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Job shop scheduling model for non-identic machine with fixed delivery time to minimize tardiness

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Abstract. Scheduling non-identic machines problem with low utilization characteristic and fixed delivery time are frequent in manufacture industry. This paper propose a mathematical model to minimize total tardiness for non-identic machines in job shop environment. This model will be categorized as an integer linier programming model and using branch and bound algorithm as the solver method. We will use fixed delivery time as main constraint and different processing time to process a job. The result of this proposed model shows that the utilization of production machines can be increase with minimal tardiness using fixed delivery time as constraint.

Keywords: Scheduling, non-identic machine, mathematical model, branch and bound, tardiness, utility, fixed delivery time

1. Introduction

This paper will present mathematical model for scheduling jobs in job shop environment on non-identic press machines to minimize total tardiness, where jobs can only be delivered at certain fixed delivery dates. Mensendiek, Gupta and Herrmann (2014) observed that scheduling problems with fixed delivery dates are often found in industry, for example when a manufacturing company relies on the timetables of shipping.

One of Indonesia manufacturing company that use press machine as production machine has a problem to schedule 59 press machines to produce many variety products (up to 70 types of product). Each machine has different processing time for one type job because several factors like age of machine, maintenance and press strength capacity of the machine. We define this condition as non-identic machine. This company has to do delivery on time because this occasion already become company's core value for the customer. Bahagia (2007) stated that nowadays manufacture industry must always efficient and can accommodate wish and needs from costumers by fixing its manufacture systems. One way to improve manufacture systems from manufacturing company that has many press machines with different processing time, many variety products and many delivery date is to make scheduling.

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Scheduling has some objectives like increase utilization from resource that used, reduce idle time, reduce mean flow time and reduce tardiness value (Bedworth & Bailey, 1987). For a company that want minimal lateness for its production and delivery, scheduling can be a solution because tardiness value can be minimize. In this paper we would propose a scheduling method for accommodate allocation of using production machine so utilization of total machine that used for production can be increase. This method will consider delivery schedule as its constraint. Objective function that we want to reach is to minimize tardiness from each job. An allocation of job to a machine will be infeasible if the processing time assigned to a delivery block exceeds the capacity.

The remainder of the paper is structured as follows; In Section 2, we present mathematical programming formulation that using branch and bound algorithm to generate optimal solution in software. In Section 3, we will do validating mathematical programming formulation with some hypothetic data. Next section, Section 4 will tell us about the numerical result of the testing mathematical model using five scenarios of different usage of production machine. We use data of processing time up to 1470 data, data of delivery date up to four schedule and number of job up to 70 jobs. Section 5 summarizes the findings and suggests some fruitful directions for future research.

2. Mathematical Programming Formulation

Define a set of jobs, J, where each job $j = 1,\ldots,$ n has a given integer processing time p_{ij} and due date d_j is to be scheduled non-preemptively on m non-identical parallel machines. Each machine $i = 1,\ldots, m$ is ready at time zero and process all jobs. Because the machine is not identic, the processing time of the job will be different one to another depend on the machine, so p_{ij} will be processing time for job j in machine i. The due dates are the times when a customer asks to receive a job and hence a job has to be processed and delivered before or at due date to avoid tardiness penalties. There is K, set of given fixed delivery dates $k = 1,\ldots, s$, where deliveries assumed occur at times $\Delta_1 < \ldots \leq \Delta_s$. Delivery capacity at each delivery date is assumed to be infinite and the time for delivery is neglected.

Each job is processed and completed in a press machine, its completion time C_j will same as its p_{ij}. The job delivery time D_j is defined to be $D_j = \min_{k\epsilon K}\{\Delta_k \mid \Delta_k \geq C_j\}$ so the job is delivered at the first delivery date after its completion. Its tardiness is defined as $T_j = \max\{0,\ D_j - d_j\}$. All of these explanations are defined into system element (Table 1).

Table 1 System element

System Element	Note
Tardiness value	Output
Total machine	Input
Total job	Input
Total delivery block	Input
Processing time for each machine	Input
Delivery capacity	Component
Production capacity	Component
Due date for each job	Input
Setup time for each job	Input
Delivery date	Input
Completion time for each machine	Component
Allocation status for each machine	Component

Hall, Leksaona and Potts (2011) give overview that in an optimal solution of a minimize tardiness problem, all jobs assigned to one block on one machine can be processed in arbitrary sequence. Due to this overview, the problem of minimize tardiness can be solved by identifying an optimal assignment of jobs to delivery blocks. Define $x_{ijk} \in \{0, 1\}$ as status of job j is assigned to delivery block k on machine i. The mathematical programming formulation for this problem can be stated as follows.

$$Min\ Z = \sum_j T_j \quad (1)$$

subject to :

$$\sum_i \sum_k x_{ijk} = 1 \quad \forall j \quad (2)$$

$$T_j \geq 0 \quad \forall j \quad (3)$$

$$T_j \geq \sum_i \sum_k \left(x_{ijk}\Delta_k\right) - d_j \quad \forall j \quad (4)$$

$$\sum_j \sum_{h:h\leq k} x_{ijh}p_{ij} \leq \Delta_k \quad \forall i,k \quad (5)$$

$$x_{ijk} \in \{0,1\} \quad \forall i,j,k \quad (6)$$

Objective function of the mathematical formulation is to minimize total tardiness of each job using (1) formulation. Tardiness of each job j must be larger or equal to zero (3) and came from equation (4). The constraint (2) will make sure that each job j is assigned to exactly one delivery block k on one machine i. Constraint (5) make sure that total processing time of a job j in machine i to a delivery block k and all preceding delivery blocks h on one machine i does not exceed that block's delivery date. Last constraint (6) showed that decision variable that assigned a job j in a machine i to a delivery block k (x_{ijk}) must be in binary form.

3. Validating Optimal Solution Procedures

To solve this mathematical programming formulation, we used branch and bound algorithm as optimal solution algorithm solver. Branch and bound algorithm is a systematic method that implicitly count all combination opportunities from decision variables. We chose LINGO 11 as solver software because this software could generate branch and bound algorithm and retrieve optimal solution from this mathematical programming formulation. Using this software with branch and bound algorithm, a scheduling can be found by finding an optimal sequence of all jobs where each job is assigned to the next available machine in ascending order of its position in the job sequence. Each job must be delivered at the first delivery block at or after its completion. A solution will become infeasible if any job is completed after the last delivery date.

Validating optimal solution must be done to check the output result from the model can represent the real system. We used hypothetic data for this step. The hypothetic data are shown in Table 2, Table 3 and Table 4.

Table 2 List of machine for validating step

Machine	Press Strength	Machine Capacity
1	300 Ton	12 hours per day
2	200 Ton	12 hours per day
3	200 Ton	12 hours per day
4	160 Ton	12 hours per day

Table 3 Job list for validating step

Job	d_j	p_{ij} (hours)			
		Machine 1	Machine 2	Machine 3	Machine 4
1	36	7	7	7	6
2	36	3	3	3	3
3	36	8	8	7	7
4	24	4	4	4	4
5	24	7	7	7	6
6	24	3	3	3	3
7	24	3	3	3	3

Table 4 List of delivery schedule

Delivery Block (k)	Delivery Date
1	14
2	28
3	38

The result of validating step shown in gantt chart form (Figure 1). Output from LINGO 11 was decision variables x_{ijk} for each job j in a machine i for a delivery date so tardiness will be minimal.

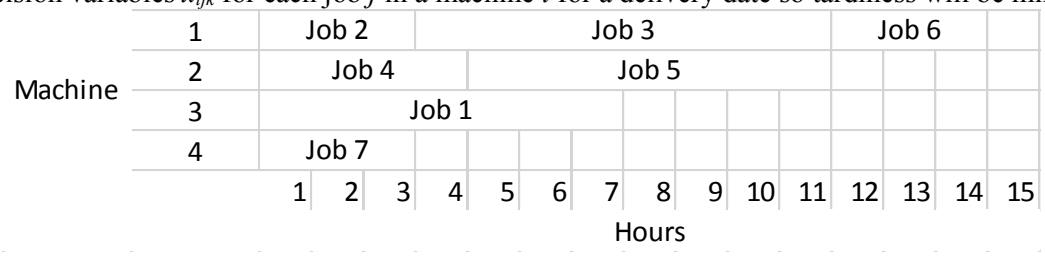


Figure 1 Gantt chart of validating result

Using hypothetic data, there is no job that exceed its due date, so tardiness is zero. All of jobs can be scheduled into a machine to fulfil a delivery date. At one timeline, each machine only can process a job, so there is no job that overlapping with another job in one machine.

4. Result

Using mathematical model, we elaborate results of five scenarios. These five scenarios are divided by total machines and press strength.

Table 5 Testing scenario

Machine Category			Scenario				
Ma-chine	**Classifi-cation**	**Press Strength (Ton)**	**1 (12 machines)**	**2 (13 machines)**	**3 (14 machines)**	**4 (15 machines)**	**5 (16 machines)**
1	High strength	300	√	√	√	√	√
2		200	√	√	√	√	√
3		200	√	√	√	√	√
4	Mid strength	160	√	√	√	√	√
5		160	√	√	√	√	√
6		150	√	√	√	√	√
7		150	√	√	√	√	√
8		150	√	√	√	√	√
9		110	√	√	√	√	√
10		110	√	√	√	√	√
11		110	√	√	√	√	√
12		110	√	√	√	√	√
13		100	X	√	√	√	√
14		80	X	X	√	√	√
15		80	X	X	X	√	√
16		80	X	X	X	X	√

We define some parameters like delivery time for each job, processing time for each job in a specific machine and due date for each job. Parameter specifications for mathematical programming formulation are depicted in Table 6.

Table 6 Parameter specifications for mathematical programming formulation

Parameter	**Value**
Number of delivery date	4
Number of processing time all jobs in machine	1470
Number of due date	70

There are some differences from the result of the formulation testing for each scenario. Using LINGO 11 as optimization software, Table 7 will show the differences for each scenario.

Table 7 Testing result for each scenario

Scenario	**Computation Time**	**Tardiness Value**	**Total Iteration**	**Mean Efficiency**
12 machines	1 second	No feasible solution	8	-
13 machines	149 hours	62	1.846.608.695	96,98%
14 machines	13 seconds	0	103.235	89,82%
15 machines	20 seconds	0	228.969	85,71%
16 machines	2 seconds	0	3.130	77,64%

Mathematical programming formulation use branch and bound algorithm to generate optimal solution. Scenario using 13 machines need very long computation time because fathoming process cannot be done. All of sub problem in this scenario are not fathomed, so the software need more time to check the alternative of the sub problem.

Based on testing result from 5 scenarios, scenario using 14 machines is the best scenario because with 14 machines problem can be solved with minimal tardiness value and not very long

computational time. The average efficiency from scenario using 14 machines can be categorized as high efficiency too (89,82%).

From testing result, we can define advantage and disadvantage of using this mathematical model. The advantage and disadvantage of using this mathematical model is stated in Table 8.

Table 8 Advantage and disadvantage of mathematical model

Advantage	Disadvantage
Mathematical model consider delivery schedule in generating production scheduling	Mathematic model cannot accommodate parameter i.e. release time for a job
Computation time will be faster if production capacity (total available machine) is large	Setup time is not implicitly state in mathematical model. Process time in this paper formulation alr eady include setup time
Scheduling for non-identic parallel machine can be made with this mathematic model	Model cannot count a more specific calculation using cost as objective

5. Conclusion

In this paper we propose optimal solution procedure using mathematical model for the practically relevant problem of scheduling jobs in job shop environment on non-identic parallel machines to minimize total tardiness subject to fixed delivery dates. This mathematical programming formula will use a branch and bound algorithm. This approaches can provide optimal solution up to 70 jobs with different processing time from 16 machines. We suggest practitioners that want optimal solution to use this mathematical model because it can process big sized problem computation and can reduce machine usage with tardiness concern.

Mathematical model for scheduling non-identic parallel machine will generate schedule with allocating job into a machine to fulfil a specific delivery time. Optimal solution generating will use software LINGO 11 and branch and bound algorithm. Objective function for this paper is minimize tardiness value of each job. The best schedule that we proposed is using scenario 14 machines from 59 machines for scheduling 70 job with 4 delivery dates. This scenario will have 0 as tardiness value and reasonable 13 seconds as computational time. Because the reduction of the machine that use for production, efficiency value of the total machine increase up to 89% for using 14 machines, increase up to 71% related to when still using 59 machines.

This paper opens several avenues for further research. First, it appears promising to further improve the performance of mathematical programming approach. Next mathematical model can consider release date of a job as a parameter and use cost as objective function. Improvement can be made with considering setup time into a new parameter too. This model can be used to evaluate the benefits of integrated decision making from incoming material, production until delivery of the product.

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