On radial flow between parallel disks

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Abstract. Approximate analytical solutions are presented for converging flow in between two parallel non rotating disks. The static pressure distribution and radial component of the velocity are developed by averaging the inertial term across the gap in between parallel disks. The predicted results from the first approximation are favourable to experimental results as well as results presented by other authors. The second approximation shows that as the fluid approaches the center, the velocity at the mid channel slows down which is due to the struggle between the inertial term and the flowrate.

1. Introduction

Converging flow is defined by fluid flowing from the periphery towards the center. As the fluid flows radially towards the center, the velocity increases in order to satisfy continuity equation. Analytical, numerical and experimental studies have been carried out by others in order to characterize the phenomena [1]-[5]. Lee and Lin [1] presented a simplified solution by linearizing the Navier-Stokes equation by replacing radial velocity with mean radial velocity which resulted in dimensionless pressure gradient equation. According to Savage [2], Livesey J.L. considered the inertial term and used integral approach and the assumption of parabolic velocity profile in order to solve the problem. Savage [2] obtained a series solution by perturbing the creeping-flow solution. Vatistas [3] also included the inertial term and linearized the inertial term in the momentum equation by taking radial velocity from the continuity equation averaged over the gap. Then, the linearized differential equation was solved using method of separation variables. The author derived the static pressure distribution, radial component of the velocity as well as the friction coefficient [3]. In paper [4], the authors presented a numerical solution which resulted in the characterization of the static pressure distribution and the radial velocity. Zitouni and Vatistas [5] solved the problem by using the power series solution. This paper focuses on an approximate analytical solution to a nonlinear problem for radial flow in between parallel disks. Equations for static pressure distribution and velocity profiles are derived by averaging the inertial term over the space between the disks.

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2. Analysis

2.1. First Approximation

The flow is assumed to be laminar, incompressible, non-swirling and purely radial. The continuity equation and Navier-Stokes momentum equation in cylindrical coordinates are transformed into nondimensional form by using the following parameters:

$v_r = \frac{v_r^*}{v_0^*}$ $v_z = \frac{v_z^*}{v_0^*}$ $r = \frac{r^*}{R_0^*}$

$z = \frac{z^*}{h^*}$ $Re = \frac{\rho v_0^* h^*}{\mu}$ $\xi = \frac{h^*}{R_0^*}$

$P = \frac{P^*}{\rho^* v_0^{*2}}$; $P^* = P_r - P_{inlet}$

where ρ = density μ = dynamic viscosity P_r = current pressure

Note: (*) symbolizes dimensional parameters for the velocity component and pressure distribution in r and z direction

The equations are simplified and can be written as:
Continuity equation

$$\frac{1}{r}\frac{\partial v_r r}{\partial r} + \frac{\partial v_z}{\partial z} = 0 \tag{1}$$

r momentum equation

$$-v_r \frac{dv_r}{dr}\xi - v_z \frac{dv_r}{dz} = +\frac{dP}{dr}\xi - \frac{1}{Re}\frac{d^2 v_r}{dz^2} \tag{2}$$

where
v_r = radial velocity v_z = axial velocity P = dimensionless pressure

Re = Reynolds number r = dimensionless radial coordinate

z, ξ = dimensionless axial coordinate

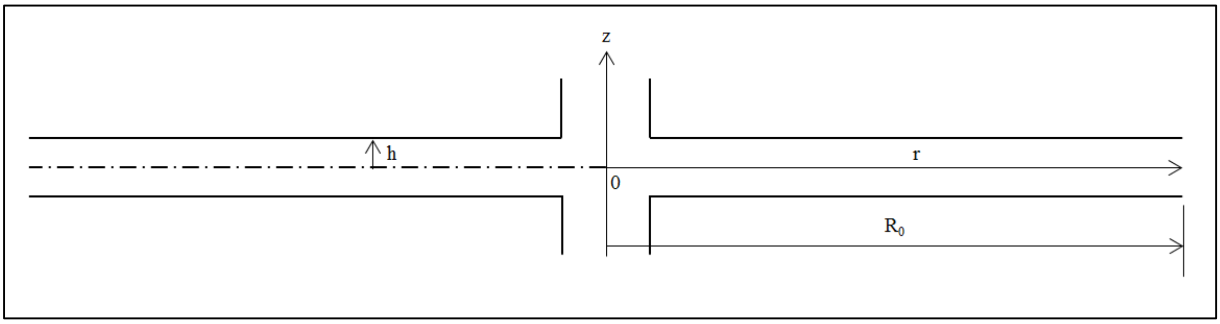

Figure 1. Cross section of flow between parallel disks.

For the first approximation, the nonlinear convective acceleration term is linearized by averaging across the gap between the parallel disks,

$$B(r) = \int_0^1 \left(-v_r \frac{dv_r}{dr}\xi - v_z \frac{dv_r}{dz}\right) dz \tag{3}$$

Then (2) can be written as:

$$\frac{1}{Re}\frac{d^2 v_r}{dz^2} = \frac{dP}{dr}\xi - B(r) = A(r) \tag{4}$$

The velocity function can be calculated by integrating (4) twice with respect to the distance between the parallel disks as shown:

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$$\frac{1}{\mathrm{Re}}v_r = \frac{A(r)z^2}{2} + c_1 z + c_2 \tag{5}$$

where c_1 and c_2 are constants of integration. The evaluation of the constants of integration is made using the following boundary conditions:

$\mathrm{BC1}: \ z = 0, \dfrac{dv_r}{dz} = 0$

$\mathrm{BC2}: \ z = 1, v_r = 0$

From continuity equation,

$$rv_r = D = \text{constant} \tag{6}$$

Integration of (6) with respect to z (from 0 to 1) yields,

$$\therefore A(r) = \frac{3D}{r\mathrm{Re}} \tag{7}$$

Thus,

$$v_{r(1)} = \frac{3D}{2r}(z^2 - 1) \tag{8}$$

Based on (8), it is a parabolic velocity profile. Thus, (3) is reduced to:

$$B(r) = \int_0^1 -v_r \frac{dv_r}{dr}\xi\, dz$$

$$\therefore B(r) = \left(-\frac{8}{15}\right)\frac{\xi \mathrm{Re}^2 A(r)A'(r)}{4} \tag{9}$$

Then from (7) and (9) yield

$$\frac{dP}{dr} = \frac{3D}{r\mathrm{Re}\xi} + \frac{6D^2}{5r^3}$$

$$\Delta P = P_r - P_i\,;\ P_i = \text{inlet static pressure}$$

Thus, the pressure distribution is given by:

$$P_r - P_{i=1} = \ln r\frac{3D}{\mathrm{Re}\xi} - \frac{3D^2}{5r^2} + \frac{3D^2}{5(1)^2} \tag{10}$$

where $\mathrm{Re}\xi = \text{Reduced Reynolds number}, \overline{\mathrm{Re}}$

2.2. Second Approximation

Second approximation is carried out in order to check the accuracy of the first approximation.
r momentum equation can be written as:

$$-v_r\frac{dv_r}{dr} - \frac{v_z}{\xi}\frac{dv_r}{dz} = +\frac{dP}{dr} - \frac{1}{\overline{\mathrm{Re}}}\frac{d^2v_r}{dz^2} \tag{11}$$

Substituting the inertial term with results obtained from the first approximation and integrating (11) twice together with boundary condition BC1 and BC2 results in:

$$\therefore v_{r(2)} = \overline{\mathrm{Re}}\left(\frac{dP}{dr}\times\frac{z^2}{2} - \frac{9z^6}{120r^3} + \frac{18z^4}{48r^3} - \frac{9z^2}{8r^3} + \frac{33}{40r^3} - \frac{1}{2}\times\frac{dP}{dr}\right) \tag{12}$$

From Continuity Equation,

$$r\int_0^1 -v_r\, dz = \int_0^1 D\, dz$$

$$\frac{dP}{dr} = \frac{3D}{r\overline{\mathrm{Re}}} + \frac{54}{35r^3} \tag{13}$$

Integrating with respect to r, the pressure distribution for the second approximation is given by:

$$P_r - P_{i=1} = \frac{-27}{35r^2} + \frac{3D}{\overline{\mathrm{Re}}}\ln r + \frac{27}{35} \tag{14}$$

Substitute pressure gradient (13) into the velocity profile (12),

$$v_{r(2)} = \frac{-3D}{2r} + \frac{3\overline{Re}}{56r^3} + \frac{3Dz^2}{2r} - \frac{99\overline{Re}z^2}{280r^3} + \frac{3\overline{Re}z^4}{8r^3} - \frac{3\overline{Re}z^6}{40r^3} \quad (15)$$

where D=1 after normalizing in terms of flow rate as shown:

$$rv_r = D = 1 \quad (16)$$

3. Results and Discussion

The analytical solutions are validated by comparing the equation with equations developed by other authors and experimental data. The first approximation results in a parabolic velocity profile as shown in figure 2.

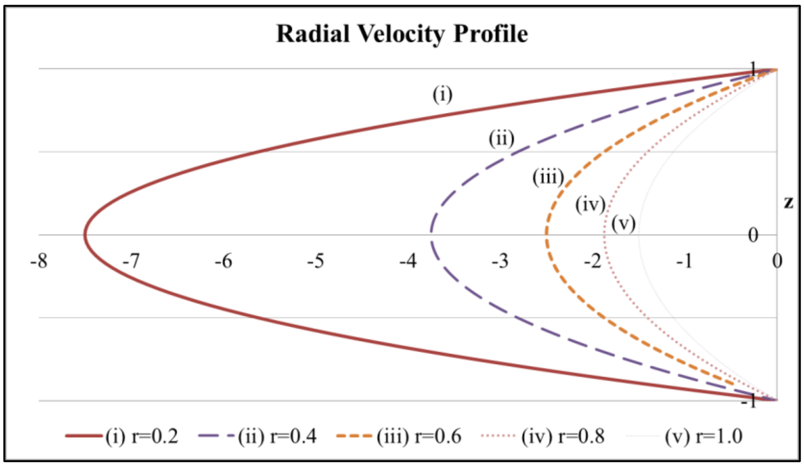

Figure 2. 1st Approximation radial velocity profile.

However, the second approximation shows a different flow kinematics as shown in figure 3. Based on the calculations, the profile flattens as the fluid flows towards the center. At radial distance (r) less than 0.3, it can be seen the maximum velocity profile is no longer at the mid channel. This phenomenon is caused by the struggle between inertial term and flowrate and is verified as the continuity equation holds.

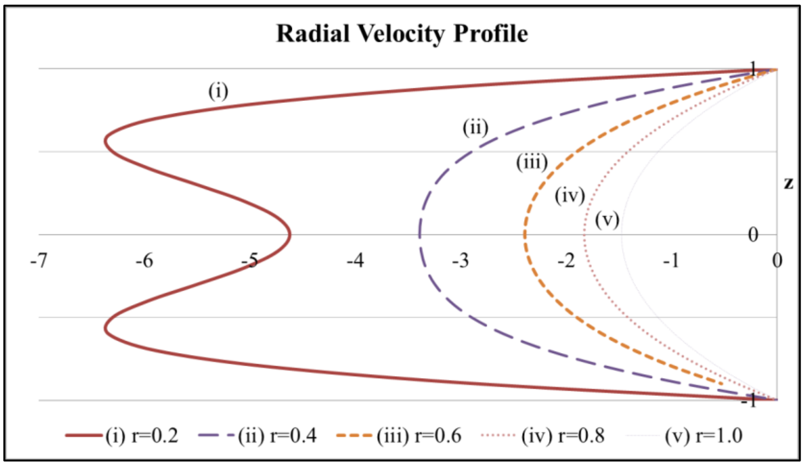

Figure 3. 2nd Approximation radial velocity profile.

The static pressure distribution obtained in this analysis is compared with results from Vatistas [3], Lee &Lin [1], Livesey [2] and Savage and Kwok & Lee which were presented in Vatistas, Ghila &

Zitouni [4] and are in favor with their solution as shown in figure 4 and figure 5. The first approximation has the exact same solution as obtained by Kwok and Lee [4] and Livesey [2]. The second approximation is very close to the results obtained by Savage [4] and shows a larger change in pressure as compared to the first approximation. This is due to the higher wall shear stress experienced by the second profile as compared to the parabolic profile obtained from the first approximation. Equation (17) and (18) shows shear stress for first approximation and second approximation respectively.

$$\tau = \frac{3z}{rRe} \tag{17}$$

$$\tau = \frac{1}{Re}\left(\frac{3z}{r} - \frac{99\overline{Re}z}{140r^3} + \frac{3\overline{Re}z^3}{2r^3} - \frac{9\overline{Re}z^5}{20r^3}\right) \tag{18}$$

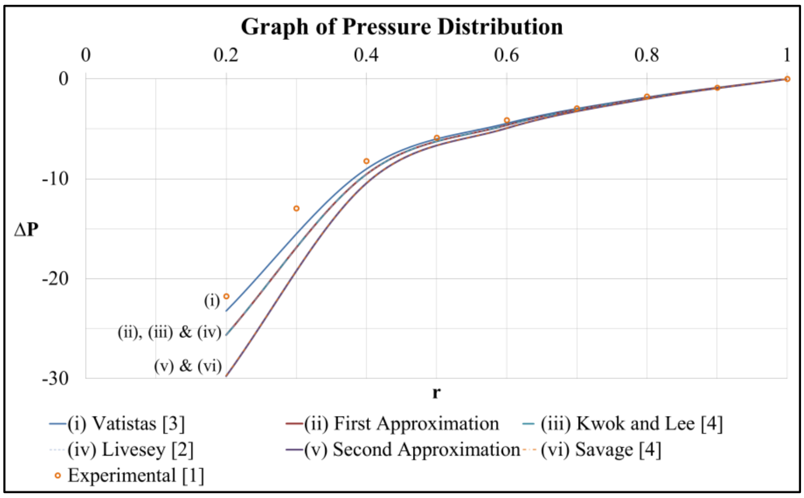

Figure 4. Pressure distribution of water along the radius ($\overline{Re} = 0.43$).

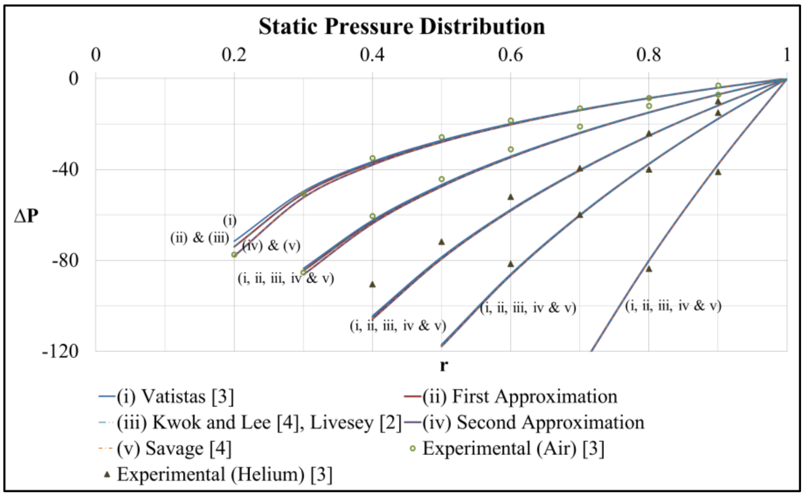

Figure 5. Static pressure distribution using different reduced Reynold's number.

Table 1 shows pressure distribution comparison presented by Vatistas, Ghila & Zitouni [4] and complimented by our results. However, there are differences between Savage and Livesey's results as cited by Savage [2] and not exactly the same as presented by Vatistas, Ghila & Zitouni [4].

Table 1. Pressure distributions.

	$\mathbf{\Delta P}$
Vatistas [3]	$\dfrac{6\bar{r}^2\ln r-\overline{Re}(1-r^2)}{2r^2\overline{Re}}$
Kwok and Lee [4] Livesey [2]	$0.6-\dfrac{0.6}{r^2}-\dfrac{3\ln r}{\overline{Re}(r^2-1)}+\dfrac{3r^2\ln r}{\overline{Re}(r^2-1)}$
Savage [4]	$0.77-\dfrac{0.77}{r^2}-\dfrac{3\ln r}{\overline{Re}(r^2-1)}+\dfrac{3r^2\ln r}{\overline{Re}(r^2-1)}$
1st Approximation	$\dfrac{15r^2\ln r-3\overline{Re}(1-r^2)}{5\overline{Re}r^2}$
2nd Approximation	$\dfrac{-27}{35r^2}+\dfrac{3}{\overline{Re}}\ln r+\dfrac{27}{35}$

4. Conclusion

The approximate analytical solution for Navier-Stokes on radial flow between parallel disks is developed. The pressure and radial velocity distribution are obtained by averaging the inertial term across the gap between the parallel disks. This approach allows one to develop the second approximation for the velocity profile and pressure gradient. The static pressure distributions are in agreement with experimental results and results obtained by other authors. Thus, the analytical method is verified. This study also shows that in contrary to the other authors, the velocity profile is not constantly in a parabolic shape as the fluid flow towards the center and it is verified as the continuity equation holds.

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