

# Energy Approach-Based Simulation of Structural Materials High-Cycle Fatigue

A F Balayev<sup>1</sup>, A V Korolev<sup>1</sup>, A V Kochetkov<sup>2</sup>, A I Sklyarova<sup>1</sup>, O V Zakharov<sup>1</sup>

<sup>1</sup> Yuri Gagarin State Technical University, Saratov, Russia,

<sup>2</sup> Perm National Research Polytechnic University, Perm, Russia

E-mail: zov17@mail.ru

**Abstract.** The paper describes the mechanism of micro-cracks development in solid structural materials based on the theory of brittle fracture. A probability function of material cracks energy distribution is obtained using a probabilistic approach. The paper states energy conditions for cracks growth at material high-cycle loading. A formula allowing to calculate the amount of energy absorbed during the cracks growth is given. The paper proposes a high-cycle fatigue evaluation criterion allowing to determine the maximum permissible number of solid body loading cycles, at which micro-cracks start growing rapidly up to destruction.

## 1. Introduction

Durability is one of the most important product performance criterion. That is why the crucial task is to evaluate durability under high-cycle loading subject to structural materials high-cycle fatigue. High-cycle fatigue was studied in many papers [1-8] which deal with destruction of solid bodies due to development of micro-cracks. These studies proposed durability evaluation criteria based on deterministic approach to description of destruction process considering random distribution of micro-cracks. In fact, not only distribution of micro-cracks in materials but the process of their growth and distribution of internal energy is random as well.

In our opinion, in order to determine the conditions of micro-cracks occurrence and development and to choose a criterion of evaluation of residual life of a real solid body in high-cyclic loading it is necessary to use a probabilistic approach to describe energy process of cracks development.

## 2. Problem Statement

Let's consider a plate of volume  $V$  stretched out and contracted alternately under a load  $P$  (Figure 1). The plate includes multiple cracks distributed at random. When approximating the crack for the first time, one can take it as flat, elliptical.

Let's take the theory of A. Griffiths as a basis for energy criterion of brittle body fracture. According to the paper [1] the energy of a crack can be determined using the following formula:

$$w_l = 2 \cdot \gamma_s \cdot s = 2 \cdot \pi \cdot \gamma_s \cdot l^2, \quad (1)$$

where  $\gamma_s$  is the crack material specific surface energy;  $s$  is the crack surface area;  $l$  is the crack equivalent length equal to the average  $D$ .

The material includes a large number of random micro-cracks of different size. Therefore, the Rayleigh law can describe the distribution of micro-crack length, and the exponential law can describe the density of the cracks energy distribution:



$$f_l(w_l) = \lambda \cdot e^{-\lambda \cdot w_l} = \frac{e^{-w_l / (8 \cdot \gamma_s \cdot l_o^2)}}{8 \cdot \gamma_s \cdot l_o^2}, \quad (2)$$

where  $l_o$  is the expected value  $l$ .

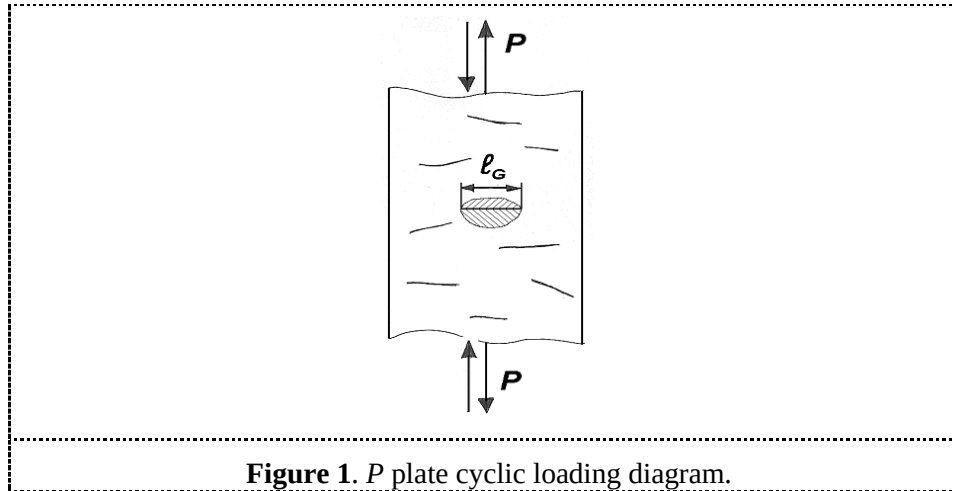


**Figure 1.** *P* plate cyclic loading diagram.

The considered volume  $V$  of the material contains a random integral nonnegative  $n$  number of cracks, which distribution is described by the Poisson law. Then, a probable occurrence of micro-cracks with total energy  $W_n$  in the volume  $V$  of the material is described with the probability function:

$$f_V(W) = \sum_{n=1}^{\infty} f_W(W_n) \cdot H_n = \lambda \cdot e^{-\lambda \cdot \left( \sum_{k=1}^n w_{lk} \right) \cdot V \cdot \rho_o} \sum_{n=0}^{\infty} \frac{\left( \lambda \cdot \left( \sum_{k=1}^n w_{lk} \right) \cdot V \cdot \rho_o \right)^n}{(n!)^2}, \quad (3)$$

where  $H_n$  is the possible presence of  $n$  number of micro-cracks ( $n = 1, 2, 3, \dots$ ) with the total energy  $W_n$  in the volume  $V$  of the material;  $\rho_o$  is the average density of the cracks distribution in the volume  $V$  of the material;  $w_{lk}$  is the energy of the  $k$ -th crack.

Let's consider the plate stretched out and contracted alternately under the symmetrical cyclic load  $P$  (Figure 1). The plate stretching-out requires the amount of work determined with the following formula:

$$U = \frac{\sigma^2}{2 \cdot E} \cdot V, \quad (4)$$

where  $\sigma$  is the stretching-out stress occurring in the plate under the  $P$  force.

The energy gained by the plate is concentrated in the material defects and is distributed among them. If the energy gain portion is proportional to the area of the crack, then the energy gain of the crack of  $l$  size is equal to:

$$u_l = \varphi_s \cdot s = \pi \cdot \varphi_s \cdot l^2, \quad (5)$$

where  $\varphi_s$  is the specific portion of the external energy  $U$  falling to a unit of free micro-cracks surface area.

Similarly with the equations (1) and (2) for the crack energy, distribution of the crack energy gain is expressed with the following formula:

$$f_l(u_l) = v \cdot e^{-v \cdot u_l} = \frac{e^{-u_l / (8 \cdot \varphi_s \cdot l_o^2)}}{8 \cdot \varphi_s \cdot l_o^2}. \quad (6)$$

Assuming that the energy required for stretching-out (4) will be absorbed by the micro-cracks, the expected value of the probability function of the total energy gain is as follows:

$$\varphi_s = \frac{\sigma^2}{16 \cdot E \cdot \rho_o \cdot l_o^2}. \quad (7)$$

A crack can grow under the impact of the energy  $u_l$  gain if energy is sufficient for this. The crack may remain unchanged if the energy gain is insufficient to form a new free surface. Hence, if, as a result of external impact on the material, the energy  $u_l$  gained by the crack is less than the standardized value  $u_g (u_l < u_g)$ , the crack does not grow. If the energy gained by the crack is larger than this value ( $u_l \geq u_g$ ), the crack starts growing. The value of this energy  $u_g$  does not depend on the size of the crack.

In this case, external forces impact on the material is as follows. Only a portion of all the micro-cracks being under the external load in the volume  $V$  can grow. To ensure the growth this portion of the cracks will absorb the additional energy  $u_l$  attributable to them. The remaining part of the cracks will not grow, and, consequently, after unloading and elastic recovery of the material the part of the additional energy attributable to them dissipates into the surrounding space.

### 3. Simulation results

Based on the equation (6) the part of the external energy absorbed by the micro-cracks can be described as follows:

$$P_g = \int_{u_g}^{\infty} f_l(u_l) du = e^{-\nu \cdot u_g}. \quad (8)$$

According to the formula (1) upon receiving additional energy  $u_l \geq u_g$  (equation (5)) the micro-cracks gain value with initial energy  $w_l$ , will be equal to:

$$\Delta l = l_1 - l = \sqrt{\frac{w_l}{2 \cdot \pi \cdot \gamma_s}} \cdot \left( \sqrt{1 + \frac{u_l}{w_l}} - 1 \right) = l \cdot \left( \sqrt{1 + \frac{\Phi_s}{\gamma_s}} - 1 \right). \quad (9)$$

During cyclic deformation a stretching-out half-cycle is followed by a contraction half-cycle, which energy is calculated in the same way.

Occurrence and development of the cracks in structural materials are accompanied by cracks setting under the loads opposite to those favoring cracks growth. This issue was studied in detail in the following papers [2, 3]. According to these papers, the crack setting requires much more energy than its development, therefore cracks development process will outpace their setting. Theoretical and experimental studies [4, 5] have shown that high hydrostatic pressure or substantial biaxial contraction in the elliptic pore causes reduction of a minor axis, while the major axis stays unchanged in terms of dimensions.

Thus, the micro-cracks setting requires certain conditions, which are difficult to ensure under a simple cyclic load. Therefore, we can assume that the micro-cracks can grow in the cyclic alternating contraction and stretching-out, if the energy concentrated in them exceeds a certain threshold, while in the micro-cracks contraction half-cycle nothing happens.

If in one loading cycle the micro-crack increased in size by a value determined with the equation (9), then in the  $i$ -th loading cycle the micro-crack length gain will be:

If microcrack at one loading cycle has increased in size by an amount determined by the equation (9), then the  $i$ -th cycle of loading length of microcracks increment will be:

$$\Delta l_i = l \cdot \delta^i - l = l \cdot \left( 1 + \frac{\Phi_s}{\gamma_s} \right)^{\frac{i}{2}} - l \quad \text{при } u_l \geq u_g, \quad (10)$$

while the micro-crack length will increase  $\delta$ -fold.

Since the external deformation energy is absorbed only by the cracks, which energy exceeds the  $u_g$ , then the cracks, which did not start growing in the first loading cycles, would not grow in the subsequent cycles too. Therefore, the value of the external energy absorbed by the micro-cracks is the sum of the energy gains of all the micro-cracks in the  $i$ -th cycle which original energy exceeded the  $u_g$ :

$$\sum u_{li} = \pi \cdot \varphi_s \cdot \sum l_i^2 = \pi \cdot \varphi_s \cdot \delta^{2i} \cdot \sum l^2. \quad (11)$$

It appears from the equation (11) that the gain of the energy absorbed with the micro-cracks is proportional to the square in parentheses. Initial energy consumption to deform the stretching-out forms the  $U$  value from the formula (4). Then the total energy loss to deform the micro-cracks in the  $k$  cycles is defined with the following formula:

$$\sum \Delta U_i = U \cdot P_g \cdot \sum_{i=1}^k \delta^{2i}. \quad (12)$$

According to the formula (12) the relative energy loss in the  $i$ -th cycle will be equal to:

$$\Delta_i = \delta^{2i}. \quad (13)$$

If we know the permissible length of the micro-crack  $l_G$ , which dramatically increases probable destruction of an item, then using the formulas (10) and (13) we can determine the number of maximum permissible cycles of elastic loading:

$$k = \frac{\ln l_G - \ln l}{\ln \delta} = \frac{2i \cdot (\ln l_G - \ln l)}{\ln \Delta_i}. \quad (14)$$

The resulting formula (14) can be used as a criterion for the residual life of the item:  $k-i$ .

#### 4. Experimental studies

We carried out experimental studies of vibromechanic relaxation of residual stresses of the item shaped like a rod with dia. = 4 mm and length of  $L = 120$  mm made of structural steel 70 GOST 14959-79.

The rod was bent under the concentrated load, which resulted in generation of internal stresses added to the initial residual stresses. The cyclic changes of the internal stresses were obtained by uniform rotation of the rod, which resulted in harmonic oscillations of the rod bent axis relative to its rotation axis. Rotation speed, treatment time and bending moment determining the internal stresses were taken as variable parameters.

During the experiment, it was found that if the number of loading cycles is less than the value determined with the formula (14), the stress is relieved from the item only partially. The number of the loading cycles was defined by the duration of treatment and rotation speed. If the vibromechanic treatment time exceeds the specified value, new stresses can appear in the item and material cyclic strength decreases. The results obtained are shown in Figure 2. The figure shows dependence of the residual stresses in the item  $\sigma_r(k, t)$  at the item rotation speed of  $t = 160 \text{ min}^{-1}$  on the treatment time  $t$  for three values of relative stress  $k_\sigma = \sigma_r / \sigma_m$  (0.6; 0.8; 0.9), where  $\sigma_m$  is the maximum stress of the item elastic deformation,  $\sigma_l$  is the yield strength of the item material.

The Figure 2 shows that during the vibromechanic treatment the residual stresses first decrease down to zero, and then increase. The decrease of the residual stresses is due to the fact that during the vibromechanic stabilization the energy generated in the treatment process gradually absorbs the potential energy generating residual stresses in the item. As soon as the potential energy of the item material becomes equal to zero, further vibromechanic treatment causes increase in the potential energy of the item material and generates internal stresses.

#### 5. Findings

The mathematical simulation of cracks occurrence mechanism with the use of probabilistic and energy approaches resulted in the following findings:

- brittle fracture energy is exponentially dependent on a solid body destroyed volume;
- increase in the volume of a body increases the probability of its destruction;
- crack length gain per one loading cycle is characterized by power-law dependence on the value of the energy gained during such cycle;

- increase in the energy absorbed by the micro-cracks is proportional to the square of the micro-cracks size gain.

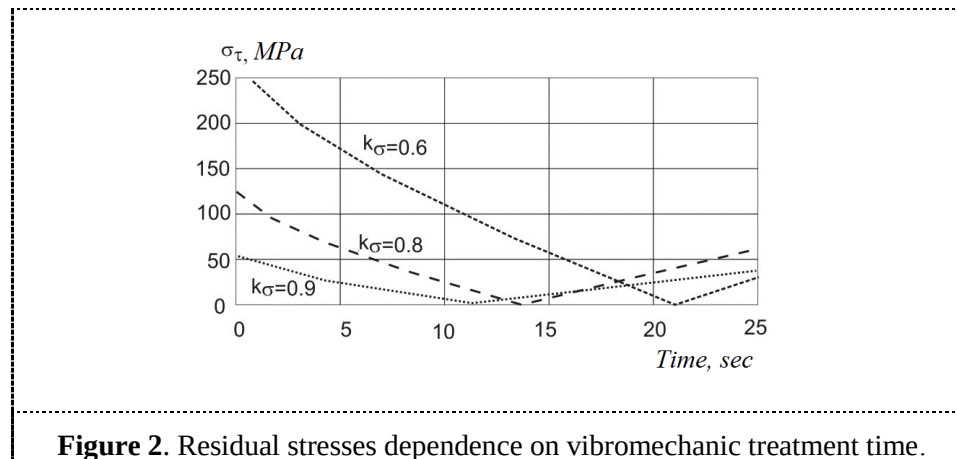


**Figure 2.** Residual stresses dependence on vibromechanic treatment time.

According to the results of the experimental studies, it was found that increase in the number of loading cycles causes relaxation of internal stresses that characterizes accumulation of internal energy in the cracks. After complete relief of the residual stresses, while continuing the cyclic loading process, internal stresses start to increase. It characterizes crossing of critical energy threshold and subsequent rapid growth of the micro-cracks followed by the destruction of the material.

The experimental results confirm the adequacy of the mathematical models obtained and criteria of evaluation of the material residual life.

## 6. Conclusion

The above method allows to evaluate high-cycle fatigue and residual life of structural materials based on the energy approach to description of the cracks occurrence mechanism. The results can be used in designing of steel structures and in development of new technological methods of residual stresses relaxation.

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