

Make or buy analysis model based on tolerance allocation to minimize manufacturing cost and fuzzy quality loss

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Abstract. The specification of tolerances has a significant impact on the quality of product and final production cost. The company should carefully pay attention to the component or product tolerance so they can produce a good quality product at the lowest cost. Tolerance allocation has been widely used to solve problem in selecting particular process or supplier. But before merely getting into the selection process, the company must first make a plan to analyse whether the component must be made in house (*make*), to be purchased from a supplier (*buy*), or used the combination of both. This paper discusses an optimization model of process and supplier selection in order to minimize the manufacturing costs and the fuzzy quality loss. This model can also be used to determine the allocation of components to the selected processes or suppliers. Tolerance, process capability and production capacity are three important constraints that affect the decision. Fuzzy quality loss function is used in this paper to describe the semantic of the quality, in which the product quality level is divided into several grades. The implementation of the proposed model has been demonstrated by solving a numerical example problem that used a simple assembly product which consists of three components. The metaheuristic approach were implemented to OptQuest software from Oracle Crystal Ball in order to obtain the optimal solution of the numerical example.

1. Introduction

Profitability is one of the most important factors which can be used to indicate the success of a manufacturing company in running the business. Profitability will be higher if companies have a high sales with low production costs. A high sales will be achieved if the company offers good quality product at a low price. The challenge for a manufacturing company to attain this goal is how the quality of product can be improved at the lower cost.

Tolerance may affect the quality of product as well as the cost of manufacturing. Tolerance is expressed as the permissible amount of deviation in design parameters and process variables which able to meet the given standard or certain functional requirements [1]. A tighter tolerances can result in excessive process costs, while loose tolerances may lead to increase waste and assembly problems. Improper tolerance specification may also result in inferior product performance and loss of market share [2]. Therefore, to obtain a high profitability, the company should carefully pay attention to the component or product tolerance so they can produce a good quality product at the lowest cost.

The most common tolerance specification problem encountered by engineering designers is tolerance allocation, which is the distribution of the specified assembly tolerance among the



components of the assembly [2]. In quality engineering, tolerance allocation reflects a trade-off between customer requirements and the ability of producers to satisfy them. Thus, the loss of both customers and producers can be balanced [3].

Chase, et al [2] presents a procedure for tolerance allocation based on quantitative estimates of the manufacturing cost, which permits the selection of component tolerances in mechanical assemblies for minimum cost of production. Feng, et al [4], in the other hand, have developed a model of supplier selection and tolerance allocation to minimize the manufacturing cost and quality loss. Kumar, et al [5] use a pattern search algorithm in finding the optimal tolerance allocation to minimize the total manufacturing cost and asymmetric quality loss.

Based on those previous researches [2, 4-5], tolerance allocation has been widely used to solve problem in selecting particular process or supplier. But before merely getting into the selection process, the company must first make a plan to analyze whether the component must be made in house, purchased from a supplier, or used the combination of both. Suppliers as the main source to supply the components have several uncertainties such as price, quality, quantity, and shipment. Hence, the company need to supply the products by in-house production not to rely on their supplier. The company must first pursue the fulfillment of components by fully working on in-house production despite the limited capacity and capability. The company needs to make sure that using the source from the supplier would be their very last option, although inevitably the company has no alternative but to find another resource outside.

Rosyidi, et al [6] have developed make or buy analysis model to minimize total cost that consists of production or purchase cost, quality loss, and scrap cost. Their model also considers the allocation of the components with several constraints such as quality, the production and supplier capacity, and process capability. Their study used the concept of Taguchi loss to determine the quality loss resulting from the decisions.

The previous studies [4, 6] show that the Taguchi loss function has been widely used in the calculation of quality loss. However, Taguchi loss function can not deal with both the asymmetrical situations and the fuzzy factors in quality semantics. Therefore, fuzzy logic is used to describe the semantic of the quality, in which the product quality level is divided into several grades such as very good, good, not good, not poor, poor, very poor [7]. Cao, et al [7] have developed a fuzzy quality loss function model.

Based on these conditions, different from previous studies [2, 4-5], we consider the combined decision, in-house production (*make*) and purchase from the supplier (*buy*) and allow for more than one processes or suppliers selected. In this paper, we used manufacturing cost and quality loss minimization function to determine the selected process and/or supplier. This model can also be used to determine the allocation of components to the selected processes or suppliers. Tolerance, process capability and production capacity are three important constraints that affect the decision. This paper will incorporate the fuzzy quality loss function in the optimization model.

2. Methods

2.1. System Description

In this case, a product assembly consists of O components which can be supplied by in-house production (*make*), purchased from suppliers (*buy*), or using the combination of both. Each component in this system is manufactured using a single process. There are H alternative processes that can be selected in *make* decision. Whereas in *buy* decision, there are U alternative suppliers that can be selected. Each alternative, either for processes or suppliers, can produce more than one types of components. The demand for each type of component can be met by more than one type of processes or suppliers.

Apart from taking the decision whether to *make* or *buy*, this model can also be used to determine the allocation for each selected process or supplier. Manufacturing cost, production capacity and tolerance for each process and supplier are differs from one to another.

In this paper, minimizing manufacturing cost and fuzzy quality loss are used as the objective function. Quality loss is defined as the characteristics of the product quality that can not meet customer needs and satisfaction. Taguchi argues that there is a loss (in the form of cost and quality) when a quality characteristic deviates from the target value, although it is still within the specifications or the specified tolerances [3]. Taguchi has developed a quadratic loss function to estimate the quality loss $L(y)$ as expressed in equation (1). The cost-related numerical constant is denoted by k , y is quality characteristics of final product and T is the target value of y .

$$L(y) = k(y - T)^2 \quad (1)$$

The quadratic loss function can not deal with both the asymmetrical situations and the fuzzy factors in quality semantics. Therefore, fuzzy logic is used to describe the semantic of the quality [7]. A serial of fuzzy sets \tilde{A} are defined to indicate the quality levels from $i = 1$ (the good quality level) to $i = Q$ (the worst quality level). The fuzzy quality loss $L_{\tilde{A}}(y)$ can be expressed as in equation (2).

$$L_{\tilde{A}}(y) = \sum_{i=1}^Q R_i L_i \quad (2)$$

R_i denotes the normalized expected probability to the quality level i and L_i is cost sets which defined as quality loss in monetary terms of each quality level. R_i can be formulated as in equation (3).

$$R_i = \frac{r_i}{\sum_{i=1}^Q r_i} \quad (3)$$

$$r_i = \int_{-\infty}^{+\infty} \mu_{\tilde{A}_i}(y) f(y) dy \quad (4)$$

The expected probability of the quality grade i is denoted by r_i . The density function of the probability distribution of quality characteristic y is expressed as $f(y)$. The subject function of quality characteristic y for each quality level is expressed as $\mu_{\tilde{A}_i}(y)$. A number of different types of subject function or membership function have been proposed for fuzzy logic such as triangular, trapezoidal, gaussian, two-sided gaussian, bell-shaped, sigmoid-right, sigmoid-left, difference-sigmoid, product-sigmoid, polynomial-Z, polynomial-S, and polynomial-PI [8].

Furthermore, each alternative process and supplier will generate variances. The total variance (σ_A^2) must be less or equal to the dimensional tolerance specifications of assembly products. The variance σ_A^2 can be calculated by considering the process capability index. Process capability means the ability of the process to meet technological or other requirements. According to [9] the process capability index can be divided into several types, such as C_p , C_{pk} and C_{pm} . This paper uses process capability index C_p that can be expressed in equation (5). USL denotes upper specification limit, LSL is lower specification limit and σ is standard deviation.

$$C_p = \frac{USL - LSL}{6\sigma} \quad (5)$$

2.2. Model Notations

The following notations are used in the model:

- m : component index, $m = 1, 2, \dots, O$
- k : machine index, $k = 1, 2, \dots, H$
- l : supplier index, $l = 1, 2, \dots, U$
- i : quality level, $i = 1, 2, \dots, Q$
- O : set of components
- H : set of machines
- U : set of suppliers

Q	: set of quality levels
M_{mk}	: cost of manufacturing for the m^{th} component manufactured in the k^{th} machine
S_{ml}	: purchased cost for the m^{th} component supplied by the l^{th} supplier
b_{mk}	: binary number for the m^{th} component manufactured in the k^{th} machine
b_{ml}	: binary number for the m^{th} component supplied by the l^{th} supplier
c_{mk}	: production capacity for the m^{th} component in the k^{th} machine
c_{ml}	: supplier capacity for the m^{th} component in the l^{th} supplier
X_{mk}	: amount of the m^{th} component manufactured in the k^{th} machine
Y_{ml}	: amount of the m^{th} component ordered to the l^{th} supplier
T_{mk}	: tolerance for the m^{th} component manufactured in the k^{th} machine
T_{ml}	: tolerance for the m^{th} component supplied by the l^{th} supplier
T_R	: assembly tolerance limit
L_i	: quality loss cost at the i^{th} quality level
n_m	: the number of the m^{th} component in final assembly
D	: demand of assembly product
C_p	: process capability index
TC	: total cost
$\mu_{Ai}(y)$: subject function of quality characteristic y at the i^{th} quality level
$f(y)$: density function of the probability distribution of quality characteristic y
σ_m^2	: variance of component
σ_A^2	: variance of assembly product

3. Result and Discussion

3.1. Objective Function

The objective function of the model in this paper is to minimize the total cost (TC) which consists of manufacturing cost, purchased cost and fuzzy quality loss. The complete expression of the objective function can be described as in equation (6).

$$\text{Min } TC = \sum_{m=1}^O \left(\sum_{k=1}^H (M_{mk} X_{mk} b_{mk}) + \sum_{l=1}^U (S_{ml} Y_{ml} b_{ml}) \right) + \left(\sum_{i=1}^Q \left(\frac{\int_{-\infty}^{+\infty} \mu_{Ai}(y) f(y) dy}{\sum_{i=1}^Q \int_{-\infty}^{+\infty} \mu_{Ai}(y) f(y) dy} \right) DL_i \right) \quad (6)$$

3.2. Model Constraints

There are five constraints considered in this paper: maximum assembly tolerance, production and supplier capacity, demand of assembled product, the minimum selected process or supplier, and binary number.

3.2.1 Maximum Assembly Tolerance. This constraint is required to ensure the quality of the assembly product. The tolerance of all components in the assembly product should not exceed the target of assembly product tolerance set by the company that can be formulated as in equation (7). The assembly tolerance is obtained from the sum of component tolerances which can be expressed in equation (8). According to [10] this paper uses combined variance to represent the variance of each component from the different sources as it can be described in equation (9).

$$\sigma_A^2 \leq \left(\frac{T_R}{3C_p} \right)^2 \quad (7)$$

$$\sigma_A^2 = \sum_{m=1}^O \sigma_m^2 \quad (8)$$

$$\sigma_m^2 = \frac{\sum_{k=1}^H \left(\frac{T_{mk}}{3C_p}\right)^2 X_{mk} + \sum_{l=1}^U \left(\frac{T_{ml}}{3C_p}\right)^2 Y_{ml}}{X_{mk} + Y_{ml}} \quad \forall m \quad (9)$$

3.2.2 Production and Supplier Capacity. This constrain shows the limitations of machine used in the processes or suppliers. It can be referred that in allocating the needed components, it is possible to select more than one machines or suppliers if the capacity of the machine or supplier are not sufficient. Constraint for production and supplier capacity can be formulated as in equation (10) and (11), respectively.

$$\sum_{m=1}^O X_{mk} b_{mk} \leq c_{mk} \quad \forall k \quad (10)$$

$$Y_{ml} b_{ml} \leq c_{ml} \quad \forall m, l \quad (11)$$

3.2.3 Demand of Assembled Product. The demand constraint indicates that the total components ordered (m) is equivalent to the components needed for assembly.

$$\sum_{k=1}^H X_{mk} b_{mk} + \sum_{l=1}^U Y_{ml} b_{ml} = n_m D \quad \forall m \quad (12)$$

3.2.4 Minimum Selected Process or Supplier. This constraint indicates that there is at least one selected process and/or supplier for each component to ensure the availability of each component. A set of binary coefficients, having a value of zero or one, are used to select the processes and suppliers.

$$\sum_{k=1}^H b_{mk} + \sum_{l=1}^U b_{ml} \geq 1 \quad \forall m \quad (13)$$

3.2.5 Binary Number. This constraint is used to represent the selection process (1 if selected and 0 otherwise). Equations (14) and (15) are used to represent binary number for process and supplier selection, respectively.

$$b_{mk} \in [0,1] \quad \forall m, k \quad (14)$$

$$b_{ml} \in [0,1] \quad \forall m, l \quad (15)$$

3.3. Numerical Example and Analysis

A numerical example is provided to demonstrate the application of the model. The numerical example in this paper is based on numerical example from [7] which used assembly product that consists of three components, revolution axis, end shield nut, and sleeve. For more detail about the assembly drawing and dimension, please refer to [7]. The dimensions x_1 , x_2 , and x_3 are 38 mm, 42 mm, and 80 mm, respectively. To maintain normal performance, the critical dimensions (x_0) is sets to be 0.2 mm.

In this numerical example there are three level of quality, i.e. good, general, and poor. The quality loss for each level is defined as shown in table 1.

Table 1. Description of fuzzy quality sets and quality loss

Quality Level, i	Fuzzy Quality Sets, \tilde{A}	Quality Loss, L_i (IDR)	Quality Semantic
1	\tilde{A}_1	$L_1 = 0$	Good quality
2	\tilde{A}_2	$L_2 = 60,000$	General quality
3	\tilde{A}_3	$L_3 = 74,000$	Poor Quality

Adapted from [7], the triangular and trapezoidal-type subject functions are adopted in this numerical example, as shown in figure 1, 2 and 3. The corresponding subject functions are listed below:

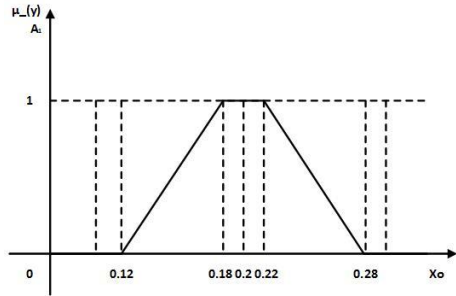


Figure 1. Subject function for good quality level ($\mu_{\tilde{A}_1}(y)$)

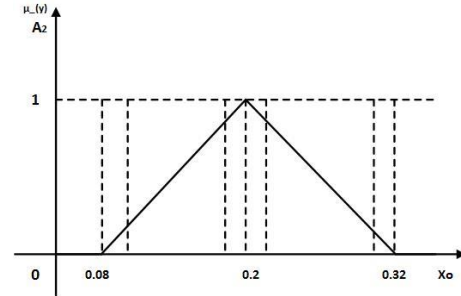


Figure 2. Subject function for general quality level ($\mu_{\tilde{A}_2}(y)$)

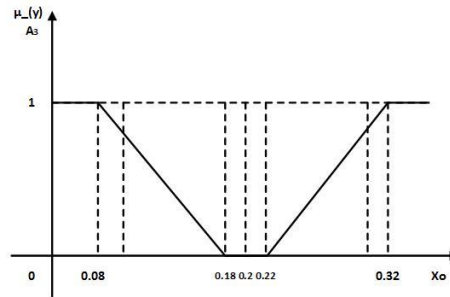


Figure 3. Subject function for poor quality level ($\mu_{\tilde{A}_3}(y)$)

$$\mu_{\tilde{A}_1}(y) = \begin{cases} 0 & y < 0.12 \\ (50y - 6)/3 & 0.12 \leq y < 0.18 \\ 1 & 0.18 \leq y \leq 0.22 \\ (14 - 50y)/3 & 0.22 < y \leq 0.28 \\ 0 & y > 0.28 \end{cases} \quad (16)$$

$$\mu_{\tilde{A}_2}(y) = \begin{cases} 0 & y < 0.08 \\ (25y - 2)/3 & 0.08 \leq y < 0.2 \\ (8 - 25y)/3 & 0.2 \leq y < 0.32 \\ 0 & y > 0.32 \end{cases} \quad (17)$$

$$\mu_{\tilde{A}_3}(y) = \begin{cases} 1 & y < 0.08 \\ 1.8 - 10y & 0.08 \leq y < 0.18 \\ 0 & 0.18 \leq y < 0.22 \\ 10y - 2.2 & 0.22 \leq y < 0.32 \\ 1 & y > 0.32 \end{cases} \quad (18)$$

Each machine and supplier have different characteristics in terms of manufacturing costs and component tolerance as shown in table 2 and table 3. Furthermore, the capacity of machines and suppliers are presented in table 4. In the numerical example, the company received an order of 300 units of final product. It is assumed that the machines and suppliers have a process capability index of $C_p = 1$ for each component.

Table 2. Manufacturing cost and the respective component tolerance for each machine

Machine	Component 1		Component 2		Component 3	
	Tolerance (mm)	Cost (IDR)	Tolerance (mm)	Cost (IDR)	Tolerance (mm)	Cost (IDR)
1	0.065	17,000	0.065	18,000	0.085	19,500
2	0.1	14,500	0.1	16,000	0.13	17,500

Table 3. Purchasing cost and the respective component tolerance for each supplier

Supplier	Component 1		Component 2		Component 3	
	Tolerance (mm)	Cost (IDR)	Tolerance (mm)	Cost (IDR)	Tolerance (mm)	Cost (IDR)
1	0.06	17,500	0.06	19,000	0.08	21,000
2	0.075	16,500	0.075	17,500	0.12	19,000
3	0.12	14,000	0.12	15,500	0.15	17,000

Table 4. Production capacity for each supplier and machine

Component	Capacity (unit)				
	Supplier 1	Supplier 2	Supplier 3	Machine 1	Machine 2
1	100	100	100		
2	100	100	150	300	300
3	125	125	100		

To solve the numerical example, we used OptQuest optimization software from Oracle Crystal Ball. The metaheuristic approach were implemented to OptQuest software from Oracle Crystal Ball in order to obtain an optimal or near- optimal solution of the numerical example. The metaheuristic approach [11-12] has been widely used to obtain optimal solution of this case. The metaheuristic approach was used to guide its search algorithm toward better solutions. This approach uses a form of adaptive memory to remember which solutions worked well before and recombines them into new, better solutions [13].

OptQuest will carry out the searching process until it finally reaches some termination criteria, either a limit on the amount of time devoted to the search or a maximum number of simulations. The searching process will be terminated early if the desired confidence level or number of non-improving solutions is met [13]. To solve the numerical example, the searching process will be terminated if there is no satisfying solution after 5000 iterations.

Table 5 shows the optimum results using OptQuest by Oracle Crystal Ball in 354 of the 5354 iterations. The performance chart of the optimum results using OptQuest is shown in figure 4. According to component allocation result, the standard deviation value of assembly product for $C_p=1$ is 0.056, whereas the set permissible value is 0.067. The value of R1, R2, and R3, are 0.4, 0.42, and 0.18, respectively. The total cost resulted from the solution is IDR 26,601,285, which consists of manufacturing cost, purchase cost and fuzzy quality loss represents by value in IDR 10,275,000, IDR 4,775,000, and IDR 11,551,285, respectively.

Table 5. Optimization result

Component	Capacity (unit)				
	Supplier 1	Supplier 2	Supplier 3	Machine 1	Machine 2
1	0	0	50	0	250
2	0	100	150	0	50
3	0	0	0	300	0

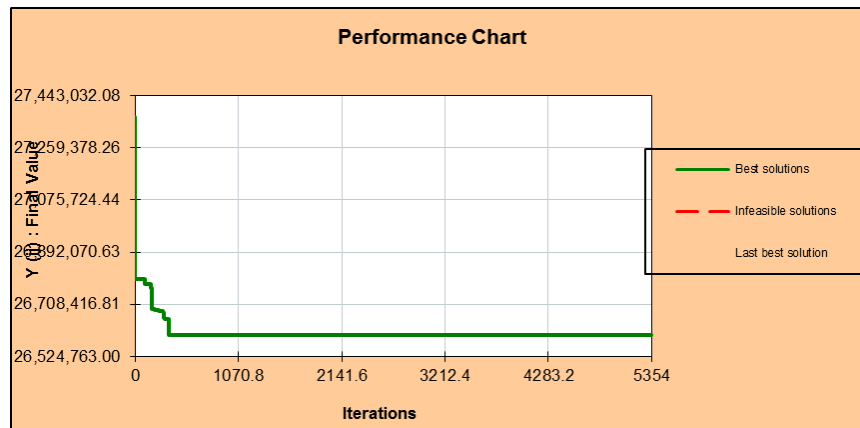


Figure 4. Performance chart of the optimum results using OptQuest

4. Conclusions

This paper proposed the development of optimization model of process and supplier selection by considering tolerance, process capability and production capacity. This model can also be used to determine the allocation of components to the selected processes or suppliers. The objective function of this model is to minimize the total cost (*TC*) which consists of manufacturing or purchasing cost and fuzzy quality loss. Fuzzy quality loss function is used in this paper to describe the semantic of the quality, in which the quality level is divided into several grades. Subjective evaluation engineering is required to determine the subject function in the fuzzy quality loss approach. A numerical example is given to show the implementation of the model. In future research, this model can be expanded by incorporating scrap cost, lateness cost, and a multi-stage manufacturing process.

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