

An analytical study of various telecommunication networks using markov models

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Abstract. The main aim of this paper is to examine issues relating to the performance of various Telecommunication networks, and applied queuing theory for better design and improved efficiency. Firstly, giving an analytical study of queues deals with quantifying the phenomenon of waiting lines using representative measures of performances, such as average queue length (on average number of customers in the queue), average waiting time in queue (on average time to wait) and average facility utilization (proportion of time the service facility is in use). In the second, using Matlab simulator, summarizes the finding of the investigations, from which and where we obtain results and describing methodology for a) compare the waiting time and average number of messages in the queue in M/M/1 and M/M/2 queues b) Compare the performance of M/M/1 and M/D/1 queues and study the effect of increasing the number of servers on the blocking probability M/M/k/k queue model.

1. Introduction

This paper represents the analytical study of different queuing Markov models' a type of single server performances with suitable applications on telecommunication networks. In general, queuing systems may be characterized by complex input process, service time and distribution and queue disciplines.[1] paper also analysing the development of multivariant models based on the Markov chains applications in the field of reliability,economics, survival analysis, engineering, social sciences, environmental studies, biological sciences, etc. In practice, such queuing processes and disciplines are often amendable to analysis.

2. Literature Review

In telecommunication networks queuing theories are modelled in a variety of network engineering principles and techniques. [2] consider the discrete-time process that the length of off-periods property taken into account which are idependent interval distribution with an arbitrary distribution. [3] emerging Value at Risk(VaR) as one of the measure in markt place to examine the high –order effect of Mark chain through backtesting. There are different testing made in Markov process, [4] is a statistic test with the stationary that to divide whole sequence of events into subintervals and then

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compute and comparing the transition probability matrix amongst each intervals. Testing and analysis plays a vital role to improve the performances of the network systems. [5] defines through specification test constructing a non-parametric simultaneous confidence band for conditional regression functions.[6] proposing the new model that Differential Output-ports Choosing Probability scheme , it is applied to optical burst switching networks that the blocking probability and the delay time will change along with the changeable ports choosing probability and the ratio between different length burst.

3. Basic Notation Representing Queuing Models

A commonly used shorthand notation, called Kendall notation, for such single queue models describes the arrival process, service distribution, the number of servers and the buffer size(waiting line) as follows:

Arrival process/service distribution/ number of servers / waiting room

The complete notation expressed as (a/b/c): (d/e/f) where,

a =arrival distribution,

b=departure distribution,

c=number of parallel service channel in the system,

d=service discipline,

e=maximum number of customers allowed in the system,

f=calling source or population

Commonly used characters for the first two positions in the shorthand notations are: M(Markovian – Poisson for the arrival or Exponential for the service time) , G(General), D(Deterministic), GI (General and Independent) and Geom(Geometric). The fourth positions used for the number of buffer places in addition to the number of servers and its usually not used if the waiting room is unlimited.

Symbols for d,

FCFS=First Come, First Served,

LCFS=Last Come, First Served,

SIRO=Service In Random Order,

GD=General service Discipline.

The symbols e and f represent a finite (N) or infinite (∞) number of customers in the system and calling source respectively. For instance, (M/E/1):(FCFS/N/∞) represents a Poisson arrival (exponential interarrival), Erlangen departure, single server, First Come First Served discipline, maximum allowable customers N in the system and infinite population model.

4. Telecommunication System Analysis

In this telecommunication queuing system, exploring the effect of having more than one server in a queuing system. Queuing networks have been analysed and solved under different assumptions and constraints [7]. To get a good feel for how does such a system behave, comparing the waiting time and average number of messages in the single server system, such as M/M/1 with that of a Multiple server's system such as M/M/2, M/M/D and M/M/k/k systems. As like [2], the number of iterations taken into account in the MatLab simulator to measure the performance and analyse it.

4.1 M/M/1 vs M/M/2

To compare the performances of an M/M/1 and M/M/2, plot the mean total time in the system against the utilization factor in both systems. The total time a customer spends in a system(W_s) is given by,

$$W_s = W_q + \frac{1}{\mu} \quad (1)$$

While considering, λ is arrival rate, μ is service rate and ρ is traffic intensity or utilization factor.

For M/M/1, The total time a customer spends in the queue(W_q) is given by,

$$W_q = \frac{\rho}{\mu(1-\rho)} \quad (2)$$

For $\mu=1$, Equation (2) becomes,

$$W_q = \frac{\rho}{1-\rho} \quad (3)$$

For M/M/2, and $\mu=1$, W_q is given by,

$$W_q = \frac{\rho^2}{1-\rho^2} \quad (4)$$

Use MatLab simulator, plotting Equation (1), the values of that are of interest are $\rho < 0.2$ and $\rho > 0.9$

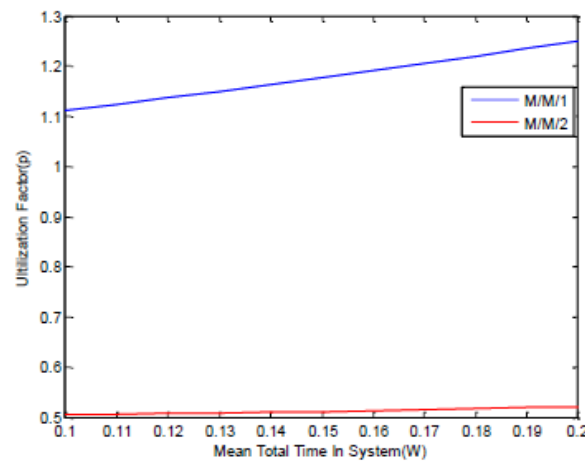


Figure 1. Mean total against Utilization Factor $\rho < 0.2$

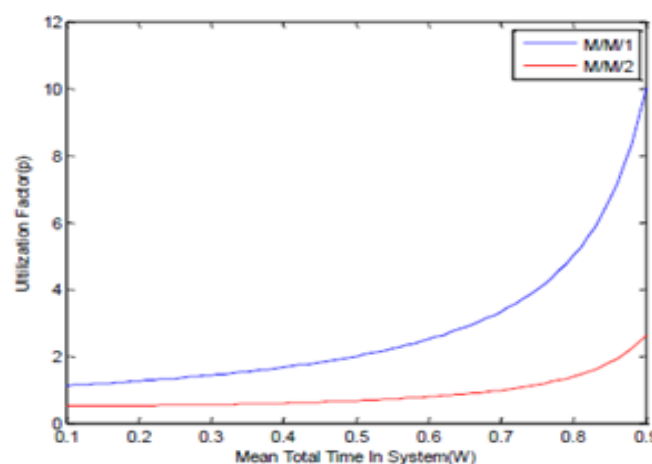


Figure 2. Mean total against Utilization Factor $\rho > 0.9$

The above two figures, Figure 1 represents the utilization factor against mean total time for the value taken as greater than 0.2 and Figure 2 represents the utilization factor against mean total time for value taken as less than 0.9. Utilization factor represents the performance of any system being one by

the system. For a lower utilization factor, the mean time in the system is very low. As the utilization factor increases, in the M/M/1 queue, the mean total time in the system increases greatly while in the M/M/2 queue, the mean total time in the system only has a slight increment. Therefore, M/M/2 queue has a superior performance than M/M/1 queue. Matlab simulator working controls are given below,

```
p = 0:0.01:0.99; % the utilization factor
w1 = []; % an array to store the values
w2 = []; % an array to store the values
lamda1 = 0:0.01:0.99;
lamda2 = lamda1 * 2;
% a for loop to do the calculations for more than a single value
for k = 1:100
w1 = [w1 (lamda1(k)/(1-lamda1(k)))+1];
w2 = [w2 (lamda2(k)^2)/(4 - (lamda2(k)^2))+0.5];
end
%plotting the graph
plot(p,w1,'r',p,w2,'b');
Title('Average time in the system for M/M/1 And M/M/2 queues when \mu = 1');
Xlabel('Utilization \rho');
Ylabel('Average Time In System');
tleg = legend('M/M/1','M/M/2');
```

4.2 M/M/1 vs M/M/2 and $\lambda=1$

Repeating the same process, however, this time needs different equations to work with, since the service rate in an M/M/2 system is twice that of an M/M/1 system, use the following equations;

For M/M/1, W_s is given by,

$$W_s = \frac{\rho^2}{2(1-\rho)} + \frac{1}{2} \quad (5)$$

For M/M/2, W_s is given by,

$$W_s = \frac{\rho^3}{(1-\rho^2)} \quad (6)$$

Plotting graph on the simulator will get $\rho < 0.2$ and $\rho > 0.9$ as below, Figure 3 represents the utilization factor against mean total time for the value taken less than 0.2 and Figure 4 represents the utilization factor against mean total time for the value taken greater than 0.9.

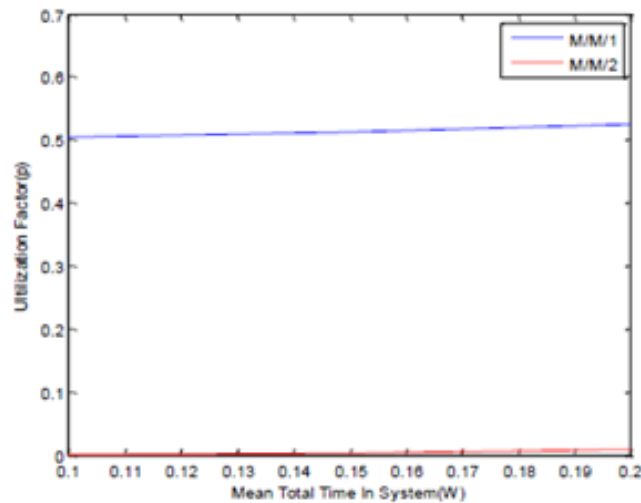


Figure 3. Utilization factor against Mean total time in the system for $\rho < 0.2$

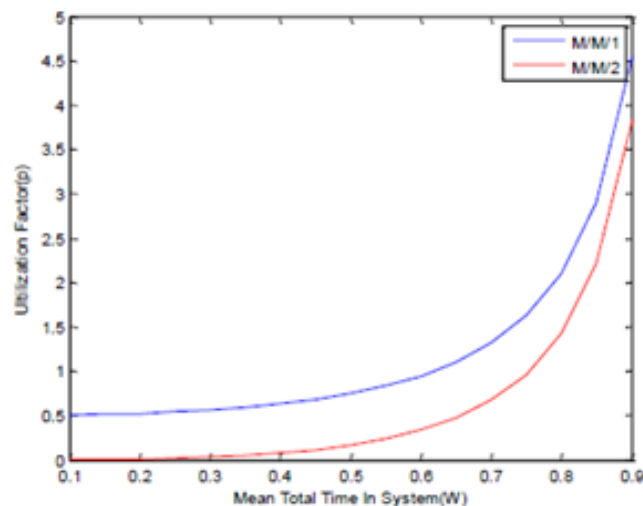


Figure 4. Utilization factor against Mean total time in the system for $\rho > 0.9$

The expected waiting time for the systems is almost same with increasing values of utilization factor. Although the M/M/2 system performed better, both systems relatively equal, because of the arrival rate is 1 for both systems. Nevertheless, the biggest difference can be observed for smaller values of ρ . In Matlab simulator working controls are given below,

```
p = 0:0.01:0.99;%utilisation factor
W1 = []; % an array to store the values
W2 = []; % an array to store the values
mu = 50:-0.5:0.5;
%a for loop to repeat the calculations for more than a single value
for k = 1:100
W1 = [W1 (1/(2*mu(k)*((2*mu(k)) -1))) + (1/(2*mu(k)))];
```

```

W2 = [W2 (1/(mu(k)*((4*mu(k)^2) -1))) + (1/(2*mu(k)))];
end
%plotting the graph
plot(p ,W1,'r', p, W2, 'g');
Title('Average time in system for M/M/1 and M/M/2 Queues when \lambda = 1, M/M/1 \mu = 2 (M/M/2 \mu)');
Xlabel('Utilization \rho');
Ylabel('Average time tn system');
tleg = legend('M/M/1','M/M/2');
set(tleg, 'Location', 'NorthWest');

```

4.3 M/M/1 vs M/M/2 and $\mu = 1$, Number of customers in the system(L_s)

By applying the Little's formula:

$$L_s = \lambda W_s$$

Using Equations(1), (3) and (4) and using the fact that $\mu=1$, get the following expressions:

For M/M/1, L_s is given by,

$$L_s = \frac{\rho}{1 - \rho} \quad (7)$$

For M/M/2, L_s is given by,

$$L_s = \frac{\rho}{1 - \rho^2} \quad (8)$$

Again, plotting the number of customers in the system against utilization factor reveals that at low values of utilization factor, the number of customers in the system is almost zero for both M/M/1 and M/M/2 queue. With increasing utilization factor, the number of customers also increases. It's an seen that the M/M/1 model has more customers in the system that the M/M/2 model, hence the number of servers, the best in performance. In Matlab simulator working controls are given below,

```

p = 0:0.01:0.99; %utilisation factor
W1 = []; % an array to store the values
W2 = []; % an array to store the values
lamda1 = 0:0.01:0.99;
lamda2 = lamda1 * 2;
N1 = [];
N2 = [];

%a for loop to repeat the calculations for more than a single value
for k = 1:100
W1 = [W1 (lamda1(k)/(1-lamda1(k)))+1];
W2 = [W2 (lamda2(k)^2)/(4 - (lamda2(k)^2))+0.5];
N1(k) = lamda1(k) * W1(k);
N2(k) = lamda2(k)* W2(k);
end

%plotting the graph
plot(p,N1,'r',p,N2,'b');
Title('Average Number of Customers In System for M/M/1 And M/M/2 Queues when \mu = 1');
Xlabel('Utilisation \rho');
Ylabel('Average Number of Customers In System');
tleg = legend('M/M/1','M/M/2');
set(tleg, 'Location', 'NorthWest');

```

4.4 M/M/1 vs M/D/1 and $\mu=1$

Comparing the performances of M/M/1 with M/D/1 in terms of average waiting time in the queue. An expression for M/M/1 was derived earlier, namely, equation(3), and for M/D/1 and $\mu=1$, will have,

$$W_q = \frac{\rho}{2\mu(1-\rho)} \quad (9)$$

A graph containing the two plots is shown below,

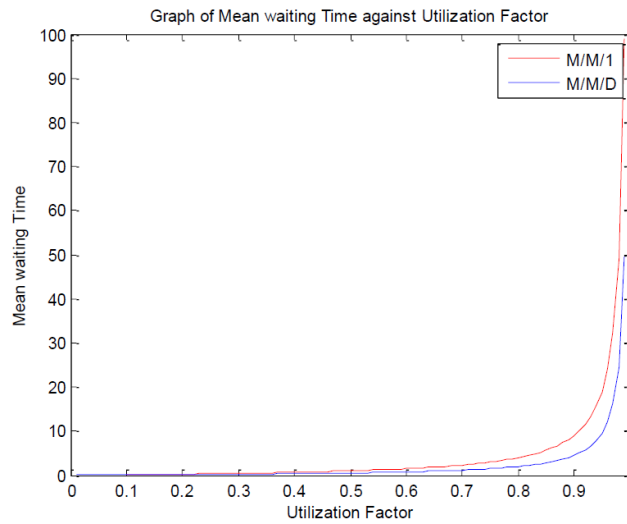


Figure 5. Utilization factor against Average waiting time

From the graph, understanding that for the both M/M/1 and M/M/D queueing models, the average waiting time in the queue is approximately until about $\rho=0.3$ for M/M/1 and $\rho=0.5$ for M/M/D after which it starts increasing exponentially. Also adding that as utilization factor increases, the average waiting time for M/M/1 is greater than M/M/D. Therefore M/M/D has a better performance than M/M/1 model. In Matlab simulator working controls are given below,

```
p = 0:0.01:0.99;
W1 = [];
W2 = [];
lamda1 = 0:0.01:0.99;
for k = 1:100
W1 = [W1 (lamda1(k)/(1-lamda1(k)))+1];
W2 = [W2 (lamda1(k)/(2*(1-lamda1(k)))+1];
end
plot(p,W1,'r',p,W2,'b');
Title('Average Time In System for M/M/1 And M/D/1 Queues when \mu = 1');
Xlabel('Utilization \rho');
Ylabel('Average Time In System');
tleg = legend('M/M/1','M/D/1');
set(tleg, 'Location', 'NorthWest');
buffer = plot(pD_Array,WD_Array,'b');
legend('M/M/1','M/M/D');
title('Graph of Mean waiting Time against Utilization Factor')
xlabel('Utilization Factor');
ylabel('Mean waiting Time');
```

4.5. M/M/k/k, Probability of Loss

In this session, investigates the effect of increasing the load of an M/M/k/k model while the number of servers remains constant. A graph is generated with different plots for 5,10,15 servers describing the probability loss(Erlang B) for traffic intensity per server ranging between 0 and 1. The Erlang B formula is given by,

$$\text{Erlang } B_k = \text{load} * \text{Erlang } B_k - \frac{1}{k + (\text{Load} * \text{Erlang } B_k - 1)}$$

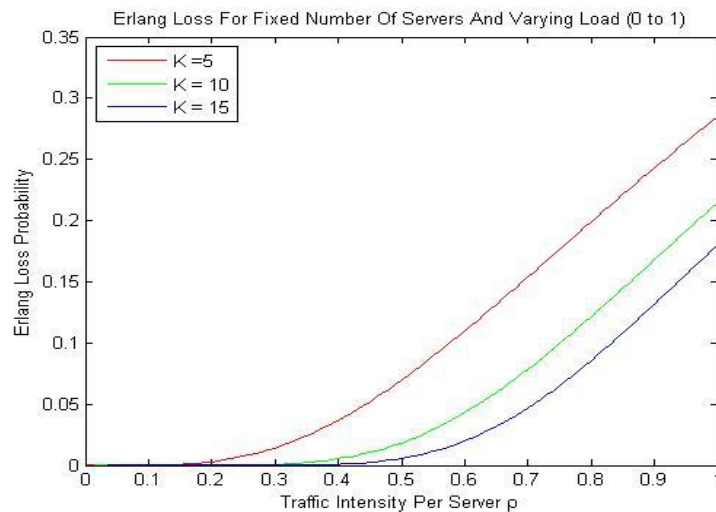


Figure 6. Erlang Loss Probability against Traffic Intensity

Based on the graph, the lower the blocking probability is seen by the system with the highest if servers $\rho=A/k$, A being the load and k being the number of servers. As the number of servers increases, then the load also increases by the same factor. Given the explanation refers to the Erlang B formula given above in the procedure, it can be deduced that as the number of servers, k increases, then the denominator will keep increasing therefore the probability of the system being full at any time $E_k(A)$ keeps reducing. [8] using blocking probability extend analysis with single layer unicast to multilayer cast call. Using Erlang 's fixed point approximation [9] finds new way to estimate blocking probability in overflow loss networks. Blocking probability is a basic quantity that is used in many network design and application tasks[10]. In Matlab simulator working controls are given below,


```

FILE 1:
function y = erlangB (A, k)
e = 1;
for i = 1:k
e = (A * e)/(i + (A * e));
end
y = e;
FILE 2:
load1 = 0:0.05:5;
load2 = 0:0.1:10;
load3 = 0:0.15:15;
fixedK1 = 5;
fixedK2 = 10;
fixedK3 = 15;
p = 0:0.01:1;
for k = 1:101
Bm1(k) = erlangB(load1(k), fixedK1);
Bm2(k) = erlangB(load2(k), fixedK2);
Bm3(k) = erlangB(load3(k), fixedK3);
end
clf;
plot(p, Bm1, 'r', p, Bm2, 'g', p, Bm3, 'b');
Title('Erlang Loss For Fixed Number Of Servers And Varying Load (0 to 1)');
Xlabel('Traffic Intensity Per Server \rho'); Ylabel('Erlang Loss Probability');
tleg = legend('K =5','K = 10', 'K = 15');
set(tleg, 'Location', 'NorthWest');

```

4.6. $M/M/k/k$ and Probability of Loss with $\rho \in [0, 10]$

In this session, repeating the investigation effects of increasing the load A of an $M/M/k/k$ model while the number of servers remains constant. The graph from Matlab simulator shown below,

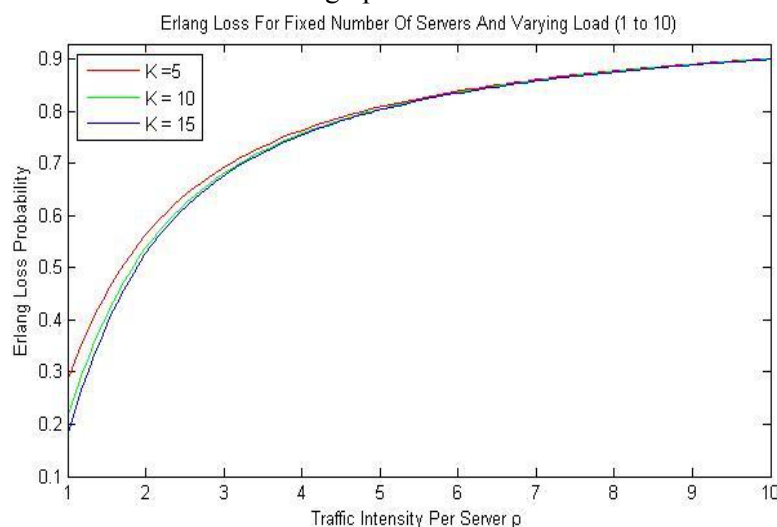


Figure 7. Erlang Loss for fixed servers against fixed traffic intensity

From the graph, it shows that the traffic intensity per server increases, the Erlang loss plots tend to converge. It is also seen that as the load increases, the blocking probability also increases, therefore reducing the performance of the system. Thus, the system with five servers has the last performance as

load increases. Never prefer such a design of the system because of its poor performance that comes as a result of the high blocking probability. In Matlab simulator working controls are given below,

```
load1 = 5:0.45:50;
load2 = 10:0.9:100;
load3 = 15:1.35:150;
fixedK1 = 5;
fixedK2 = 10;
fixedK3 = 15;
p = 1:0.09:10;
for k = 1:101
Bm1(k) = erlangB(load1(k), fixedK1);
Bm2(k) = erlangB(load2(k), fixedK2);
Bm3(k) = erlangB(load3(k), fixedK3);
end
clf;
plot(p, Bm1, 'r', p, Bm2, 'g', p, Bm3, 'b');
Title('Erlang Loss For Fixed Number Of Servers And Varying Load (1 to 10)');
Xlabel('Traffic Intensity Per Server \rho');
Ylabel('Erlang Loss Probability');
tleg = legend('K =5','K = 10', 'K = 15');
set(tleg, 'Location', 'NorthWest');
```

5. Conclusions

From results analysis, understanding that M/M/2 queuing model has a better performance than M/M/1 model based on average waiting time and utilization factors of both systems. A M/M/D model given more performance measure than an M/M/1 model using the same variable as above. The M/M/D performance can be approximated to that of M/M/2 model. Therefore, the more the number of servers in a system, get more and better performance. It can also be deduced that while keeping the utilization factor constant and increasing the number of servers, the blocking probability reduces thus better performances.

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