

# Competing risk models in reliability systems, a weibull distribution model with bayesian analysis approach

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**Abstract.** In reliability theory, the most important problem is to determine the reliability of a complex system from the reliability of its components. The weakness of most reliability theories is that the systems are described and explained as simply functioning or failed. In many real situations, the failures may be from many causes depending upon the age and the environment of the system and its components. Another problem in reliability theory is one of estimating the parameters of the assumed failure models. The estimation may be based on data collected over censored or uncensored life tests. In many reliability problems, the failure data are simply quantitatively inadequate, especially in engineering design and maintenance system. The Bayesian analyses are more beneficial than the classical one in such cases. The Bayesian estimation analyses allow us to combine past knowledge or experience in the form of an apriori distribution with life test data to make inferences of the parameter of interest. In this paper, we have investigated the application of the Bayesian estimation analyses to competing risk systems. The cases are limited to the models with independent causes of failure by using the Weibull distribution as our model. A simulation is conducted for this distribution with the objectives of verifying the models and the estimators and investigating the performance of the estimators for varying sample size. The simulation data are analyzed by using Bayesian and the maximum likelihood analyses. The simulation results show that the change of the true of parameter relatively to another will change the value of standard deviation in an opposite direction. For a perfect information on the prior distribution, the estimation methods of the Bayesian analyses are better than those of the maximum likelihood. The sensitivity analyses show some amount of sensitivity over the shifts of the prior locations. They also show the robustness of the Bayesian analysis within the range between the true value and the maximum likelihood estimated value lines.

**Keywords:** Competing risks, Likelihood function, Posterior/prior function, Hazard function, and Net/Crude/partial crude probability

## 1. Introduction

Reliability theory concerns the ability of a component or a system, either a life creature or an equipment, to be functioning during its expected length of life. The studies of reliability theory become more important because they are generally more expensive than measurement data in most quality control process.



In reliability theory, the most important problem is to determine the reliability of a complex system from the reliability of its components. The weakness of most reliability theories is that the system and its components are described and explained as simply functioning or failed. In many real situations, the failures may be from many causes depending upon the age and the environment of the system and its components.

In physical systems, the failure may be a phenomenon within a single item such as the tread wear, puncture, or defective side walls of an automobile tire. Or it can happen in physically distinct components of a system such as the CPU, hard-drive, or monitor, of a computer system. In bio-medicine, the failures may be the deaths of human beings caused by heart diseases, accidents, pulmonary diseases, pneumonia, or other causes of deaths. In engineering, this phenomena is commonly called as *competing risks*. The competing risks theory describes how many causes of failure act together to affect the performance of a system [1].

In reliability theory, one of estimating the parameters of the assumed failure models is another problem. The estimation may be based on data collected over censored or uncensored life tests. Data may also be obtained from past engineering experience or from handbook data, if available. Analysis of data from life tests may use any of the classical parameter estimation methods such as maximum likelihood, matching moments, regression, etc. The classical sampling methods have been considered inappropriate in many instances due to its exclusively use of data. In contrast, Bayesian estimation methods allow an analyst to combine past knowledge or experience in the form of an apriori distribution with life test data to accomplish the task with smaller sample sizes [2].

This paper will apply Bayesian estimation methods to competing risk systems of the independent variables. Although Bayesian estimation has been used widely in reliability analysis, the application so far has been limited to single risk models. Competing risks models have been studied for over two centuries; the estimation methods, however, have been classical. The combination of the two is long overdue and and this paper contributes to it, for engineers and scientist to design a better product reliability and maintainability.

The cases are limited to the models with independent causes of failure by using the Weibull distribution as our model. This paper begins with an introduction and followed by reviewing some related literatures in this area. The Weibull distribution function due to each specific risk and their system likelihood function are presented in the next section. For the Bayesian analysis, we develop the posterior and the prior functions and followed by point, interval, and reliability estimations. The estimations of net and crude probabilities are also included. A simulation is conducted for this distribution with the objectives of verifying the models and the estimators and investigating the performance of the estimators for varying sample size. The simulation data are analyzed by using Bayesian and the maximum likelihood analyses.

## 2. Literature Review

The theory of competing risks has been in use in bio-medicine for a long time. It began in 1760 with Daniel Bernoulli when he tried to determine mathematically what would happen to a population mortality structure at different ages if smallpox were eliminated from that population. Most of the currently accepted techniques were developed during the 19th century when the problem had been of great interest and importance to actuaries for over 100 years.

Mahekan was the first scientist to formulate the competing risks and studying its practical applications and Neyman named this technique for the first time as the theory of competing risks [3,4]. A review and critique of some of the models used in the competing risk analysis, such as Mahekan's and Neyman's models has been reported [5]. Chiang in his book, described and classified the types of probability of failure from a specific cause into three categories: crude, net, and partial crude probabilities [6]. Some other important researches in this area were a brief description of failure-modes and mixed-population approaches related to reliability, a study of the history of actuarial and statistical contributions to the theory of competing risks, and a model for the reliability and performance analysis of systems with multi-state components and dependent failures [7,8,9]. And also some the methods used in analyzing survival data with competing risks has been discussed by using SAS software to implement the appropriate nonparametric methods for estimating cumulative incidence functions and for testing differences between these functions in multiple groups [10]. For the risk-based design of complex engineering systems a study of the challenges in the design process, the

mathematical modeling of their topological architecture, and their unique behaviors has been conducted [11]. A model was proposed in which it was applied to accelerated degradation data from plastic substrate active matrix light-emitting diodes (AMOLEDs), along with sensitivity analysis. The reliability of the model was estimated with competing risks analyses from linear degradation and failure time data [12].

Some scientists have made their contributions in the use of Bayesian analysis in reliability studies. They used this analysis to estimate the structural reliability by considering the structural failure, fatigue crack initiation and propagation, and crack-detection capability that are characterized by a number of random variables [13,14,15]. One of proposed methods was for Bayesian reliability assessment of a coherent structure of independent components using failure data available on either the structure, its components, or both [13]. Another scientist developed a method of Bayesian reliability analysis to establish an optimal inspection schedule of a structure with multiple components [14]. A fuzzy-Bayesian analysis was developed for the decision regarding proper inspection schedule to maintain structural integrity [15].

Bayesian reliability analyses of serial systems of binomial sub-systems and components were presented in order to estimate the overall reliability of a certain air-to-air heat-seeking missile system containing five major sub-systems with up to nine components per sub-system [16,17]. It was followed by a study of a Bayesian procedure for estimating the reliability of a complex system of independent binomial series or parallel sub-systems and components [18]. They noted that the method is used to estimate the unavailability on demand of the low-pressure coolant injection system in a certain US commercial nuclear-power boiling-water reactor. There were also a study was performed in the Bayesian reliability analysis of a parallel system by using operational experience about the components or sub-units of the system [19]. For Bayesian reliability analysis, a new class of prior distributions was introduced for reliability growth tests under Binomial data under the monotone model [20]. Comparisons are made by using two examples. By taking the advantage of graphic representation and uncertainty reasoning of Bayesian network, a method of reliability modeling and assessment of multi-state system with common cause failure was proposed [21]. It took into account of the influence of common cause failure to system reliability and the widespread presence of multi-state system in engineering practices. A Bayesian framework was developed to assess the reliability and performance of multi-state systems that consists of multiple multi-state components of which the degradation follows a Markov process [22]. A Bayesian analysis of a simple step-stress was used for analyzing lifetime data obtained from accelerated life testing experiments model under Weibull lifetimes [23]. Monte Carlo simulations were performed to see the effectiveness of the proposed method, and a data set has been analyzed for illustrative purposes[24].

### 3. Weibull Distribution and Its Likelihood Function

In this paper, we describe the use of the Weibull distribution function in the competing risks analysis. This distribution function is chosen as a model because of its widely used in reliability analysis. The characteristic of this distribution is the choice of decreasing, constant, or increasing hazard function. With a constant hazard, it reduces to the exponential distribution. In this type of data, not only the failure times are recorded but also the failed or the survival items for each cause of failure. In the sections below, we will describe the likelihood function for the uncensored and type II censored life tests. The posterior function is then described as well as the use of uniform distribution as the prior. A computer simulation has been conducted to evaluate the performance of the models and estimators. The simulation uses the MATLAB computer software. The results are presented in the section below.

The probability distribution function (p.d.f.) of the length of life  $t_i$  due to a risk  $C_i$  is given by [1]

$$g_i(t_i; \lambda_i, \beta_i) = \lambda_i \beta_i t_i^{\beta_i - 1} \exp(-\lambda_i t_i^{\beta_i}), \quad (1)$$

Where  $i = 1, \dots, k; t_i \geq 0$  and  $\lambda_i, \beta_i > 0$ . The parameter  $\lambda_i$  is known as the *scale parameter*, and  $\beta_i$  is the *shape parameter*. In this paper, we will use the terms “*individual*” for every function that refers to an individual risk.

Let us consider a life test of  $m$  items in which  $n$  items have failed and  $s$  items have survived. Suppose  $n_i$  items have failed and  $s_i$  have survived from cause  $C_i$  where  $n = \sum_{i=1}^k n_i$ ,  $s = \sum_{i=1}^k s_i$ ,  $m_i =$

$n_i + s_i$ , and  $m = \sum_{i=1}^k m_i$ . The failed items due to each cause are in the time order of  $t_{i1}, t_{i2}, \dots, t_{in_i}$  and the surviving items have been operating for  $t_{i(n_i+1)}, t_{i(n_i+2)}, \dots, t_{im_i}$ , respectively.

The likelihood function of the system is given by [1]

$$L(\lambda_1, \lambda_2, \dots, \lambda_k) \propto \prod_{i=1}^k \left\{ \lambda_i^{n_i} \beta_i^{n_i} \prod_{j=1}^{n_i} t_{ij}^{\beta_i-1} \exp\left(-\lambda_i \left(\sum_{j=1}^{n_i} t_{ij}^{\beta_i} + \sum_{j=n_i+1}^{m_i} t_{ij}^{\beta_i}\right)\right) \right\} \times \prod_{\substack{l=1 \\ l \neq i}}^k \exp\left(-\lambda_l \left(\sum_{j=1}^{n_l} t_{lj}^{\beta_l} + \sum_{j=n_l+1}^{m_l} t_{lj}^{\beta_l}\right)\right) \quad (2)$$

$$\propto \prod_{i=1}^k \left\{ \lambda_i^{n_i} \beta_i^{n_i} \left(\prod_{j=1}^{n_i} t_{ij}^{\beta_i-1}\right) \exp(-\lambda_i w_i) \right\} \propto \prod_{i=1}^k L_i(\lambda_i) \quad (2)$$

where

$$L_i(\lambda_i) \propto \lambda_i^{n_i} \beta_i^{n_i} \left(\prod_{j=1}^{n_i} t_{ij}^{\beta_i-1}\right) \exp(-\lambda_i w_i), \quad (3)$$

is an individual likelihood function, and

$$w_i = \sum_{l=1}^k \sum_{j=1}^{n_l} t_{lj}^{\beta_l} + \sum_{l=1}^k \sum_{j=n_l+1}^{m_l} t_{lj}^{\beta_l}, \quad (4)$$

is then a sufficient statistic for the estimation of the scale parameter  $\lambda_i$ .

For the uncensored life test, the statistic in the equation above becomes

$$w_i = \sum_{l=1}^k \sum_{j=1}^{n_l} t_{lj}^{\beta_l}, \quad (5)$$

where  $m = n = \sum_{i=1}^k n_i$  and  $s_i = 0$ .

In the case of a type II item-censored life test, the test is terminated after  $n$  items have failed or  $s$  items have survived. The statistic in equation (4) above becomes

$$w_i = \sum_{l=1}^k \sum_{j=1}^{n_l} t_{lj}^{\beta_l} + s \sum_{l=1}^k t_{ln_l}^{\beta_l}. \quad (6)$$

### 3. The Posterior and Prior Functions

We limit our model for the case where *the scale parameter  $\lambda_i$  is a random variable and the shape parameter  $\beta_i$  is known*. This type of a sampling model works for a single mode of failure [24]. This section extends his work to the case of competing risks by using uniform distribution as the prior. We calculate the point and interval estimators of the parameter  $\lambda_i$ . The estimation of the hazard functions is also included. It is followed by the estimation of the reliability and the net, crude and partial crude probabilities.

The individual posterior distribution is given by

$$g(\lambda_i | w_i) = \frac{L_i(\lambda_i) f(\lambda_i)}{\int_0^\infty L_i(\lambda_i) f(\lambda_i) d\lambda_i} = \frac{\lambda_i^{n_i} e^{-\lambda_i w_i} f(\lambda_i)}{\int_0^\infty \lambda_i^{n_i} e^{-\lambda_i w_i} f(\lambda_i) d\lambda_i}, \quad (7)$$

where  $f_i(\lambda_i)$  is the individual posterior distribution.

We have the individual uniform prior distribution given by

$$f(\lambda_i) = \begin{cases} \frac{1}{a_i - b_i} & b_i < \lambda_i < a_i \\ 0 & \text{elsewhere} \end{cases}. \quad (8)$$

The individual posterior distribution is given by

$$g_i(\lambda_i | w_i) = \frac{\lambda_i^{n_i} \exp(-\lambda_i w_i)}{\int_{b_i}^{a_i} \lambda_i^{n_i} \exp(-\lambda_i w_i) d\lambda_i}, \quad b_i < \lambda_i < a_i. \quad (9)$$

which simplifies to

$$g_i(\lambda_i | w_i) = \frac{w_i^{n_i+1} \lambda_i^{n_i} \exp(-\lambda_i w_i)}{\Gamma(n_i + 1, a_i w_i) - \Gamma(n_i + 1, b_i w_i)}, \quad b_i < \lambda_i < a_i. \quad (10)$$

#### 4. Point Estimation

The individual posterior mean is given by

$$E_i(\lambda_i|w_i) = \frac{\Gamma(n_i + 2, a_i w_i) - \Gamma(n_i + 2, b_i w_i)}{w_i [\Gamma(n_i + 1, a_i w_i) - \Gamma(n_i + 1, b_i w_i)]}, \quad (11)$$

with individual posterior variance

$$\text{Var}(\lambda_i|w_i) = \frac{\Gamma(n_i + 3, a_i w_i) - \Gamma(n_i + 3, b_i w_i)}{w_i^2 [\Gamma(n_i + 1, a_i w_i) - \Gamma(n_i + 1, b_i w_i)]} - \left( \frac{\Gamma(n_i + 2, a_i w_i) - \Gamma(n_i + 2, b_i w_i)}{w_i [\Gamma(n_i + 1, a_i w_i) - \Gamma(n_i + 1, b_i w_i)]} \right)^2. \quad (12)$$

The individual hazard function can be estimated by

$$h_i(t) = E_i(\lambda_i|w_i) \beta_i t^{\beta_i - 1}. \quad (13)$$

The estimate of the hazard function for the system is the given by

$$h(t) = \sum_{i=1}^k E(\lambda_i|w_i) \beta_i t^{\beta_i - 1}. \quad (14)$$

#### 5. Interval Estimation

Equations for the  $100(1 - \gamma)\%$  two-sided Bayes confidence interval (TBCI) for each individual posterior mean are

$$\text{Prob}(\lambda_i \leq \lambda_{i,L} | w_i) = \frac{\Gamma(n_i + 1, \lambda_{i,L} w_i) - \Gamma(n_i + 1, b_i w_i)}{\Gamma(n_i + 1, a_i w_i) - \Gamma(n_i + 1, b_i w_i)} = \frac{\gamma_i}{2}. \quad (15)$$

and

$$\text{Prob}(\lambda_i \geq \lambda_{i,U} | w_i) = \frac{\Gamma(n_i + 1, a_i w_i) - \Gamma(n_i + 1, \lambda_{i,U} w_i)}{\Gamma(n_i + 1, a_i w_i) - \Gamma(n_i + 1, b_i w_i)} = \frac{\gamma_i}{2} \quad (16)$$

where  $\lambda_{i,L}$  and  $\lambda_{i,U}$  are the lower and the upper bounds of the interval, respectively. To find the TPBI for each  $\lambda_i$ , these equations can be solved numerically

#### 6. Reliability Estimation

The reliability estimation of the system can be obtained as follows

$$R_{\text{sys}}(t|\lambda_i) = \prod_{i=1}^k R_i(t|\lambda_i) = \exp\left\{-\sum_{i=1}^k E(\lambda_i|w_i) t^{\beta_i}\right\}. \quad (17)$$

#### 7. The Net, Crude, and Partial Crude Probabilities

This section describes the estimation of the Chiang's probabilities of failure in the Weibull model [6].

Suppose that  $C_i$  is the only risk present. The value of the *net probability*  $q_{\text{net}}$  can be obtained as follows

$$q_{i,\text{net}} = 1 - \exp\left\{-\int_0^t \sum_{i=1}^k E(\lambda_i|w_i) \beta_i x^{\beta_i - 1} dx\right\}. \quad (18)$$

The *crude probability* is the probability of failure of the system from risk  $C_i$  in the presence of all other risks,  $C_1, C_2, \dots, C_k$  is given by

$$Q_{i,\text{crude}} = \int_0^t \left\{ E_i(\lambda_i|w_i) \beta_i x^{\beta_i - 1} \exp\left(-\int_0^x \sum_{i=1}^k E(\lambda_i|w_i) \beta_i y^{\beta_i - 1} dy\right) \right\} dx. \quad (19)$$

*Partial crude probability* is the probability of failure if risk  $C_1$  is eliminated while  $k - 1$  risks remain, and is given by

$$Q_{i,1,\text{prt.crude}} = \int_0^t E_i(\lambda_i|w_i) \beta_i y^{\beta_i - 1} \exp\left\{-\int_0^x \left[ \sum_{i=1}^k E(\lambda_i|w_i) \beta_i y^{\beta_i - 1} - E_1(\lambda_1|w_1) \beta_1 y^{\beta_1 - 1} \right] dy\right\} dx \quad (20)$$

Equation (20) can be modified if we want to eliminate  $l$  risks for  $1 < l < k$  from the system.

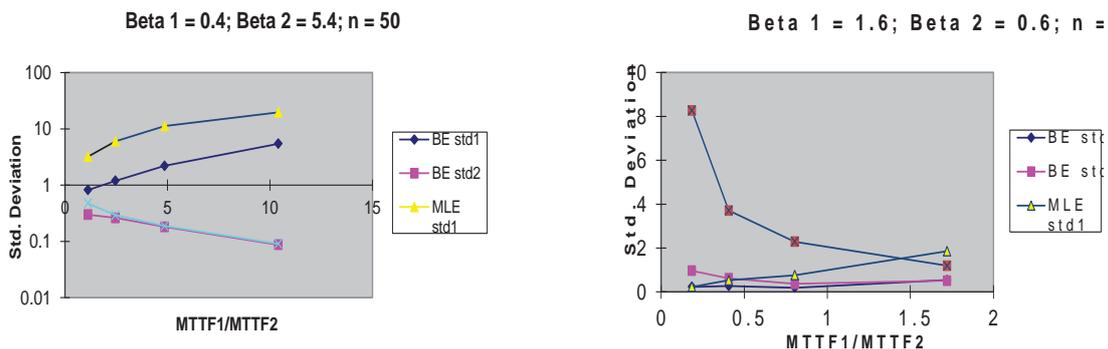
#### 8. Simulation of the Weibull Case

The simulation of the Weibull case is designed for the analysis for two causes of failure. We perform the simulation not only for the Bayesian analysis, but also for the maximum likelihood. The objectives of the simulation are to verify the models and the estimators and to investigate the performance of the estimators for varying sample size. Other objectives are to compare the results of these two analyses and to investigate the sensitivity of the Bayesian estimators to the locations of the uniform prior.

We simulate the uncensored life tests of items for varying sample sizes using the MATLAB computer programs for this purpose. For the Bayesian analyses, the simulation first creates the Weibull failure times for each risk. In this simulation process, we use four pairs of the shape parameters  $\beta_1$  and  $\beta_2$  in combination with 16 pairs of the scale parameters  $\lambda_1$  and  $\lambda_2$ . The shape parameters are the known parameters. In the real situations, these numbers should be based on the past experience of the model. The values of the scale parameters are randomly selected from within the prior distribution limits. In reality, the values of  $\lambda_1$  and  $\lambda_2$  should be unknown. They are the estimated values. We simply pretend to have them.

We run the simulation programs of the Bayesian analysis for sample sizes 30, 50, 100, and 150. We conduct a similar simulation for the maximum likelihood analysis. They result in the failure time data sets for each case, 20 replications, and four sets of sample size. The second results are the estimated values of the parameters  $\lambda_1$  and  $\lambda_2$  and the mean times to failure of the first risk ( $MTTF_1$ ) and of the second risk ( $MTTF_2$ ).

Examples of the charts that show the relationship between the true  $(MTTF_1)/(MTTF_2)$  ratio values and the standard deviations are given in Figure 1. These charts are plotted for the Bayesian and the maximum likelihood analysis results. The charts are constructed for each sample size.



**Figure 1. Example of  $(MTTF_1)/(MTTF_2)$  vs. Standard Deviation for different values of  $\beta$**

These charts show that the change of the  $(MTTF_1)/(MTTF_2)$  ratio values will change the values of both standard deviations in an opposite direction. The smaller the true mean time to failure value of one risk relative to another, the bigger the chance is for this risk to fail first. With more failure time data available, the smaller the standard deviation for that risk, the larger the standard deviation is for another. These changes can be seen clearly if the true  $(MTTF_1)/(MTTF_2)$  ratio values are much smaller or much larger than one.

For a perfect information on the prior distribution, the standard deviations of the estimated values of parameters for the Bayesian analyses are smaller than those in the maximum likelihood analyses. In this condition, the results show that the estimations of the Bayesian analyses are better than those of the maximum likelihood analyses.

Examples of the charts for the sensitivity analysis purposes are shown in Figure 2. In each of these charts, we have the true value of the scale parameter, the estimated value of the maximum likelihood analysis and the estimated values of the Bayesian analysis for different locations of the uniform prior distribution. Be-

tween two points in the charts represent 50 and 100 percent shifts in the prior locations of cases 2.1 and 3.4, respectively.

We can see from these charts that there are lacks of sensitivity over the shifts of the prior locations. These charts also show the robustness of the Bayesian analysis within the range between the true value and the maximum likelihood estimated value lines.

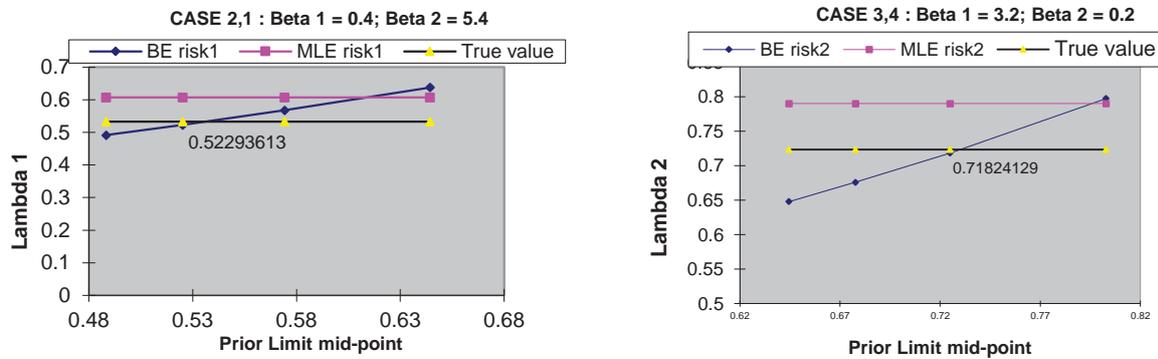


Figure 2. Example of prior limit mid-point vs.  $\lambda_1$  for different values of  $\beta$ .

## 9. Conclusions

The primary objective of this paper is to investigate the applications of the Bayesian estimation methods to competing risks by using the Weibull distribution as the model. A simulation for two risks is conducted to verify the analyses and the estimators. The likelihood function of the system is separable into the likelihood functions of each risk; each of these is treated individually. This individual function, combined with the individual prior, is then used to make inferences of the parameters of interest. The simulation results show that the change of the true value of one parameter relatively to another's will change the values of both risk's standard deviations in an opposite direction. The smaller the true value of one risk relative to another, the bigger the chance is for the this risk to fail first. With more failure time data available, the smaller the standard deviation for that risk, the larger the standard deviation is for another. For a perfect information on the prior distribution, the standard deviation values of the Bayesian analyses are smaller than those in the maximum likelihood. In this condition, the estimation methods of the Bayesian analyses are better than those of the maximum likelihood.

The sensitivity analyses show some amount of sensitivity over the shifts of the prior locations. They also show the robustness of the Bayesian analysis within the range between the true value and the maximum likelihood estimated value lines. In general, the use of the Bayesian methods in the competing risks analyses gives us several advantages. These analyses allow us to use past knowledge or experience in the form of an a priori distribution in combination with failure time data. They allow us to accomplish the task with smaller sample sizes, especially in the situations of performing a life test is expensive and/or time consuming. The censoring types of life test are applied here. The real application of this method is to evaluate the reliability of a system in which that system successfully performs its intended function for a given period of time under specified conditions. This method is very useful to evaluate detailed and complex systems such as space exploration, military applications, and commercial uses.

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