Multi-objective Mixed Integer Programming approach for facility layout design by considering closeness ratings, material handling, and re-layout cost

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Abstract. Facility layout becomes one of production system factor that should be managed well, as it is designated for the location of production. In managing the layout, designing the layout by considering the optimal layout condition that supports the work condition is essential. One of the method for facility layout optimization is Mixed Integer Programming (MIP). In this study, the MIP is solved using Lingo 9.0 software and considering quantitative and qualitative objectives to be achieved simultaneously: minimizing material handling cost, maximizing closeness rating, and minimizing re-layout cost. The research took place in Rekayasa Wangdi as a make to order company, focusing on the making of concrete brick dough stirring machine with 10 departments involved. The result shows an improvement in the new layout for 333,72 points of objective value compared with the initial layout. As the conclusion, the proposed MIP is proven to be used to model facility layout problem under multi objective consideration for a more realistic look.

1. Introduction

In a production system, there are a lot of processes and occurrences that must be considered and planned. One of production system factor that must be managed well is the facility layout, because it is designated for the production location, where machines, human, assets, material, and product move along processes. Having a bad designed or arranged facilities may inflict a disarranged movement of production entity, which may cause an increasing chance of error, increasing transportation/handling time, not ergonomic work place, and many causes that can inflict costs. Considering the importance of proper facility layout design and planning, development toward this field has been conducted until now.

Modeling the plant layout becomes an approach that often to be chosen as it can save the time and cost to discover the effect of layout modifications. The model can be built either by manual or computer based. Koopmans and Beckmann (1957) proposed a basic mathematical model for assignment problems by using linear programming that is known as Quadratic Assignment Problems (QAP) designed for layout with equal area [1]. A Mixed Integer Programming (MIP) based mathematical model for facility layout problems has been developed by previous researcher [2]. This MIP method was known as the most optimal mathematical model for facility layout design, which then is developed more by Meller et al. (1998) [3] and Sherali et al. (2003) [4]. From QAP and MIP, the model then expanded with many approaches and aided by computer programming. CRAFT that was developed by Armour and Buffa in 1963 was the famous computer based heuristic algorithm for

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plant layout optimization considering minimum material handling cost [5]. There was also another computer-based algorithm with adjacency ratings among facilities such as Computerized Layout Planning (CORELAP), Automated Layout Design Program (ALDEP) and BLOCPLAN, the hybrid one that able to process both optimal and heuristic algorithms. Even farther, intelligent approach such as Genetic Algorithm had also been implemented for optimizing layout that became an interesting discussion nowadays [6], [7], [8], [9].

Although many approaches are intended to mimic the layout condition and changes, they still cannot represent the real world in some ways. For example, some approaches only try to focus only in one indicator like material handling cost. Indeed material handling have a big contribution toward operation handling cost that is up to 50 to 60 percent of product cost, and take 50 percent of production time [10]. But sometimes it is not representative as in the real business company will consider other costs or importance. There are safety and ergonomic issues, for instance, which become a constraint for 2 facilities to be closed even their material handling cost is high, and vice versa. Some researchers have conducted research concerning these objectives simultaneously [11], [12].

Although there are already several researches regarding to multi-objective layout problems, yet there is still no research concerning about re-layout cost together with those objectives. Meanwhile, sometimes facility re-layout is also not feasible as it has a very high installment cost and production loss for stopping the machine. Thus, these considerations need to be brought together in the optimization approach in order to get a more realistic result. This research is conducted by building an optimization model to develop a new layout by minimizing material handling cost, re-layout cost, and maximizing closeness ratings simultaneously. The model will be built based on MIP approach in order to get the optimal result of the layout.

2. Related works

Literature study for facility layout problems have been conducted [13], [14], and [15]. Both [13] and [15] emphasized the opportunities for multi objective as the further research object in facility layout problems is needed to resume more realistic considerations and fill the research gap. This initiate the research to be focusing in multi objective problems.

A multi objective algorithm for facility layout problem by considering quantitative and qualitative objectives at once is developed [16]. They proposed a probability of superiority as additional criteria for proofing that one layout is better than the other. There is also another research conducted [17]. In that study, multi-pass halving and doubling procedure method are used to get the importance of each objective subjectively to the decision maker. A prior test was also conducted to get the consistency of the paired comparison matrix.

Using another approach, a heuristic approach for determining the sequence of common linear machine with multi products under different operation sequences and facilities is proposed [18]. The criteria assessed are total flow distance by products, number of machines in linear sequence, and total investment cost of the machines.

Multi objective layout problems become an interesting topic as actually there are a lot of things can be considered in the facility layout problems and the need for a simultaneously calculation. In other side, there is still a lack of research concerning multi objective facility layout problems with MIP as the method. Researches applying MIP usually only focus toward one objective such as material handling.

Heragu and Kusiak (1991) proposed some MIP techniques named as M1, M2, and M3 [19]. The first technique: M1, focuses on minimizing the total cost required in trips between facilities for a single row layout. Besides, both M2 and M3 focus on multi row layout, where in M2 all departments are assumed to have the same area and M3 can be for unequal department's areas. These models have only a few constraints which can ease the calculation, but it is assumed that there is no restriction on the shape of the building of the facilities.

Meller (1998) also proposed another MIP equations for facility layout problem [3], where this equation seems to be more realistic rather than Heragu and Kusiak (1991) had proposed. He enhanced the model proposed by Montreuil (1990) [2], by redefining Monteruil's binary variables and tightening the department area constraints. He also proposed some general classes of valid inequalities thus the range of problem solved can be enhanced, from *n* = 5 departments (as conducted by Montreuil) to be *n* = 9 departments.

Afterward, Meller's model which known as FLP2 and FLP2+ is enhanced by Sherali et al. (2003) [4]. The new proposed model can solve layout problems more accurate and efficient. Another MIP formulation also proposed by Konak et al. (2005) [20], designed for facility layout design using flexible bays. The application of MIP model for facility layout is an interesting area as this method is proofed to be the best-most optimal optimization techniques for facility layout problems, although it only can solve small amount of problems in departments.

As can be seen from the previous researches mentioned, there are still no previous paper discussing a multi objective facility layout problem considering material handling cost, adjacency ratings, and re-layout cost simultaneously. Thus, a research that involves those 3 objectives is conducted here. It uses MIP approach in order to get the optimal solution of the problem. From this research, it is expected that a better layout arrangement can be derived by the proposed mathematical model.

3. Model Description

The parameters involved are as follows:

w = weighting score of each objective
c_{ij} = cost of material handling from facilities *i* to *j*
f_{ij} = frequency of material flow from facilities *i* to *j*
Pl_i = production loss from moving facilities *i*
t_{ij} = time spent for moving facilities *i* to *j*
$cm1_{ij}$ = cost of moving facilities *i* to *j* (where it is varied to distance)
$cm2_{ij}$ = fixed cost of moving facilities *i* to *j* (where it is not influenced by distance)
r_{ij} = adjacency score between facilities *i* to *j*
a_i = area of department *i*, where $a_i > 0$
$\propto_i$ = maximum permissible ratio between the longest and shortest side of department *i*. i.e., $\max_s\{l_i^s\} / \min_s\{l_i^s\} \leq \propto_i$
lb_i = lower bound of department's length
ub_i = upper bound of department's length
H^s = maximum permissible total length of a rectangular building (building criteria)
s = x, y coordinate

The decision variable of the equations are as follows:

l_i^x = half-length of department *i* having a rectangular shape (correspond to *x* ordinate)
l_i^y = half-width of department *i* having a rectangular shape (correspond to *y* ordinate)
c_i^x = centroid of department *i* in *x* ordinate
c_i^y = centroid of department *i* in *y* ordinate
d_{ij}^s = distance between facilities *i* and *j*

$$z_{ij}^s \begin{cases} 1, if\ department\ i\ is\ forced\ to\ preced\ department\ j\ in\ direction\ s \\ 0, otherwise \end{cases}$$

The multi-objective model to be achieved is like in the following model:

$$\min Z = \sum_{i=1}^{n-1} \sum_{j=1+1}^{n} w_1 O_1 - w_2 O_2 + w_3 O_{3a} + \sum_{i=1}^{n} w_3 O_{3b} \quad (01)$$

$$O_1 = \sum_{i=1}^{n-1} \sum_{j=1+1}^{n} c_{ij} f_{ij} \left(d_{ij}^x + d_{ij}^y\right) \quad (02)$$

$$O_2 = \sum_{i=1}^{n-1} \sum_{j=1+1}^{n} r_{ij} \left(d_{ij}^x + d_{ij}^y\right) \quad (03)$$

$$O_{3a} = \sum_{i=1}^{n-1} \sum_{j=1+1}^{n} cm1_i \left(d_{ij}^x + d_{ij}^y\right) \quad (04)$$

$$O_{3b} = \sum_{i=1}^{n} cm2_i + \left(pl_i \; t_i\right) \quad (05)$$

As can be seen, the Objective 1 (equation 02) focuses on the material handling cost minimization, the Objective 2 (equation 03) focuses on qualitative objective: maximizing adjacency score between facilities, and the objective 3 (equation 04) and (equation 05) focus on minimizing the movement of facilities in re-lay outing. Specifically, Objective 3a (equation 04) concerns about the minimizing cost of moving machine where distance is influencing the cost, while objective 3b (equation 05) concern about the cost of moving machine where distance is not influencing and added by production loss of moving machine. In other word, Objective 3b tends to fixed cost.

The constraints implemented are based on the Sherali et al.'s (2003) MIP model with a derived polyhedral outer-approximation for the area constraints [4], which are enhanced from Meller's (1998) FLP2 equation [3].

$$\text{Subject to:} \, a_i l_i^x + 4\bar{x}^2 l_i^y \geq 2a_i\bar{x} \quad \forall \; lb_i^x \leq \bar{x} \leq ub_i^x, \text{i} \quad (06)$$

$$\bar{x} = lb_i^x + \frac{\lambda}{(\Delta-1)}\left(ub_i^x - lb_i^x\right) \quad \forall \; \lambda = 0, 1, \ldots, \Delta - 1, \text{ for any selected integer } \Delta \geq 2 \quad (07)$$

$$\sum_{s=x}^{y}\left(z_{ij}^s + z_{ji}^s\right) \geq 1 \quad \forall \; i < j \quad (08)$$

$$z_{ij}^s + z_{ji}^s \leq 1 \quad \forall \; j > i, s \quad (09)$$

$$c_i^s + l_i^s \leq c_j^s - l_j^s + H^s\left(1 - z_{ij}^s\right) \quad \forall \; i \neq j, s \quad (10)$$

$$d_{ij}^s \geq c_i^s - c_j^s \quad \forall \; f_{ij} \neq 0, j > i, s \quad (11)$$

$$d_{ij}^s \geq c_j^s - c_i^s \quad \forall \; f_{ij} \neq 0, j > i, s \quad (12)$$

$$l_i^s \leq c_i^s \leq H^s - l_i^s \quad \forall \; i, s \quad (13)$$

$$\overline{lb_\iota} \leq l_i^s \leq min\left\{\overline{ub_\iota}, H^s/2\right\} \quad \forall \; i, s \quad (14)$$

$$0 \leq d_{ij}^s \leq H^s - \left(lb_i + lb_j\right) \quad \forall \; i \neq j, s \quad (15)$$

$$ub_i = min\left\{\sqrt{\propto_i a_i}, max_s\{H^s\}\right\}/2 \quad \forall \; i, s \quad (16)$$

$$lb_i = a_i/(4ub_i) \quad \forall \; i \quad (17)$$

$$c_i^s \geq 0 \quad (18)$$

$$z_{ij}^s \in \{0,1\} \forall \; i \neq j, s \quad (19)$$

Equation (06) and equation (07) are used for area constraint for each department *i*. The constraints come from the derived polyhedral outer-approximation. The number of discretization points Δ are varied from 5 to 50 tangential supports. The higher the tangential supports are, the more accurate the result will be. Equation (08) and equation (09) ensure that *i* and *j* are separated in at least one direction, while equation (10) prevents overlapping of *i* and *j* in direction *s* by using the 0-1 value from z_{ij}^s. Equation (11) and equation (12) are the linearization of distance's absolute value from the difference between centroid *i* and *j* in direction s. Equation (13) ensures that department i is inside the building or facility. Equation (14) forces the length of i department to be not exceeding the boundary set, while equations (15) and equation (16) denote the value of lower bound and upper bound of the length of each department i. Equation (17) indicates the constraints of the distance that its value must not exceed the building length minus by the lower bound of i and *j* departments. Meanwhile, equation (18) forces the centroid to be not in negative value and equation (19) restricts the z_{ij}^{s} value to be binary (0-1) as one of the decision variables.

Besides FLP2 equation above, there are still some additional equations can be added to enhance the solving process to be faster and more accurate. Here, *p-q* strategy supported with B2 and V2 valid inequalities are implemented:

$$c_p^s \leq c_q^s \quad \forall\ s = x, y \tag{11}$$

$$z_{qp}^{x} = z_{qp}^{y} = 0 \tag{12}$$

$$\sum_{s=x}^{y} \left(c_q^s - c_p^s\right) \geq min\left\{lb_p^x + lb_q^x, lb_p^y + lb_q^y\right\} \tag{13}$$

$$\text{B2: } d_{ij}^s \geq (lb_i^s + lb_j^s) \ \ \forall\ f_{ij} \neq 0, j > i, s \tag{14}$$

$$\text{V2: } d_{ij}^s \geq (l_i^s + l_j^s) - min\left\{ub_i^s + ub_j^s, H^s\right\}(1 - z_{ij}^s - z_{ji}^s) \ \forall\ f_{ij} \neq 0, j > i, s \tag{15}$$

By having a pair of *p* and *q* critical departments based on a maximum total interaction or any other essences, equation (20), equation (21), and equation (22) above, named as position *p-q* method, can be implemented for reducing problem symmetry. The equation requires centroid of *p* department to be the south and west of *q* departments. In this case, *p* and *q* departments will be chosen to a pair of departments having the largest flow. It is need to be noticed that this position *p-q* method only can be implemented if there is no fixed departments.

Equation (23) and equation (24) above indicate the valid inequalities developed [3] that Sherali et al. (2003) [4] found as the equation bring a better performance compared to the other inequalities, where in this equation Sherali et al. already modified the B2 and V2 with their tighter bounds. These equations are implemented here as it is found that B2 and V2 constraints are effective to be used in conjunction with position *p-q* method.

This model was intended to a normal condition where there are no fixed departments or fixed-width-departments. For handling such that further problem, the detailed model are provided in [3] or [4].

4. A case study

The company observed in this study is located in Yogyakarta, Indonesia. The company focuses on producing machines for industries and personal usage related to manufacturing, trade, and service of food and beverage. The products are made based on the request of the costumer, where sometimes costumers also order for special needs. The company has 10 departments and located in a 26,5 m x 24,5 m of rectangle building. Table 1 shows department's dimension, their location and cost of moving.

Table 1. department's dimension and their location

No	Dept.	L (m)	H (m)	c_i^x	c_i^y	Cost of moving machine (IDR/meter)
1	Cutting machine	12	8	16,55	5,725	7.200.000 (fixed)
2	Cutting dept.	3	4	15,2	6,05	Equal to material handling cost/meter
3	Drill	3	2,5	15,1	6,375	Equal to material handling cost/meter
4	Welding	8	2,5	17,05	6,275	Equal to material handling cost/meter
5	Circle	9	4,5	16,4	6,825	Equal to material handling cost/meter
6	Grinding	6	2,5	15,65	6,375	Equal to material handling cost/meter
7	Bender	3	2,5	15,825	6,625	Equal to material handling cost/meter * 2
8	Roll plate	4	2	16,85	6,5	7.200.000 (fixed)
9	Painting	6	6	15,65	5,95	Equal to material handling cost/meter
10	Bending	7	4,5	16,3	6,375	7.200.000 (fixed)

In table 1, it can be seen that the cost of moving of Bender department is equal to material handling cost per meter multiplied by 2. It is because 2 workers are required to move the machine inside such department. Other required input data are material flow frequency and closeness rating between department that can be seen in table 2 and table 3 respectively while initial layout of the company is depicted in figure 1.

Table 2. Flow frequency between department

Dept.	1	2	3	4	5	6	7	8	9	10
1	0			2						1
2		0	3	3		3				
3		1	0	10	1	1				
4			1	0					1	
5			2	6	0	8	4			
6			5	7		0		1		
7				7			0			
8				1				0		
9									0	
10				1						0

Table 3. Closeness rating between departments

Dept.	1	2	3	4	5	6	7	8	9	10
1	0	4	3	2	2	3	2	4	1	3
2	4	0	4	2	2	3	2	2	1	2
3	3	4	0	3	3	5	2	2	1	3
4	2	2	3	0	2	4	2	2	1	2
5	2	2	3	2	0	3	3	3	1	2
6	3	3	5	4	3	0	2	2	1	5
7	2	2	2	2	3	2	0	2	1	2
8	4	2	2	2	3	2	2	0	2	2
9	1	1	1	1	1	1	1	2	0	3
10	3	2	3	2	2	5	2	2	3	0

Note: (6 = absolutely necessary, 5 = essentially necessary, 4 = important, 3 = ordinary, 2 = unimportant, 1 = undesirable)

In this study, production lost due to re-layout activity is also considered. Such consideration is to make the re-layout analysis to be more scientific. When a department is required to move, logically the production activity must be stopped and it will be lost production for the company. Table 5 shows the production lost per minute due to re-layout activity.

The moving time as illustrated in Table 6 are assumed to be fix value toward distance, because the area of production is not too big. Thus, wherever the machine will be moved to, there will be only a slight difference that in this case will be averaged. The moving time for cutting machine, roll plate, and bending above require more time if compared to other departments, because they involve big machines that need a lot of setup operation only to replace them. The company's owner estimated that by minimum, it takes 4 days to move the machines, wherever the position is.

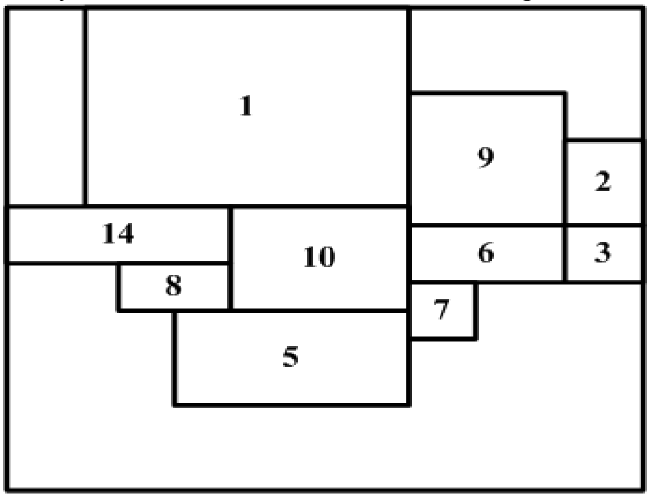

Figure 1. Initial layout of the company

Table 4. Production loss per minute for each department

Dept.	Operation Time (min)	Production loss/min	Operation time/total (%)
1	35	0,010	1%
2	103	0,030	3%
3	392	0,112	11%
4	1.640	0,470	47%
5	529	0,152	15%
6	198	0,057	6%
7	290	0,083	8%
8	30	0,009	1%
9	240	0,069	7%
10	30	0,009	1%
Sum	3.487	1	100%

Table 5. Moving time of machine

Dept.	Moving Time (min)
1	1.920
2	5
3	5
4	3
5	5
6	3
7	15
8	1.920
9	7
10	1.920

The mathematical model to solve the problem is developed under several assumptions as follows:

- The material handling cost is assumed to be 1. This is according to the manual (barehanded) material handling and no certain man to handle the material handling job. The material handling was carried out by operator in each department, which makes there is no existing value to measure the material handling cost.
- The value of Δ in area constraint equation is 20.
- The α ratio of each department uses the initial length and width of each department.
- In this case, the departments are located along a maximum permissible length $H^x = 24$ and $H^y = 18$ (in meter). This is due to some building's spaces are used for others besides the production, which makes the maximum permissible length is not equal to the building's area. These values are obtained from the same maximum lengths with the initial layout.

- Based on interview with the company's owner, the most important objective to be considered is qualitative one with score $w_2 = 0{,}5$. This high score comes from the thought that safety is in prior in the production itself. In the second place, there is material handling efficiency objective with score $w_1 = 0{,}4$, as the company realize the importance of workflow efficiency. The rest is for the cost of moving machine objective with score $w_3 = 0{,}1$.
- The p and q departments chosen for p-q strategy are the pair of departments 3-4, as it has the highest material flow.

Objective value for the new layout provided by the proposed mathematical model is IDR 2.158.887,79. The length and area of each department can be seen in the Table 7 while figure 2 shows the new layout.

Table 6. The half-length and centroid for each departments

Dept.	c_i^x	c_i^y	l_i^x	l_i^y
1	9,305733	10,48089	5,356049	4,480891
2	1,974842	16,48089	1,974842	1,519109
3	4,61782	2,25	1,5	1,25
4	18,66178	4,75	4	1,25
5	19,16178	8,25	4,5	2,25
6	17,66178	16,75	3	1,25
7	7,411782	4,75	1,25	1,25
8	22,85734	1,75	1,142659	1,75
9	11,66178	3	3	3
10	18,21143	13	3,549648	2,230891

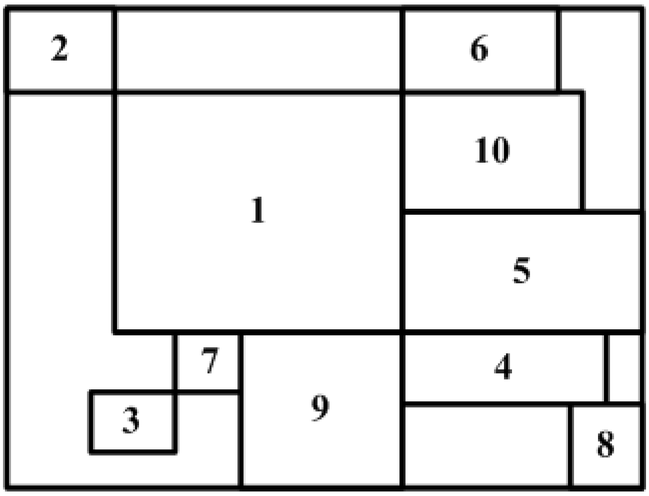

Figure 2. The new optimum layout

5. Discussion

In order to analyze the improvement of the new optimum layout, a comparison analysis with the initial layout is carried out. When the initial layout is substituted into the proposed mathematical, its objective value is 2.159.221,50. As the objective used in the model is minimization objective, the new layout is proven better than the initial layout with 333,72 point improvement. Thus, as the conclusion, the new layout still could be implemented as it has a higher objective value.

6. Conclusions and Further Research

As the conclusion, it is proven that the proposed MIP model is able to optimize multi-objective facility layout problems. The new layout not only has a minimum material handling, but also proportional to the closeness ratings and its re-layout cost. As the further research, it could be an interesting topic to modify the minimization of re-layout cost objective by considering monument or fixed departments, and integrated moving time among departments, a dynamic moving time or production loss, or others. Besides, developing meta-heuristic algorithm such as Genetic Algorithm (GA) combined with a knowledge-based system for the same objectives problem also can be an interesting area as GA is compatible to be used to solve complex problems.

7. Acknowledgements

We would like to thank to all supporting people toward this research, either directly or indirectly. Especially for our family.

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