of elastic deformations, we can derive the mathematical model without mentioned disadvantages of the mathematical model. For this purpose, let us expand matrices $[M_i]$ and $[C_i]$ in the vicinity of the programmed motion into the Taylor series at $\Delta q \to 0$ retaining the summands of this series to the first infinitesimal order inclusively:

$$[M_i](q+\Delta q) = [M_i](q) + \sum_{l=1}^{n} \frac{\partial [M_i]}{\partial q_l} \Delta q_l\,,$$

$$[C_i](q+\Delta q) = [C_i](q) + \sum_{l=1}^{n} \frac{\partial [C_i]}{\partial q_l} \Delta q_l\,.$$

Matrices $[M_i]$ and $[C_i]$ are determined by the inertial parameters of the manipulator system; therefore, we can consider that they vary weakly with small variations in the vector of generalized coordinates $\{q\}$. Consequently, we can neglect the partial derivatives standing under the summation sign in the obtained expansions and retain only zero approximations of the starting matrices, i.e., we can accept

$$[M_i(q+\Delta q)] = [M_i(q)] + O(\Delta q),$$

$$[C_i(q+\Delta q)] = [C_i(q)] + O(\Delta q).$$

By similar reasoning, we can assume that when $\Delta q \to 0$,

$Q_{Gi}(q+\Delta q) = Q_{Gi}(q) + O(\Delta q)$ and $Q_{Fi}(q+\Delta q) = Q_{Fi}(q) + O(\Delta q)$.

Let us transform Equation (1) rejecting the second infinitesimal order summands and derive

$$[M_i(q)]\{\ddot{q}\} + \{\dot{q}\}^T [C_i(q)]\{\dot{q}\} + [M_i(q)]\{\Delta\ddot{q}\} + 2\{\dot{q}\}^T [C_i]\{\Delta\dot{q}\} = Q_{Di}(q) + Q_{Gi}(q) + Q_{Fi}(q)\,. \tag{3}$$

Total elastic deformation $\{\Delta q\}$, which appears in hinges, can be represented as the following sum:

$$\Delta q_i = \Delta q_i^{ks} + \Delta q_i^{d},\ i = (1,\dots,n), \tag{4}$$

where Δq_i^{ks} is the quasi-static elastic deformation and Δq_i^{d} is the deformation corresponding to elastic vibrations in an *i*-hinge.

Using (4), we can decompose Equation (3) into three equations separating slow Δq^{ks} and rapid Δq^{d} variables:

$$\begin{cases} [M_i]\{\ddot{q}\}+\{\dot{q}\}^T[C_i]\{\dot{q}\}=Q_{Di}+Q_{Gi}+Q_{Fi},\ i=(1,\ldots,n); \\ [M_i]\{\Delta\ddot{q}^d\}+2\{\dot{q}\}^T[C_i]\{\Delta\dot{q}^d\}=-w_i\Delta q_i^d-r_i\Delta q_i^d; \\ w_i\Delta q_i^{ks}=-Q_{Di}; \end{cases} \quad (5)$$

where r_i is a damping coefficient, which reflects the energy dissipation in an *i*-hinge.

The set of three Equations (5) is a mathematical model of the manipulator system with elastic hinges at small deformations. The first equation of the system is the equation of motion of a rigid manipulator system. The second equation is the equation of small elastic vibrations $\{\Delta q^d\}$, which appear in the hinges of manipulator systems. The right side of this equation reflects elastic and dissipative properties of hinges. The third equation, according to (2), associates the quasi-statically small elastic deformations $\{\Delta q^{ks}\}$ with forces developed by drives.

3. Test simulation

Let us compare the results of numerical modeling of the dynamics of manipulator systems with elastic hinges obtained using mathematical models (1), (2), and (5). We will perform modeling by the example of the three-link manipulator system (Figure 1).

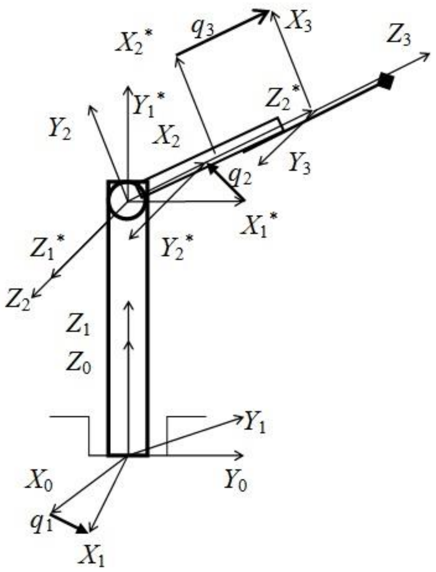

Figure 1. The three-link manipulator system.

The first link of the manipulator system under study is modeled by a thin-wall pipe with length l_1, radius R_1, and mass m_1, which is rotated around the vertical axis coinciding with the central longitudinal axis of the pipe. The second link is pivotally connected with the first one and is modeled by a thin rod with length $2l_2$ and mass m_2, which is rotated relative to the first link in the vertical plane. The third link moves steadily relative to the second link and has length l_3 and mass m_3 concentrated at the link end.

Let us investigate the deviations of motion manipulator system considering the straight-linear uniformly accelerated motion of the point associated with concentrated mass *m*3 of the third link. Let us accept the point with Cartesian coordinates $(x(0), y(0), z(0)) = (0.6, 0.0, 0.0)$ as the initial point of the motion trajectory, and the point with coordinates $(x(T), y(T), z(T)) = (0.0, 0.6, 1.2)$, where $T = 2$ s is the motion time, as the end point. Solving the inverse kinematic problem, let us find the law of motion manipulator system under study along the specified trajectory (Figure 2).

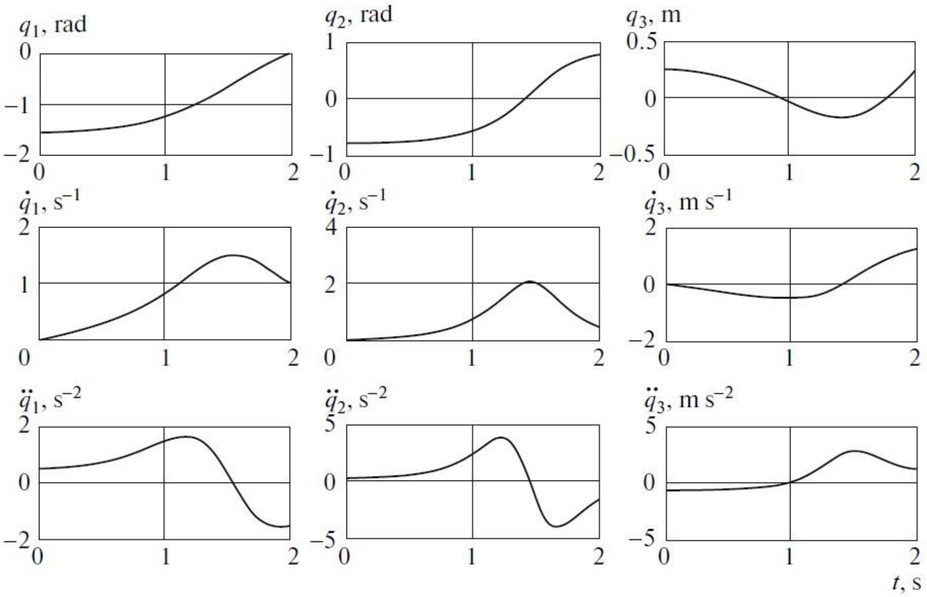

Figure 2. The law of motion manipulator system.

Let us determine the numerical values of geometric, inertial, and elastic parameters of the manipulation system under study as well as the initial values for integration variables. We accept R_1 =0.1 m, l_1 =0.6 m, l_2 =0.3 m, l_3 =0.3 m, m_1=20 kg, m_2 =10 kg, $m3$ =10 kg, w_1 =1.0e4 Nm, w_2 =1.0e4 Nm, w_3 = 1.0e5 Nm, $\Delta q^d_i(0)$ =0.0 (i =1–3), $\Delta\dot{q}_1^d(0) = \dot{q}_1$ (0.01) = 0.0075 s^{-1}, $\Delta\dot{q}_2^d(0)$ = $\dot{q}_2$ (0.01) = 0.0038 s^{-1}, $\Delta\dot{q}_3^d(0)$ = $\dot{q}_3$(0.01) = 0.0095 ms^{-1}, $\Delta q_1(0)$ = $\Delta q_1^{ks}(0.01)$ = –2.201e–4 m, $\Delta q_2(0)$ = $\Delta q_2^{ks}(0.01)$ = 0.0082 m, $\Delta q_3(0)$ = $\Delta q_3^{ks}(0.01)$ = 7.573e–4 m, and $\Delta\dot{q}_i(0) = \Delta\dot{q}_i^d(0)$ (i = 1–3).

Differential equations were integrated by the fourth-order Runge–Kutta method. The results of calculations are presented in Figures 3–5. Plot *1* corresponds to the dependence, which reflects the sum of the quasi-static and vibrational components of small elastic deviations $\Delta q_i^{ks}(t)+\Delta q_1^d(t)$ (i = 1–3). Plot *2* represents dependence $\Delta q_i^{ks}(t)$ (i = 1–3). Dependencies presented in plots *1* and *2* are calculated on the basis of the set of equations (5). Plot *3* corresponds to dependence $\Delta q_i(t)$ (i = 1–3) calculated on the basis of the set of Equations (1 and 2). Plots *1* and *3* are compared. It should be noted that as the rigidity of hinges wi (i = 1–3) increases, the difference between the results of modeling found by two compared mathematical models decreases.

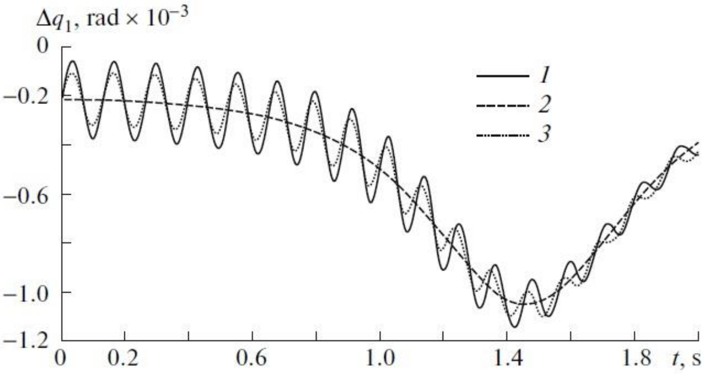

Figure 3. Plot 1. Quasi-static and vibrational components of coordinates q_1.

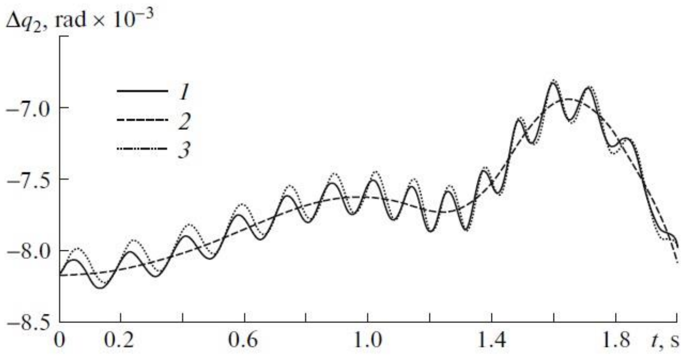

Figure 4. Plot 2. Quasi-static and vibrational components of coordinates q_2.

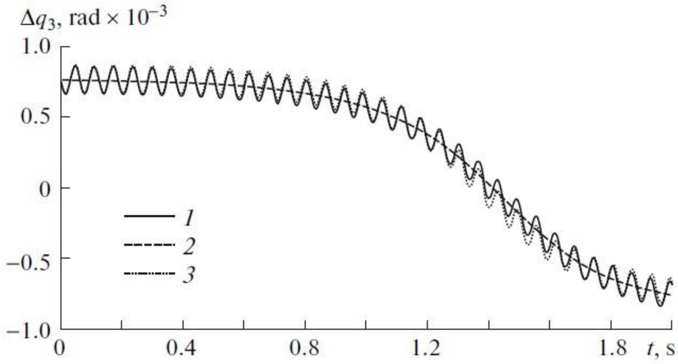

Figure 5. Plot 3. Quasi-static and vibrational components of coordinates q_2.

A detail derivation of the set of equations similar to (5) is presented in [1].

The distinctive feature of the mathematical model of manipulation systems of robots, which is based on the set of Equations (5), is the fact that this mathematical model can be simultaneously used for the synthesis of programmed motions of rigid manipulation systems and for the analysis of deviations from these motions, which appear in actual manipulation systems because of the elastic compliance in hinges.

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