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Mathematical simulation of a profile cutter as a surface of revolution

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Abstract. Various types of cutters (spherical, toroidal, etc.) are used in surface processing of parts of a transmission mechanism. The cost of a special profile tool is somewhat higher than that of such cutters. But the increase in the cost of the tool is compensated by a significant reduction in the time of processing parts. The present paper deals with a mathematical model of a profile cutter surface (as a surface of revolution) for processing parts of a cylindrical transmission gear with an eccentrically cycloidal gearing (EC-gearing). A computer program for determining radii of the cutter's circular cross sections for a given set of axial displacements was created.

1. Introduction

In the present paper we will simulate the surface construction of a so-called profile cutter for processing parts of a cylindrical gear with an eccentrically cycloidal gearing (EC-gearing) [1-2]. Profile cutters allow grinding a tooth surface 'in one go' in contrast to spherical and toroidal cutters which perform parts processing by sequential grinding of coordinate lines on the surface of a part [3-5]. The increase in the cost of the profile tool is compensated by a significant reduction in the time of parts processing. The input and output parts of a transmission mechanism are commonly called the 'gear' and the 'wheel', respectively. In the paper we will construct a surface of a profile cutter for processing gear teeth.

2. Constructing the profile cutter surface

Let us write the constants which are included in the equations of parts surfaces:
Aw – the centre-to-centre distance (the distance between parallel axes of parts rotation);
ε – the eccentricity;
z_1 – the number of gear teeth;
z_2 – the number of wheel teeth;
$n = z_2/z_1 + 1$;
ρ – the radius of the gear tooth section circle;
l – the size of a part along the axis of rotation;
r_c – the radius of the cylindrical drum with gear teeth;
K – the number of the cutter's surface sections;
μ_i – the height of the section lift $(i = 0,1\ldots K)$;
r_i – the radii of the cutter's section circle for different values of μ_i;
η – the shift of the cutter's axis.

The surface of a gear tooth in a cylindrical gear with an EC-gearing [1] is formed by circles located in parallel planes, the centres of these circles lying on a helical line which belongs to a cylinder of radius ε (a screw element). Let the axis of the wheel rotation be axis *OZ* and the axis of the gear rotation be directed along the parallel line displaced along the *OX* axis by an amount of *Aw*. Then the parametric equations of the gear tooth surface can be written as a vector function of two arguments:

$$\vec{\boldsymbol{S}}(\upsilon,\alpha)=\begin{pmatrix} Aw-\varepsilon\cos\upsilon+\rho\cos\alpha \\ -\varepsilon\sin\upsilon+\rho\sin\alpha \\ \dfrac{lr\,\upsilon\,z_1}{2\pi} \end{pmatrix}, \tag{1}$$

where $\alpha = 0,..,2\pi$, $\upsilon = 0,..,2\pi/z_1$.

The surface of the cutter will be built as a family of circles in the planes perpendicular to the axis of the cutter with the centres lying on this axis. The radius of each of these circles will be determined from the condition of its contingence with the flat curve – the section of the tooth by the plane of the circle. The ideal straight line of the cutter's axis intersects the axis of the gear rotation at a right angle and is located at equal distances from the gear teeth. In practice, for the purpose of avoiding a tooth contacting with the adjacent teeth during processing, value η is introduced which is the value of a small parallel shift of the cutter's axis (while maintaining perpendicularity of the gear's axis of rotation). Figure 1 shows the position of the cutter's axis in a flat gear section perpendicular to the axis of rotation for $z_1 = 4$. The four circles are the gear teeth sections having radius ρ, and r_c is the radius of the great circle.

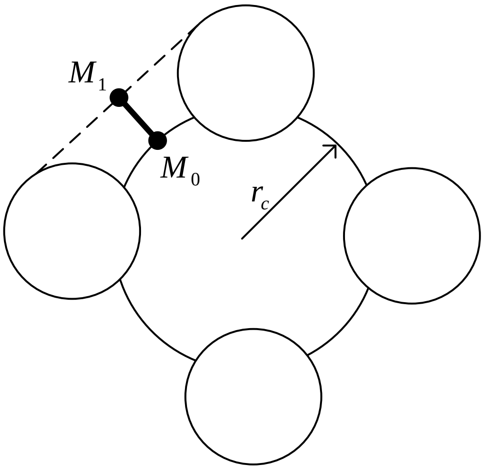

Figure 1. The section of the input part (gear) of a cylindrical transmission gear in an EC-gearing for $z_1 = 4$. The position of the profile cutter's axis and the tangent to the section circle of the first gear tooth perpendicular to the axis of the cutter is shown (dashed line).

With this arrangement of the cutter's axis, its equations in the section plane have the following form:

$$\overrightarrow{\boldsymbol{Osf}}(\mu)=\begin{pmatrix} Aw-r_c\cos\left(\frac{\pi}{z_1}\right)+\eta\sin\left(\frac{\pi}{z_1}\right) \\ r_c\sin\left(\frac{\pi}{z_1}\right)+\eta\cos\left(\frac{\pi}{z_1}\right)\end{pmatrix}+\mu\begin{pmatrix}-\cos\left(\frac{\pi}{z_1}\right)\\ \sin\left(\frac{\pi}{z_1}\right)\end{pmatrix}. \tag{2}$$

When $\mu = 0$, from (2) we obtain the coordinates of point M_0, which is passed through by the surface of the first circle of the cutter's surface section. The position of the last circle surface is defined by intersection point M_1 which is the point of the cutter's axis intersecting the tangent to the section circle of the gear tooth perpendicular to this axis (Figure 1). Let us write the equation of this tangent in the normal form:

$$\cos\left(\frac{\pi}{z_1}\right)\left[x-\left(Aw-\rho\cos\left(\frac{\pi}{z_1}\right)-\varepsilon\right)\right]-\sin\left(\frac{\pi}{z_1}\right)\left(y-\rho\sin\left(\frac{\pi}{z_1}\right)\right)=0\,. \tag{3}$$

Then distance μ_1 from point M_0 to tangent (3) is obtained by introducing the coordinates of point M_0 in (3):

$$\mu_1=\varepsilon\cos\left(\frac{\pi}{z_1}\right)+\rho\;-r_c$$

Thus, the planes perpendicular to the axis of the cutter, in which the circles are arranged, forming the surface of the cutter, pass through points M_0 and M_1 of the cutter's axis, i.e. when parameter μ in (2) varies from 0 to μ_1. The equations of these planes family can be written as:

$$\cos\left(\frac{\pi}{z_1}\right)\left[x-\left(Aw-(r_c+\mu)\cos\left(\frac{\pi}{z_1}\right)\right)\right]-\sin\left(\frac{\pi}{z_1}\right)\left(y-(r_c+\mu)\sin\left(\frac{\pi}{z_1}\right)\right)=0, \tag{4}$$

where $\mu = 0,.., \mu_1$.

Now we will get the equation of the family of curves – the sections of the gear tooth surface by the planes of family (4). To do this, we substitute x and y in (4) by the first two coordinates of vector function (1) defining the surface of the gear tooth. As a result, we obtain the relation between parameters υ, α, and μ, from which we can obtain the expression of υ through α and μ:

$$\upsilon(\alpha,\mu)=\frac{-\pi}{z_1}+\arccos\left(\frac{\rho\cos\left(\frac{\pi+\alpha z_1}{z_1}\right)+r_c+\mu}{\varepsilon}\right). \tag{5}$$

Introducing (5) in (1), we obtain the desired equations of the flat curves family – the sections of the gear tooth by the planes of family (4):

$$\overrightarrow{\boldsymbol{Sem}}(\alpha,\mu)=\begin{pmatrix} Aw-\varepsilon\cos(\upsilon(\alpha,\mu))+\rho\cos\alpha \\ -\varepsilon\sin(\upsilon(\alpha,\mu))+\rho\sin\alpha \\ \dfrac{lr\,\upsilon(\alpha,\mu)\,z_1}{2\pi}\end{pmatrix}. \tag{6}$$

These particular curves must be touched by the circles forming the surface of the profile cutter. These circles are located in the planes of family (4) and their centres lie on the axis of cutter (2) and radii r vary depending on parameter μ of family (4). From this, we can write the equations of such circles family (with arbitrary radius r as yet) in the following form:

$$\overrightarrow{\boldsymbol{Okr}}(\beta,\mu)=\begin{pmatrix} Aw-(r_c+\mu)\cos\left(\dfrac{\pi}{z_1}\right)+(r\cos\beta+\eta)\sin\left(\dfrac{\pi}{z_1}\right) \\ (r_c+\mu)\sin\left(\dfrac{\pi}{z_1}\right)+(r\cos\beta+\eta)\cos\left(\dfrac{\pi}{z_1}\right) \\ r\sin\beta \end{pmatrix}. \tag{7}$$

In formulas (5)-(7) $\mu = 0,..,\mu_1$. In each section μ, circle (7) must touch curve (6), i.e. the scalar product of the vectors must be zero:

$$\left(\overrightarrow{\boldsymbol{M}}_0-\overrightarrow{\boldsymbol{Sem}}(\alpha,\mu),\ \overrightarrow{\boldsymbol{Sem}}'(\alpha,\mu)\right)=0, \tag{8}$$

Here $\overrightarrow{\boldsymbol{M}}_0=\overrightarrow{\boldsymbol{Osf}}(0)$ and $\overrightarrow{\boldsymbol{Sem}}'(\alpha,\mu)$ indicates the derivative of vector function $\overrightarrow{\boldsymbol{Sem}}(\alpha,\mu)$ with respect to parameter α:

$$\overrightarrow{\boldsymbol{Sem}}'(\alpha,\mu)=\begin{pmatrix} \varepsilon\sin(\upsilon(\alpha,\mu))\upsilon'(\alpha,\mu)-\rho\sin\alpha \\ \varepsilon\cos(\upsilon(\alpha,\mu))\upsilon'(\alpha,\mu)-\rho\cos\alpha \\ \dfrac{lr\,\upsilon'(\alpha,\mu)\,z_1}{2\pi} \end{pmatrix},$$

$$\upsilon'(\alpha,\mu)=\frac{\rho\sin\left(\dfrac{\pi+\alpha z_1}{z_1}\right)}{\left(\varepsilon^2-\left[\rho\cos\left(\dfrac{\pi+\alpha z_1}{z_1}\right)+r_c+\mu\right]^2\right)^{\frac{1}{2}}}.$$

For the purpose of solving equation (8) a computer program was created. For each value $\mu_i = \dfrac{\mu_1}{K}i$ (K is a set number of sections of the cutter's surface) this program defines the initial approximation for finding the root of the function in the left-hand part of equation (8) and then, using built-in function *root* of the MathCad pack, defines root α_i. Next, to determine the radius of circle r_i which touches the section of the gear tooth corresponding to μ_i, we require the presence of a common point of these curves, i.e. we equate the coordinates of vector functions (6) and (7). The equality of the second and third coordinates gives a system of two equations:

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$$r\cos\beta = \frac{-\varepsilon\sin(\upsilon(\alpha,\mu)) + \rho\sin\alpha - (r_c+\mu)\sin\left(\frac{\pi}{z_1}\right)}{\cos\left(\frac{\pi}{z_1}\right)} - \eta\,,$$

$$r\sin\beta = \frac{lr\,\upsilon(\alpha,\mu)\,z_1}{2\pi},$$

from where we find the desired radii of the circles for each value of μ_i:

$$r_i = \left(\left\{\frac{-\varepsilon\sin(\upsilon(\alpha_i,\mu_i)) + \rho\sin\alpha_i - (r_c+\mu_i)\sin\left(\frac{\pi}{z_1}\right)}{\cos\left(\frac{\pi}{z_1}\right)} - \eta\right\}^2 + \left\{\frac{lr\,\upsilon(\alpha_i,\mu_i)\,z_1}{2\pi}\right\}^2\right)^{\frac{1}{2}}.$$

Thus, the surface of the profile cutter is obtained as a family of circles of form (7), i.e. as the surface of revolution:

$$\overrightarrow{\boldsymbol{Fr}}(\beta, i) = \begin{pmatrix} Aw - (r_c+\mu_i)\cos\left(\frac{\pi}{z_1}\right) + (r_i\cos\beta+\eta)\sin\left(\frac{\pi}{z_1}\right) \\ (r_c+\mu_i)\sin\left(\frac{\pi}{z_1}\right) + (r_i\cos\beta+\eta)\cos\left(\frac{\pi}{z_1}\right) \\ r_i\sin\beta \end{pmatrix}.$$

Figure 2 shows a profile cutter, modelled using the above-described scheme. It shows the plane of the cutter's section perpendicular to its axis. In this plane, the section line of the gear tooth which is touched by the circle of the cutter's section is depicted.

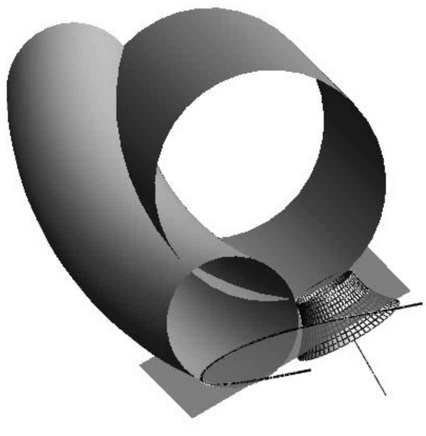

Figure 2. A profile cutter for processing the surface of a gear tooth in a transmission gear with an EC-gearing. The section plane perpendicular to the axis of the cutter and the line of the gear tooth section by this plane are shown.

3. Conclusion

We obtained the parametric equations of the circles family in the planes perpendicular to the cutter's axis of rotation, with the centres lying on this axis. Each circle is found from the condition of touching the flat curve which is the circle plane section of the tooth. The constructed family of circles forms the surface of the profile cutter. The problem was solved for cylindrical gears with an EC-gearing. The work provided substantial assistance in manufacturing EC-engagement parts for gears of various types.

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