

# Mathematical modelling in the computer-aided process planning

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**Abstract.** This paper presents new approaches to organization of manufacturing preparation and mathematical models related to development of the computer-aided multiproduct process planning (CAMPP) system. CAMPP system has some peculiarities compared to the existing computer-aided process planning (CAPP) systems: fully formalized developing of the machining operations; a capacity to create and to formalize the interrelationships among design, process planning and process implementation; procedures for consideration of the real manufacturing conditions. The paper describes the structure of the CAMPP system and shows the mathematical models and methods to formalize the design procedures.

## 1. Introduction

The researches related to the computer-aided process planning (CAPP) are relevant throughout the development of mechanical engineering. The reason for that is a regular change of machined parts range, requirements to reduce the period of manufacturing preparation, expansion of technological capabilities of enterprises.

Process planning includes several activities to determine economic and competitive methods for processing workpieces and parts; by means of the methods the parts can be produced at all stages of machine processing [1]. The computer-aided process planning system is an essential part of computer-integrated manufacturing [2]. The aim of CAPP is to automate the development of process plans to a great extent. However, there are no absolutely satisfiable solutions in automated design of machining operations, for instance, formation of the operation structure defining a sequence of manufacturing operations, selection of equipment, tools and fixtures. An approach to decision-making formalization is considered in [3]. There is a description of the decision logic schema of process planning using backward chaining reasoning.

An integration between process planning and production scheduling is also a purpose of CAPP [4]. It is necessary to solve a problem of securing the relationship between the systems of planning and implementation for this purpose achievement. A series of researches considers the possible solutions of this problem. In research [5], the authors used a genetic algorithm optimization methodology during preliminary and detailed process planning. Possible operation sequences are generated by analyzing the constraints and using a genetic algorithm. The architecture of the integrated CAD/CAPP system is presented in [6]. There is a brief discussion of the content and methodology of STEP, and STEP based strategies for CAD/CAPP information integration are pointed out.

Thus, these researches and many other works consider particular tasks for solving the problem of CAPP, but they do not deal with an end-to-end approach to a creation of the integrated CAPP system.



The first part of this paper presents a model of multiproduct machining process planning, and describes its distinctive features and advantages.

The second part considers the subsystem of the machining operation design. Its structure model is given, the models and methods for formalization of the key design procedures are proposed.

The third part describes a database on capabilities and current states of equipment and tools.

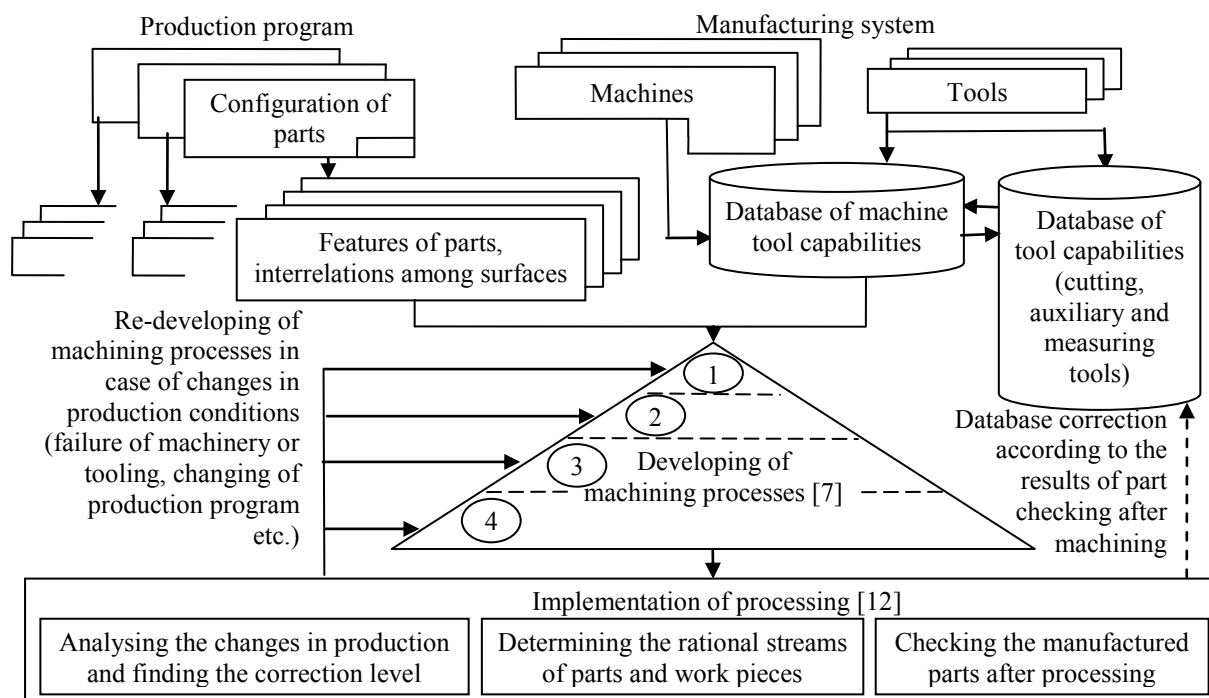
In conclusion, the article presents the main findings and proposals to use the results.

## 2. The model of the process planning system

Currently it is necessary to have a tight link between process planning and implementation. Figure 1 shows the proposed scheme of the computer-aided multiproduct process planning system (CAMPP). The basic approaches to its creation are presented in [7]. The CAMPP system has distinctive peculiarities in comparison with the existing CAPP systems:

- 1) process plans development at the level of machining operations is fully formalized [8, 9, 10];
- 2) availability of the capacity to create and to formalize the interrelations among design, process planning and process implementation [11];
- 3) provision of procedures to consider the real manufacturing conditions [12];
- 4) availability of the database on capabilities and current states of equipment and machine tools.

The production program is initial data containing the information about the manufactured parts. Every part is represented as a set of features. There are defined dimension interrelations among features and listed accuracy and surface quality requirements. The manufacturing system has necessary information about machine tool resources and equipment capabilities.



**Figure 1.** A scheme of multiproduct machining process planning.

The development of process plans has the following order:

- 1) development of the principal route to parts machining for each feature;
- 2) formation of the tuples of feasible machining methods;
- 3) selection of the machine tools;
- 4) design of machining operations [9, 10].

Each design procedure undergoes automatically three phases: generation of the possible alternatives; elimination of the unpractical alternatives; selection of the efficient alternatives for given manufacturing conditions. This order ensures the reduction of the designing time costs and improvement of the quality of design solutions due to avoidance of subjective factors, i.e. human limited capabilities.

The designed process plans come to the implementation subsystem. Three key procedures secure a regular interrelationships between design and implementation subsystems.

1. Determining the rational streams of parts and workpieces. This secures high loading of machines and low production time.

2. Analyzing the changes in production conditions and finding the correction level. In case of failure of machinery or tooling, changes in production program, the system is able to select among previously generated process plans, that is, to respond to a real manufacturing situation.

3. Checking the manufactured parts after processing. It secures the databases of machines and tools in the actual state. Therefore, the CAMPP system provides the lowest percent of production faults and the highest machining performance.

Each process planning design procedure is performed automatically through the mathematical models briefly described below.

### 2.1. The model for development of the principal route for machining of parts

The base of this model is in creating the abstract structural scheme of the machining process plan (PP). The defined sequence of process steps  $\varphi_1, \varphi_2 \dots \varphi_n$  represents this basic scheme. A process step has a set of technological parameters  $T_k = (t_{k1}, t_{k2}, \dots, t_{km})$  carrying hereditary information related to the material and surface quality of the machined part. The set of parameters depends on the selected machining method. The parameters are divided into two groups: the parameters with a constant value from beginning to end of PP and the parameters with a changeable value during PP implementation.

The control parameters are characterized by a finite number of output parameters of the product  $\xi_i$  ( $i = 1, 2, \dots, S$ ) fully describing the state of the surface at each process step. Vector of quality  $\bar{X}$  defines the surface quality as  $\bar{X} = (\xi_1, \xi_2, \dots, \xi_S) \in K$ , where  $K$  is a spatial state of the surface quality, which is Cartesian product of tolerances  $J_i$  for each  $\xi_i$ .

Then the task of creating a principal scheme to process a feature is considered as the task of finding possible routes of  $\bar{X}$  from some beginning state  $X_b \subset K$  to end state  $X_e \subset K$  by path  $L_j \subset X_N$ . In other words, the transition of the surface from  $X_b$  to  $X_e$  is realized by going through  $N_j$  process steps in area  $X_N$ .

Fields of feature characteristics obtained after processing are represented as fields of output parameters of process steps in matrix  $\Omega$ , and fields of initial characteristics as fields of input parameters in matrix  $\Delta$ . A selection of the machining method consists in finding such process step that has all characteristics  $\Omega_i^j(k)$  equaled to or exceeding  $\Delta_i^j(k+1)$ . This requirement is true for whole PP of the feature. A database on manufacturing capabilities is a base for comparison. As a result, the possible machining routes are formed for each part.

### 2.2. The model for the formation of the tuples of feasible machining methods

The formation of the tuples of feasible machining methods is a procedure related to the issue of the definition of the process steps that can be jointly accomplished.

A tuple of the feasible machining methods is bundle  $K = \langle M, D, C, E \rangle$ , where  $M$  – machining methods;  $D$  – variants of datum and fixture;  $C$  – characteristics of processed surfaces;  $E$  – variants of machines.

Petri net represents a model of the datum surface kit formation. A set of places in designed Petri net  $P = \{P_1, P_2, \dots, P_n\}$  is events to define a possibility of using either machined surface as datum for a given process step and a possibility of including two and more process steps in one tuple. Both

execution of transitions  $T=\{T_1, T_2, \dots, T_m\}$  and the net structure consider constraints related to design and engineering requirements of parts and capabilities of machines and tools.

Marking of the net is a code of machining transition, which is assigned to each initial event ( $P_1, P_2, \dots$ ). Due to execution of procedural operations, the net changes marking so that the tokens are moved through the net to form different combinations. The structure of the net supposes the creation of all possible combinations of the given process steps.

The first three phases of movement through Petri net generates a set of technological data. At the next stages, the tuples of process steps are formed and the possibility of merging of several process steps in one operation is considered.

### 2.3. The model for selection of the processing machines

The graph of queuing system states represents the implementation of the process step tuple on a separate group of machines. This graph considers the characteristics of reliability associated with the emergence and elimination of failures in the manufacturing system. A system of differential equations for the probabilistic states of the system is written in accordance with the labeled graph. The developed model was studied within the limits of the system steady state. Therefore, a system of differential equations was converted into the system of algebraic equations, the solution of which allows defining indicators of machines during parts manufacture in a given range, such as the probability of an idle machines group, the average number of a queued bundle of parts, machine utilization, etc.

The created model defines rational values of the intensities of the input and output streams of parts for the considered machine groups and determines the best alternatives of PP.

## 3. Design of machining operations

A machining operation design is a key element of the process planning system. In this case, the most difficult and responsible decisions are made. Figure 2 shows the generalized structure of machining operation design in CAMPP system. For each design procedure we propose mathematical models described below that can be used to automate the development of machining operations.

### 3.1. The model for the formation of toolkits

We proposed to solve the problem using Petri nets [13]. A set of features is  $S = \{S_1, S_2, \dots, S_s\}$ , a set of equipment items is  $E = \{E_1, E_2, \dots, E_e\}$ , a set of cutting tools is  $R = \{R_1, R_2, \dots, R_r\}$  and a set of auxiliary tools is  $V = \{V_1, V_2, \dots, V_v\}$ . So, in terms of the Petri nets theory each phase of this design procedure is defined by a set of places  $P^1, P^2, P^3, P^4, P^5, P^6$ . Events  $T^1, T^2, T^3, T^4, T^5, T^6$  occur when the decision on the choice of tooling options during each phase is made. During the Petri net transition, one decides whether it is possible to machine with either cutting tool or not. Petri net consists of several levels, where the compliance is verified with one of the parameters: an equipment group; connection parameters of equipment; the hardness of the machined surface; the material of the machined feature; the type of the feature; the accuracy; dimensions of the machined feature.

As a result, a set of feasible tooling is formed to process at the appropriate process step. A criterion of homogeneity is used to eliminate inefficient choices of the formed set of tooling. This criterion ensures the invariance of the operation structure and the high level of interchangeability among machining operations.

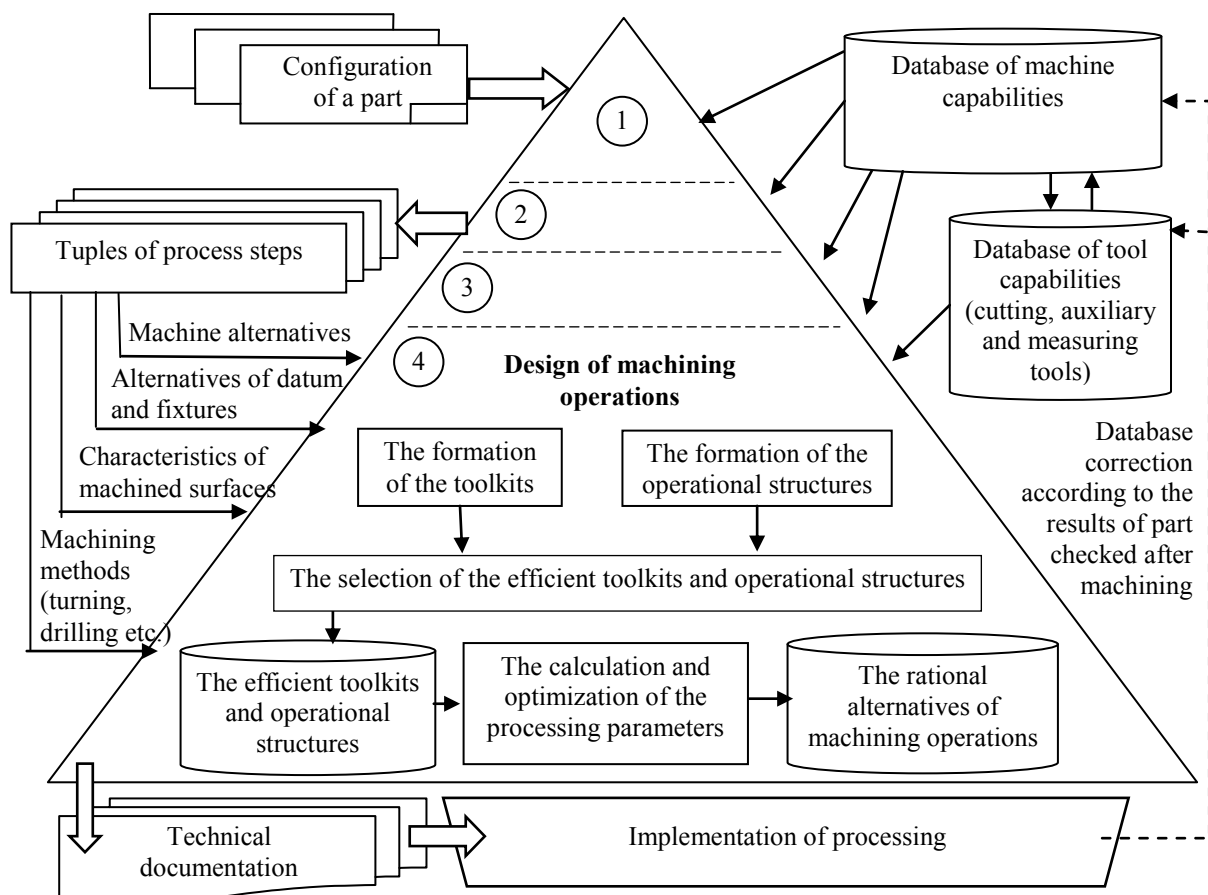
### 3.2. The model for the formation of the machining operational structure

The next stage of process planning is a formation of the rational machining operation structure. A key task is to choose a rational sequence of process steps for each tuple. We used a graph theory to modeling this design procedure.

A set of tuples with process steps  $K=\{K_1, K_2, \dots, K_n\}$  approaches the input of the design subsystem, where  $n$  – a number of tuples. A set of machined features is  $S=\{S_1, S_2, \dots, S_p\}$ , where  $p$  – a number of features. A code of the machined feature is assigned to either vertex of the graph. Each arc between

two vertices represents a processing sequence: the outgoing arc defines the feature to be machined before the feature specified in the vertex with the incoming arc. In case of absence of the fundamental difference in the processing sequence of any two features, these two respective vertices are connected by two oppositely directed arcs. The processing sequence of two similar features is shown as a loop in the vertex.

A flock of ordered sets  $K'$  is formed after selecting of possible sequences for each tuple of process steps. Each set in the flock of  $K'$  represents a possible alternative of the machining operation structure. In case there are several possible sequences for the same tuple, at the next stages of process planning it is necessary to eliminate inefficient alternatives and to choose rational sequences by the criterion of minimizing the processing time due to the combination of process steps and parallel processing.



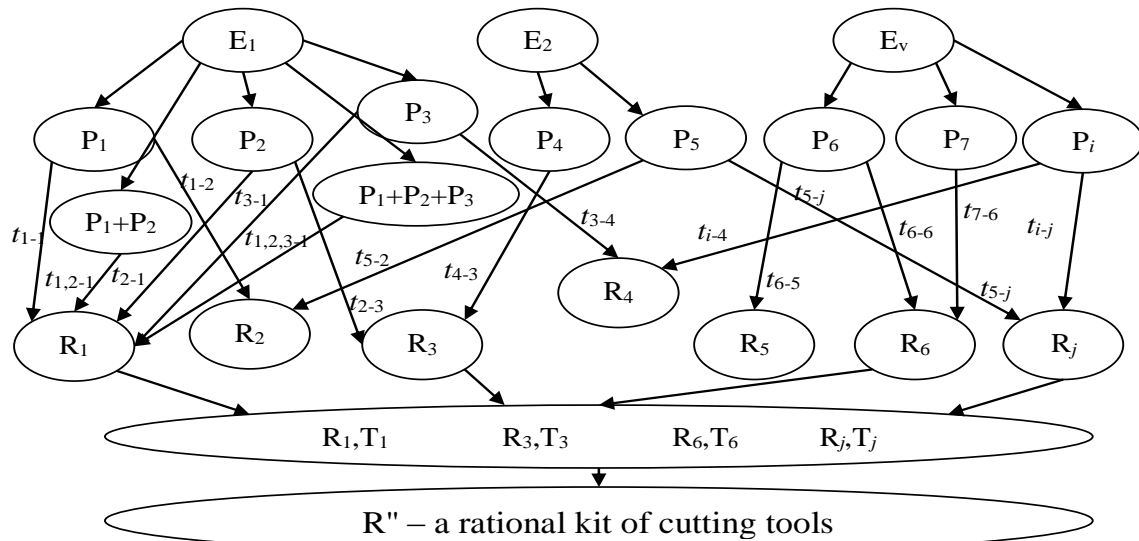
**Figure 2.** The generalized structure of the subsystem for machining operations design.

### 3.3. The model for a selection of the rational toolkits and operational structures

The problem of a selection of the rational sets of cutting and auxiliary tools for each type of equipment used in the manufacturing system based on the known methods, is time consuming. It is due to an exhaustive search of all possible alternatives along with the calculation of setup complexity and processing time (figure 6). Therefore, the mathematical apparatus of dynamic programming has been proposed to solve this problem [15]. The advantage of dynamic programming is needlessness of calculating the optimization parameters for each combination of tool alternatives. Furthermore, dynamic programming optimizes the overall system and avoids cases when the optimization of individual elements leads to inoperability of the entire system.

The set of all machining operations is represented as a system with process steps as elements of this system. The set of process steps is  $P = \{P_1, P_2, \dots, P_n\}$ , where  $n$  – a number of different process steps for

all machining operations, and a set of feasible cutting tools is  $R = \{R_1, R_2, \dots, R_m\}$ , where  $m$  – a number of different sizes of cutting tools. It is required to choose the option of cutting tools  $R_j$  for each process step  $P_i$  so that the formation of the kit of cutting tools was the most efficient in current manufacturing conditions.



**Figure 3.** The logical scheme of a selection of the rational toolkits and operational structures.

Step controls  $x_1, x_2, \dots, x_n$  are decisions on the choice of the cutting tool of any size for processing at the  $i$  process stage, where  $i=1 \dots n$ ;  $n$  – a number of process steps. In this case, the gain is equal to  $w_i$ . The control of the operation is a set of control steps  $x = \{x_1, x_2, \dots, x_n\}$ . It is necessary to find control  $x^*$ , in which total gain  $W$  becomes maximum. Herewith  $x^* = \{x_1^*, x_2^*, \dots, x_n^*\}$  is an optimal control consisting of a set of optimal control steps. Optimization parameters are:

- 1) total machining time  $t_{0\Sigma}$  for each considered machining operation that should be minimal;
- 2) the number of cutting tool changes that should be minimized.

Control  $x_i = \{j, t_{ij}, a_{ij}\}$  at the  $i$ -stage means that in this process step machining is carried out by cutting tool  $j$  with time  $t_{ij}$  and number of tool changes  $a_{ij}$ . The system state at stage  $i$  is characterized by vector  $S_i = \{j, \tau_{ij}\}$ . It means that before performing process step  $i$ , one uses size  $j$  of the cutting tool, and it has exhausted resource  $\tau_{ij}$ .

A solution of the problem begins with the conditional optimization at last stage  $n$ , when calculating conditional optimal gain  $W_n(S_n) = \max\{f_n(S_n, x_n)\}$  and finding conditional optimal control  $x_n(S_n)$  for the possible cutting tools at this stage. Further optimization is performed by determining conditional optimal solutions at previous steps. Finally, unconditional optimization should be performed, following the recommendations received at each step.

#### 4. The database of the manufacturing capabilities

The database of the manufacturing capabilities and capacities of tooling is needed to secure an automatic mode to perform the design procedures. In this regard, a special database structure is developed to reflect the relations among the elements of the manufacturing system and to allow one to keep information about the capabilities of a particular production system up to date.

The database of machine and tool capabilities:

- represents an information basis for the formalization of design procedures to develop and to implement process plans and machining operations;
- supports the compliance with specified requirements to processed parts;
- allows justifying the choice of machines and tools regarding the effectiveness of their use in specific manufacturing conditions;

- allows using the manufacturing capabilities fully;
- provides the capacity to form rational processing tools and auxiliary equipment for maximum efficiency of machining;
- creates conditions for formation of the rational machining operation structure.

All this changes the activity of the process planner: instead of process plan development, one sets up a database on manufacturing capabilities to provide compliance with the specific manufacturing system.

## 5. Conclusion

In the paper the authors have proposed approaches to creation of an integrated manufacturing preparation system based on fully automated design procedures for process planning and machining operations design. At the same time, the interoperability between the design and implementation processes is ensured, providing an opportunity to reduce the time period required for manufacturing preparation, to reduce manufacturing costs and to increase the competitiveness of organizations.

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