

Modelling a flows in supply chain with analytical models: *Case of a chemical industry*

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Abstract. This study is interested on the modelling of the logistics flows in a supply chain composed on a production sites and a logistics platform. The contribution of this research is to develop an analytical model (integrated linear programming model), based on a case study of a real company operating in the phosphate field, considering a various constraints in this supply chain to resolve the planning problems for a better decision-making. The objectives of this model is to determine and define the optimal quantities of different products to route, to and from the various entities in the supply chain studied.

1. Introduction

The modelling of supply chains is an essential tool for a better understanding of its complexity in order to optimize its functioning and improve its performances. However, a set of problems omnipresent in the industry is commonly due to the absence of a pertinent modelling. For that reason, in this study, we focus on the modelling of a supply chain of a company operating in the phosphate industry specifically, establishing models of logistics flows between a production sites and a logistics platform.

The purpose is to develop an analytical planning model in order to improve the management of daily transportation flows of products. In addition, to maximize the quantities transported of goods, therefore to optimize the routing of the products to and from the logistics platform considering all the constraints (the capacity of stocks, cadence of production, capacity of the transportation equipment and the customers' demands).

This paper is structured as follows, the first paragraph present a literature review in supply chain modelling and the planning problems. The second paragraph expose our case study and the models adopted.

2. Context of the study

2.1. Definition

According to the concept of logistics, supply chain has several definitions depending on the context and scope of study. Hereafter, we present some definitions of the supply chain from which we can derive a definition synthesis.

Supply chain is “a system whose constituted shares include material supplier, production facilities, distribution services and customers linked together via the feed forward flow of materials and the feedback flow of information” [1].

Thus, we can define the supply chain as a succession of companies, which contribute to providing a product or service for a customer. By means of exchanging of material flows, and financial information.

In addition, we can define the logistics platforms, conferring to the Council of Logistics Management, as an area within which all activities relating to transport, logistics and the distribution of goods, both for national and international transit, are carried out by various operators [2].



2.2. Supply chains Modelling

Modelling is a set of techniques that provides the ability to study and understand the structure and the operating principle of a system. The term model in a technical context is a useful presentation of some objects, it is an abstraction of a reality expressed in term of some formalism.

A model is always based on a three kind of languages: the informal language, semi-formal (graphic language) and the formal language (mathematical). Usually, the models based on an informal language is used to describe an existing situation, while models based on a formal language permits the verification of properties in new project [3, 4].

Therefore, there are different classification of models in a supply chains [5-14], a taxonomy of these models is presented in the following works:

- Harrel and Tumay in 1994 classified models of supply chains into two categories: the solutions evaluation methods and the solutions generation methods [12].
- Beamon in 1998 considered four types of models: deterministic, stochastic, economic and simulation models [13].
- Min and Zhou in 2002 considered four model classes: deterministic, hybrid, stochastic and IT models [14].
- Trilling in 2004 distinguee three categories of models: structured, systemic and object oriented models [5].
- Labarthe in 2006 grouped the approaches in three main classes namely: organizational models, analytical models and simulation models [10].

In this work, we adopt the taxonomy (See Figure 1) proposed by Labarthe [10], which grouped the different modelling approaches.

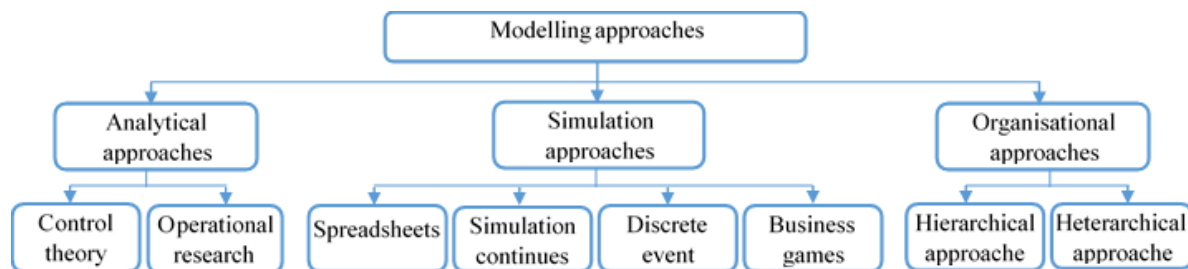


Figure 1. The Modelling supply chain approaches.

2.2.1. Analytical approach. The analytical modelling approach describes the system by mathematical equations. Consequently, two approaches are presented in this category: The control theory, which models the supply chains by differential equations and the operational research, which intended to develop the best decisions based on a set of constrains.

The resolution methods for this approaches, can be divided into two types of methods [14]: the exact methods (linear program, dynamic program...) which guarantee to find the best solution [15] and heuristics or meta-heuristics that identify approximate solutions [16].

As a result, the approach adopted in this work is development of a deterministic analytical model. The models are presented in the next sections.

2.2.2. Simulation approach. Simulation is an imitation of the operations of a real-world system over the time that involves the creation of an artificial history of the real-system [8]. This approach is typically used when it is difficult to find a relation (an equation) between different variables.

The variety of supply chain problems caused the appearance of several approaches, like the following: spreadsheets, simulation continues, discrete event simulation and business games [8].

2.2.3. Organizational approach. This approach represent supply chains using its entities, activities, processes, functions, structures and comportments. The main approaches resulting are the hierarchical approaches and heterarchical approaches.

2.3. Planning models in a supply chains

The literature on planning models research applied to supply chains is particularly extensive, several reviews of this works reflect the interest of this research area [17-24].

In addition, these models has been used to model planning problems in many types of industries, including computer manufacturing industry [25], detergent production industry [26], health product industry [27], the pharmaceutical industry [28], paper industry [29], petroleum industry [19] and textiles industry [22].

Resulting of this review, we notice the lack of reference models that formalizes this kind of problems, developing a planning model based on a case study of a real company operating in the phosphate field and modelling a planning problems in a multi-site, multi-product, multi-depots, transport multimode and multi-periods.

3. Model development

3.1. Description of the case study

In this paper, the case studied is a supply chain in the phosphate field, which composed of three main production sites, to transform the raw material (sulphur) and produce the goods (phosphoric acid and fertilizers), and a logistics platform to export these goods and import the necessary of a raw material.

The logistics platform is implemented in a seaport at 13 km from the production sites. It includes two main units; the first is for receiving the products from the production sites, while the second is for the receiving of raw materials imported from many international places.

In this supply chain, trains do the routing of products normally 24/24 hours; however, the routing is done by dump trucks in the case of surpassing the logistics platform capacity to serve customers demands. See figure 2, which present the configuration and the flows in this supply chain.

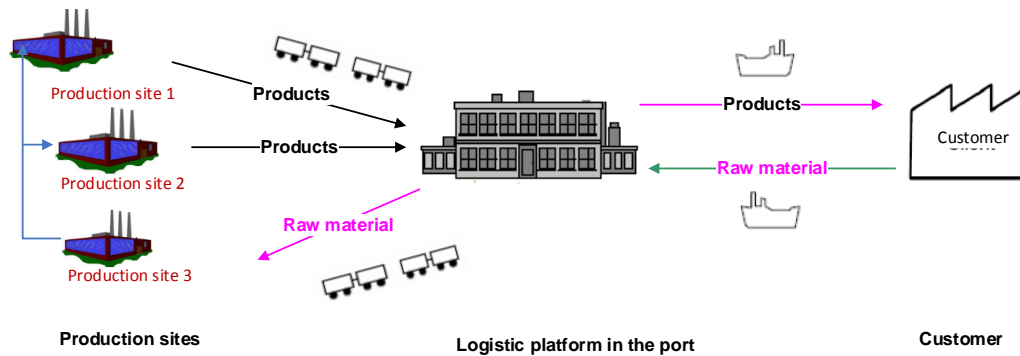


Figure 2. The supply chain studied.

3.2. Proposed mathematical model

We present a general mathematical model, used for planning the activities in this supply chain as well as decisions related to procurement and transportation of various products.

The proposed planning model is a discrete event model, and the time division in this model corresponding to the planning reference period on the routing of product is established. This period corresponds to the period allowed to load each ship. It is determined from the stems confirmed by customers.

The notations, parameters and variables used in this model are presented in the following:

3.2.1. Series

- P : Series of transported products, $p \in P$;
- T : Number of periods t in the planning horizon $t=1 \dots T$, $t \in T$;
- J : Series of the days in the planning period t , $j=1 \dots J$, $j \in J$;
- E : Series of entities in the supply chain, $e=pr$ for logistics platform and $e=u$ for the production site, $e \in E$.

3.2.2. Parameters

$S_{e,p}^{j,t}$: The stock of the product p in the entity e during the day j of period t;
 $P_{e,p}^{j,t}$: The daily production of the product p in the entity e for the period t;
 $D_{e,p}^{j,t}$: The daily demand for the period t of the product p of customer in the entity e;
 $DMax_{e,p}^t$: The maximal quantity of the product p required by the customer for the period t in the entity e.

3.2.3. Data

$SMax_{e,p}$: The maximal capacity of the stocks of the product p in the entity e;
 $SSec_{e,p}$: The quantity of product p, which corresponds to the safety stock in the entity e;
 $S_{e,p}^{0,0}$: The initial stock ($j = 0, t = 0$) of the product p in the entity e;
 $PMax_{e,p}$: The maximal production capacity of the product p in the entity e;
 $CMax_{e,p}$: The maximal capacity of the product p that can support equipment in the entity e;
 $NdMax_{Ca}$: The maximal number of daily routes that can make a truck;
 $NdMax_{Tr}$: The maximum number of daily routes that can make trains;
 $QDesMax_{Ca,p}$: The maximal quantity of product p that can be transported in each truck travel;
 $QDesMax_{Tr,p}$: The maximal quantity of product p that can be transported in each trains travel;
 M : A constant with big value $M = "+ \infty"$.

3.2.4. Decision variables

Q_p^t : The quantities of the products p in the period t transported between logistics platform and the production sites;
 $Q_p^{j,t}$: The quantities of products p transported during the day j in the period t;
 $QDes_{Tr,p}^{j,t}$: The quantities of products p transported in each trains travel;
 $QDes_{Ca,p}^{j,t}$: The quantities of product p shipped in each trucks travel;
 $Nd_{Tr}^{j,t}$: Number of travels made by the trains during the day j in period t;
 $Nd_{Ca}^{j,t}$: Number of travels made by trucks during the day j in period t;
 α_j : Binary variable to start or not the mode of transport by trucks.

3.2.5. Objective Function

The function objective (see formula 1) proposed in this model consists to maximize the quantities transported in a specified period, to feed the stocks of every entity of the studied supply chain, in order to satisfy the customers demand.

$$\text{Max}(z) = \sum_{t \in T} Q_p^t = \sum_{t \in T} \sum_{j \in J} \sum_{p \in P} Nd_{Tr}^{j,t} \cdot QDes_{Tr,p}^{j,t} + \sum_{t \in T} \sum_{j \in J} \sum_{p \in P} Nd_{Ca}^{j,t} \cdot QDes_{Ca,p}^{j,t} \cdot (1 - \alpha_j) \quad (1)$$

3.2.6. Constraints

The mathematical formulations of the constraints are described on the following:

- The constraints related to quantities transported

When the daily demand of products in the logistics platform exceeds the capacity of stocks and trains, the routing of products is done by trucks and trains at same time. For that reason, we used a binary variable that takes the following values:

$$\alpha_j = \begin{cases} 0 & ; \quad \text{If the routing of products is made by trains and trucks} \\ 1 & ; \quad \text{If the routing of products is made only by trucks} \end{cases}$$

In this model, for starting the routing of products by trucks, we considered the daily stock $S_{pr,p}^{j,t}$ in the port as an indicator for safety stock and a threshold to start routing.

So if the value of the daily stock $S_{pr,p}^{j,t}$ is below the safety stock $S_{sec,pr,p}$, the binary variable take $\alpha=1$, consequently the term of quantities transported by trucks in the objective function is annulled.

We can formulate these conditions in the following constraints (2) and (3). Therefore, in the formula 2 we can consider the value of the constant M, equal to the maximal capacity of the stocks.

$$S_{pr,p}^{j,t} - S_{sec,pr,p} \leq M \cdot \alpha_j \quad \forall t \in T, j \in J, p \in P \quad (2)$$

$$S_{pr,p}^{j,t} - S_{sec,pr,p} > 0 \quad \forall t \in T, j \in J, p \in P \quad (3)$$

In the constraint (4), we consider that the transported quantities Q_p^t in period t are equal to the sum of the daily transported quantities $Q_p^{j,t}$ during this period t.

$$Q_p^t = \sum_{j=1}^J Q_p^{j,t} \quad \forall t \in T, j \in J, p \in P \quad (4)$$

- The constraints related to stocks

To transport product, it is necessary to ensure that the quantities of products requested by the customers are available in the stocks of different entities. This condition is presented by the following constraints:

- The constraint (5) ensures that the quantities of products are always available in stocks of production sites.

$$S_{sec,u,p} < S_{u,p}^{j,t} \leq S_{Max,u,p} \quad \forall t \in T, j \in J, p \in P \quad (5)$$

- The constraint (6) ensures that the quantities of products are always available in stocks of logistic platform.

$$S_{pr,p}^{j,t} \leq S_{Max,pr,p} \quad \forall t \in T, j \in J, p \in P \quad (6)$$

- Constraint (7) specifies the daily quantity of products in the stock of production sites. It is equal to the quantity of products in the stock in the day j-1 plus the quantity produced in the day j, less the total quantity transported in the same day j.

$$S_{u,p}^{j,t} = S_{u,p}^{j-1,t} + P_{u,p}^{j,t} - Q_p^{j,t} \quad \forall t \in T, j \in J, p \in P \quad (7)$$

- Constraint (8) specifies the daily quantity of products in the stock of logistics platform $S_{pr,p}^{j,t}$. It is equal to the quantity of products in the stock in the day j-1 plus the total quantity transported in the day j, less the quantity requested by customers in the same day j.

$$S_{pr,p}^{j,t} = S_{pr,p}^{j-1,t} + Q_p^{j,t} - D_{pr,p}^{j,t} \quad \forall t \in T, j \in J, p \in P \quad (8)$$

In the precedent constraints (7) and (8), it is necessary to consider the initial quantity of products in stocks $S_{e,p}^{0,0}$.

- The constraints related to the production

To transport the requested products, it is necessary to feed stocks with needed quantities $P_{e,p}^{j,t}$ and not to exceed the maximal quantity of daily production $P_{Max,e,p}$. This condition is described in the constraint (9).

$$P_{e,p}^{j,t} \leq P_{Max,e,p} \quad \forall t \in T, j \in J, e \in E, p \in P \quad (9)$$

- The constraints related to customer's demand

The demand $D_{Max,e,p}$ is determined from the "stems" confirmed by customers, this demand is equal to the sum of daily requests $D_{e,p}^{j,t}$ in the period t (see the constraint 10).

$$\sum_{j=1}^J D_{e,p}^{j,t} = D_{Max,e,p} \quad \forall t \in T, j \in J, e \in E, p \in P \quad (10)$$

The daily demand $D_{e,p}^{j,t}$ cannot exceed the maximal capacity (daily flow) $CMax_{e,p}$ that can support the routing equipment (see the constraint 11).

$$D_{e,p}^{j,t} \leq CMax_{e,p} \quad \forall t \in T, j \in J, e \in E, p \in P \quad (11)$$

- The constraints related to the transport

The quantities $QDes_{Tr,p}$ of products transported by trains in each trip, cannot exceed the maximal capacity $QDesMax_{Tr,p}$ that can support train (see the constraint 12).

$$QDes_{Tr,p}^{j,t} \leq QDesMax_{Tr,p} \quad \forall t \in T, j \in J, p \in P \quad (12)$$

The number of routes done in day j in the period t , cannot exceed the maximal capacity of daily transportation (see the constraint 13).

$$Nd_{Tr}^{j,t} \leq NdMax_{Tr} \quad \forall t \in T, j \in J, p \in P \quad (13)$$

The same constraints presented above are applied in the case of transport by dump trucks (see the constraints 14 and 15).

$$Nd_{Ca}^{j,t} \leq NdMax_{Ca} \quad \forall t \in T, j \in J, p \in P \quad (14)$$

$$QDes_{Ca,p}^{j,t} \leq QDesMax_{Ca,p} \quad \forall t \in T, j \in J, p \in P \quad (15)$$

3.3. Formulation of the global models

The obtained model and the proposed constraints are recapitulated in the following. This global model, we permit to formalize the specific models for each product and for a chosen period to plan the routing products in the supply chain studied.

$$Max(z) = \sum_{t \in T} Q_p^t = \sum_{t \in T} \sum_{j \in J} \sum_{p \in P} Nd_{Tr}^{j,t} \cdot QDes_{Tr,p}^{j,t} + \sum_{t \in T} \sum_{j \in J} \sum_{p \in P} Nd_{Ca}^{j,t} \cdot QDes_{Ca,p}^{j,t} \cdot (1 - \alpha_j)$$

Constraints

$$S_{pr,p}^{j,t} - SSec_{pr,p} \leq M \cdot \alpha_j \quad \text{and} \quad S_{pr,p}^{j,t} - SSec_{pr,p} > 0 \quad \forall t \in T, j \in J, p \in P$$

$$Q_p^t = \sum_{j=1}^J Q_p^{j,t} \quad \forall t \in T, j \in J, p \in P$$

$$SSec_{u,p} < S_{u,p}^{j-1,t} + P_{u,p}^{j,t} - Q_p^{j,t} \leq SMax_{u,p} \quad \forall t \in T, j \in J, p \in P$$

$$S_{pr,p}^{j-1,t} + Q_p^{j,t} - D_{pr,p}^{j,t} \leq SMax_{pr,p} \quad \forall t \in T, j \in J, p \in P$$

$$P_{e,p}^{j,t} \leq PMax_{e,p} \quad \forall t \in T, j \in J, p \in P, e \in E$$

$$\sum_{j=1}^J D_{e,p}^{j,t} = DMax_{e,p}^t \quad \text{and} \quad D_{e,p}^{j,t} \leq CMax_{e,p} \quad \forall t \in T, j \in J, p \in P, e \in E$$

$$QDes_{Tr,p}^{j,t} \leq QDesMax_{Tr,p} \quad \text{and} \quad Nd_{Tr}^{j,t} \leq NdMax_{Tr} \quad \forall t \in T, j \in J, p \in P$$

$$Nd_{Ca}^{j,t} \leq NdMax_{Ca} \quad \text{and} \quad QDes_{Ca,p} \leq QDesMax_{Ca,p} \quad \forall t \in T, j \in J, p \in P$$

$$QDes_{Tr,p}^{j,t}, QDes_{Ca,p}^{j,t}, P_{e,p}^{j,t}, D_{e,p}^{j,t} \in \mathbb{R}^+ \quad \text{and} \quad Nd_{Tr}^{j,t}, Nd_{Ca}^{j,t} \in \mathbb{N}^+ \quad \text{et} \quad \alpha_j \in \{0,1\} \quad \forall t \in T, j \in J, p \in P, e \in E$$

3.4. Evaluation Model

The model developed in this study is a Mixed Integer Programming (MIP); it belongs to the class of NP complete problems. It is a difficult problem to solve, consequently one of the most methods used for resolving this type of model is the method "branch and bound".

Therefore, we are working now on solving this program, using this exact method in a solver software (IBM ILOG CPLEX). The results of this study will be the subject of a future paper.

4. Conclusion

We are interested in this work on the modelling of the logistics flows in a supply chain, specifically developing a planning model whose objective is to define the optimal quantity of products rotted in the supply chain studied (companies operating in the phosphate field). Therefore, we presented a literature review of works related to the modelling of this kind of problems, then we focused to accomplish the analytical model considering the various constraints presented in the case study.

In our future work, we propose to generalize these models for the other entities in this supply chain, we also suggest to couple our model to a simulation models for a better understanding of this kind of problems.

5. References

- [1] Tayur S, Ganeshan R and Magazine M 1999 Quantitative models for supply chain management *Kluwer Academic Publishers*.
- [2] Kate A and Vitasek V 2003 Supply Chain Visions terms and glossary Washington.
- [3] Chapurlat V Larnac M Lamine E Magnier J 1999 Definition of a formal analysis framework for existing enterprise modelling approaches *Proc. of the Annual Conf. of ICIMS-NOE* Louvain Belgium.
- [4] Braesch C Haurat A and Beving J 1995 *L'entreprise-système* La modélisation systémique en entreprise Hermès Paris France.
- [5] Trilling L et al. 2004 Investigation et comparaison des méthodes et outils d'analyses pour l'étude des systèmes hospitaliers *HRP Report*.
- [6] Pidd M 1999 Just modelling through: a rough guide to modelling Interfaces.
- [7] Min H and Zhou G 2002 *Computers & Industrial Engineering* Supply chain modelling: past, present and future **43** pp 231-249.
- [8] Kleijnen, J P C 2005 *Simulation & process modelling* Supply chain simulation tools and techniques: a survey" **1** pp 82-89.
- [9] Lauras M 2004 Méthodes de diagnostic et d'évaluation de performance pour la gestion de chaînes logistiques: application à une filiale internationale dans un groupe pharmaceutique et cosmétique *Phd report* L'institut National Polytechnique de Toulouse.
- [10] LABARTHE O 2006 Modélisation et simulation orientées agents de chaînes logistiques dans un contexte de personnalisation de masse: modèles et cadre méthodologique *Phd report* Université Laval Québec et Université Paul Cézanne Marseille.
- [11] Ganeshan R Jack E Magazine M J and Stephens P 1998 A taxonomic review of supply chain management research, in quantitative models for supply chain management *Kluwer Academic Publishers* Boston pp 841-880.
- [12] Harrel C Tumay K 1994 *Engineering & Management press* Simulation made easy.
- [13] Beamon M 1998 *Production Economics* Supply chain design and analysis: models and method **55** pp 281-294.
- [14] Harrison T P 2001 *Journal of information systems frontiers* Global supply chain design **3** pp 413-416.
- [15] Baniel F 2009 Prise en compte d'objectifs de stabilité pour l'organisation de collectes de déchets *Phd report* Université de Toulouse France p 164.
- [16] Kaddoussi A 2012 Optimisation des flux logistiques: vers une gestion avancée de la situation de crise *Phd report* Ecole Centrale de Lille France.
- [17] Marcotte S 2013 Modélisation du flux patient entre deux urgences *Proc. of the industrial engineering Conf.* France.
- [18] Gunasekaran A and Kobu B 2007 *Inter. Journal of Production Research* Performance measurements and metrics in logistics and supply chain management: a review of recent literature for research and applications **45** pp 2819-2840.
- [19] Lee Y H Kim S H 2002 *Computers and Industrial Engineering* Production distribution planning in supply chain considering capacity constraints **43** pp 169-190.
- [20] Dhaenens F C Finke G 2001 *IIE Transactions* An integrated model for an industrial production-distribution problem **33** pp 705-715.

- [21] Selim H Am C Ozkarahan I 2008 *Transportation Research Part E-Logistics and Transportation Review* Collaborative production distribution planning in supply chain: a fuzzy goal programming approach **44** pp 396–419.
- [22] Safra I et al. 2011 Planification séquentielle tactique-opérationnelle d'une chaîne logistique textile *Proc. of the industrial engineering Conf.* France.
- [23] Arabi M 2011 Intégration de la simulation et de l'optimisation pour la planification tactique forêt-usine *Proc. of the industrial engineering Conf.* France.
- [24] Lemoine D 2008 Modèles génériques et méthodes de résolution pour la planification tactique mono-site et multi-site *Phd report* Université Blaise Pascal-Clermont-Ferrand II.
- [25] Arntzen B C Brown G G Harrison T P Trafton L 1995 *Interfaces* Global supply chain management at digital equipment corporation **25** pp 69-93.
- [26] Ozdamar L and Yazgac T 1999 *Inter. Journal of Production Research* A hierarchical planning approach for a production distribution system **37** pp 3759–3772.
- [27] Jayaraman V Pirkul H 2001 *European journal of operational research* Planning and coordination of production and distribution facilities for multiple commodities **133** pp 394-408.
- [28] Matta R D and Miller T 2004 *European journal of operational research* Production and interfacility transportation scheduling for a process industry **158** pp 290-300.
- [29] Rizk N Martel A 2008 *Journal of the operational research society* Synchronized production distribution planning in a single-plant multi-destination network **59** pp 90-104.