Theoretical and experimental investigations of thickness-stretch modes in 1-3 piezoelectric composites

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Abstract. Bulk piezoelectric ceramics operating in thickness-stretch (TSt) modes have been widely used in acoustic-related devices. However, the fundamental TSt waves are always coupled with other modes, and the occurrence of these spurious modes in bulk piezoelectric ceramics affects its performance. To suppress the spurious modes, 1-3 piezoelectric composites are promising candidates. However, theoretical modeling of multiphase ceramic composite objects is very complex. In this study, a 1-3 piezoelectric composite sample and a bulk piezoelectric sample are fabricated. The electrical impedance of these two samples are compared. A simple analytical TSt vibration mode from the three dimensional equations of linear piezoelectricity is used to model the performance of 1-3 piezoelectric composites. The theoretical results agree well with the experimental results.

1. Introduction

Bulk piezoelectric ceramics operating in thickness-stretch (TSt) modes have been widely used in ultrasonic transducers, resonators, filters, vibration sensors and other acoustic wave devices. Strictly speaking, pure TSt motions for bulk piezoelectric plates are only possible in unbounded plates. However, piezoelectric devices have finite dimensions of thickness and radius or length in practical applications. Besides, due to in-plane field variation, Poisson's effect and edge effects, the fundamental TSt waves in piezoelectric plates are always coupled to the in-plane extensional wave and the symmetric thickness-shear (TSh) wave [1, 2]. The occurrence of these spurious modes in bulk piezoelectric ceramics plates is undesired, as it may interfere with the operation of the transducers.

To reduce the coupling effect and to suppress the spurious modes, many novel structured materials have been invented. 1-3 piezoelectric composites consisting of active piezoelectric rods embedded in a polymer phase, which combines the advantages of both ceramic and polymer phases, have been widely used in transducer applications [3, 4]. Compared with bulk piezoelectric ceramics, 1-3 piezoelectric composites show many advantages: lower acoustic impedance (Z), a higher coupling coefficient (k_t), a lower mechanical quality factor (Q_m) and dielectric loss ($\tan \delta$) and better design

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flexibility [5, 6]. One example is that Or and Chan [7] have used piezoelectric composite rings to suppress the undesirable radial and wall thickness modes in transducer construction. Nowadays, 1-3 piezoelectric composites can replace bulk piezoelectric materials in bio-medical transducers, underwater applications, and micropositioning systems [8].

Modeling the dynamic behavior of transducers made of 1-3 piezoelectric composites is important in real applications. It is also helpful for the technical improvement of ultrasound-related fields. However, theoretical modeling for multiphase ceramic composite objects is very complex. In the past, many models have been introduced for investigating the physical and electrical properties of 1-3 composites [5, 9, 10]. In [5, 9], 1-3 composites can be treated as an effective homogeneous medium. However, details of the thickness vibration expression and electric independence were not derived in the paper [10]. In addition, the mechanical losses of 1-3 composite piezoelectrics were neglected, and the type of thickness mode was not discerned in the analysis [9]. The finite element method (FEM), which had received considerable attention in recent years, was used to analyze and optimize the 1-3 piezoelectric composites, providing reliable descriptions of the behavior of real devices [4, 11]. However, FEM was always computationally expensive and time consuming. Thus, an analytical solution from which the behaviors of the system can be seen explicitly and directly is still needed.

In this paper, both 1-3 piezoelectric composite samples and bulk piezoelectric samples are fabricated. The electrical impedances between these 2 samples are compared. Then, the paper presents a simple analytical TSt vibration mode from the three dimensional equations of linear piezoelectricity to model the performance of 1-3 piezoelectric composite plates. The theoretical results agree well with the experimental results.

2. Impedance comparison between a 1-3 piezoelectric composite disc and a bulk piezoelectric disc

To illustrate the superior performance of 1-3 piezoelectric composites, two kinds of samples are fabricated and compared. As shown in figure 1, one disc is made of the 1-3 piezoelectric composite. The other is made of bulk piezoelectric ceramics. In this work, both samples are operated at a fundamental frequency of 0.8 MHz. The bulk piezoelectric disc is fabricated using lead zirconate titanate (PZT-5A, [12]). The 1-3 piezoelectric composite disc is fabricated using PZT-5A and epoxy. The PZT-5A unit rods used as the active component are embedded in a passive epoxy. The aspect ratio (ratio between the thickness and the lateral dimension of the PZT unit cell) of the sample is about 3, larger than 2 mentioned in [13], in order to model the thickness mode sufficiently.

Note that the fundamental TSt mode is most useful in applications [14]. The electrical impedances near the fundamental frequency of the 1-3 piezoelectric composite disc and the bulk PZT disc are compared in figure 2. The electrical impedance of the two samples is measured by an impedance analyzer (Agilent 4294A). As shown in figure 2, the impedance of the 1-3 piezoelectric composite disc
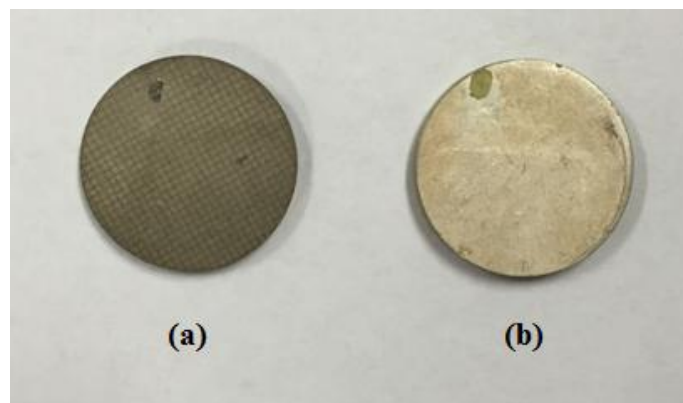

Figure 1. Samples of 1-3 piezoelectric composite disc (a) and bulk PZT disc (b).

at its anti-resonance frequency is much higher than that of the bulk PZT disc. In addition, many spurious modes occur at the low-frequency region of the bulk PZT disc, which would interfere with the performance. Moreover, figure 2 shows that the coupling coefficient (k_t) of the 1-3 piezoelectric composite disc is much higher than that of the bulk PZT disc.
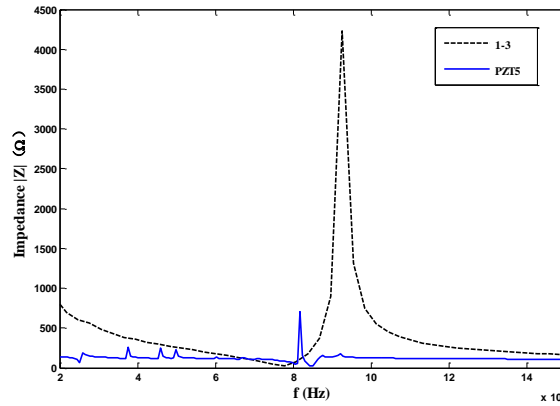

Figure 2. Electrical impedance comparison between sample (a) and (b).

3. Modeling of 1-3 piezoelectric composites operating in TSt modes

1-3 piezoelectric composites, as shown in figures 1(a) and 3, are formed by embedding piezoelectric ceramic rods in a passive polymer medium. The poling direction of the PZT rods is along the longitudinal direction. As mentioned in [5, 9], 1-3 composites can be treated as an effective homogeneous medium with a set of effective material properties. Thus, an analytical model in which the 1-3 composites are replaced by a bulk ceramic plate is proposed. Consider the bulk ceramic plate with a thickness of 2 h poled along the x_3 axis (see figure 3). The plate is bounded by two planes at $x_3 = \pm h$ that are traction-free and electroded. A time-harmonic voltage is applied across the plate thickness.
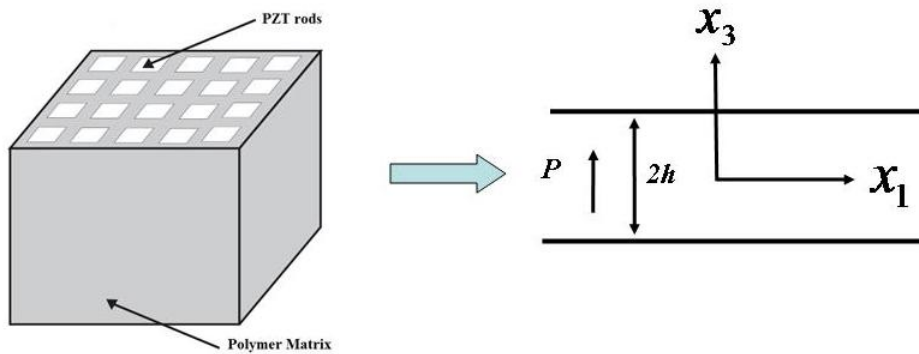

Figure 3. Schematic representation of 1-3 piezoelectric composite plate and its theoretical model.

The governing equations of three-dimensional linear piezoelectricity [15] and the boundary conditions for the 1-3 piezoelectric composite plate shown in figure 3 can be written as [16]:

$$T_{ji,j} = \rho \ddot{u}_i, \quad D_{i,i} = 0,$$
$$T_{ji,j} = c_{ijkl}S_{kl} - e_{kij}E_k, \quad D_i = e_{ikl}S_{kl} + \varepsilon_{ik}E_k,$$

$$S_{ij} = (u_{i,j} + u_{j,i})/2, \quad E_i = -\phi_{,i},$$

$$T_{3j} = 0, \quad x_3 = \pm h, \tag{1}$$

$$\phi(x_3 = h) - \phi(x_3 = -h) = Ve^{i\varpi t}.$$

where u_i is the mechanical displacement vector, T_{ij} is the stress tensor, S_{ij} is the strain tensor, E_i is the electric field vector, D_i is the electric displacement vector, and ϕ is the electric potential. c_{ijkl}, e_{kij} and ε_{ij} are the effective elastic, piezoelectric and dielectric constants of the 1-3 piezoelectric composite plate, respectively. We denote the effective mass density by ρ. We use the Cartesian tensor notation, the summation convention for repeated tensor indices and the convention that a comma followed by an index denotes partial differentiation with respect to the coordinate associated with the index. A superimposed dot represents the time derivative.

For 1-3 piezoelectric composites poled in the x_3 direction, consider the so-called thickness-stretch (TSt) vibration of the plate only depending on u_3. We assume the solutions are in the following forms:

$$u_3 = u(x_3)e^{i\varpi t}, \ u_1 = u_2 = 0, \ \phi = \phi(x_3)e^{i\varpi t}, \tag{2}$$

Modes described by Equation (2) are allowed by the three-dimensional equations of linear piezoelectricity.

4. Theoretical and experimental results and discussion

To describe the TSt vibrations of the disc, the effective constitutive relationship for this homogeneous medium was introduced in [5].

$$c_{33} = v[c_{33}^E - \frac{2\bar{v}(c_{13}^E - c_{12}^P)^2}{v(c_{11}^P + c_{12}^P) + \bar{v}(c_{11}^E + c_{12}^E)}] + \bar{v}c_{11}^P,$$

$$e_{33} = v[e_{33}^E - \frac{2\bar{v}e_{31}^E(c_{13}^E - c_{12}^P)}{v(c_{11}^P + c_{12}^P) + \bar{v}(c_{11}^E + c_{12}^E)}], \tag{3}$$

$$\varepsilon_{33} = v[\varepsilon_{33}^E + \frac{2\bar{v}(e_{31}^E)^2}{v(c_{11}^P + c_{12}^P) + \bar{v}(c_{11}^E + c_{12}^E)}] + \bar{v}\varepsilon_{11}^P,$$

$$\rho = v\rho^E + \bar{v}\rho^P.$$

where v is the volume fraction of the piezoceramics in the composite, and $\bar{v} = 1 - v$ is the volume fraction of the epoxy. The material parameters with a superimposed E and P inside are for the ceramic phase and polymer phase, respectively. The corresponding material parameters for both PZT-5A and epoxy are given in table 1.

Table 1. Material parameters for PZT-5A and epoxy.

Piezoceremics	PZT-5A [12]	Epoxy	Araldite D
c_{11}^E (10^{10} N/m^2)	12.1	c_{11}^P (10^{10} N/m^2)	0.8
c_{12}^E (10^{10} N/m^2)	7.59	c_{12}^P (10^{10} N/m^2)	0.44
c_{13}^E (10^{10} N/m^2)	7.54	$\varepsilon_{11}^P / \varepsilon_0$	4
c_{33}^E (10^{10} N/m^2)	11.1	ρ^P (Kg/m^3)	1150

e_{31}^E (C/m^2)	-5.4	$\varepsilon_0 (10^{-12} \text{F/m})$	8.85
e_{33}^E (C/m^2)	15.8		
ε_{31}^E (10^{-8} C/Vm)	0.811		
ε_{33}^E (10^{-8} C/Vm)	0.735		
ρ^E (Kg/m^3)	7750		

The calculated electrical impedance of the disc is compared with the one obtained experimentally by the impedance analyzer (Agilent 4294A). The comparison between the experimental and theoretical electrical impedances is shown in figure 4. There is excellent agreement between the experimental electrical impedance and that obtained by the theoretical TSt vibration we proposed. This means that the 1-3 piezoelectric composite can operate in nearly pure TSt vibration. Other modes can be suppressed near the fundamental TSt resonance. The model we present could be used as a guideline for transducer design.
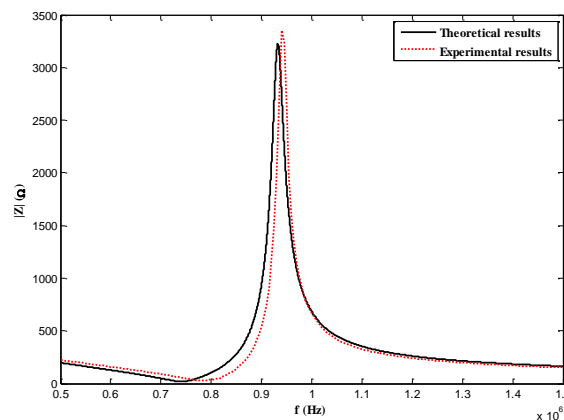

Figure 4. The comparison of electrical impedances between the experimental and theoretical results.

5. Conclusions

This paper presents the theoretical and experimental studies of the TSt modes of the 1-3 piezoelectric composite. An analytical solution of TSt vibration from the three dimensional equations of linear piezoelectricity to analyze the performance of 1-3 piezoelectric composites is performed. The electrical impedance obtained by the theoretical TSt vibration agrees well with experimental results. This means that the 1-3 piezoelectric composite can operate in nearly pure TSt vibration. This simple model of TSt vibration is sufficient to analyze 1-3 piezoelectric composites with larger aspect ratios quickly and efficiently. The results could be used as a guideline for transducer design.

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