

Shear strength and structural behaviours of HPRWO with web openings with circular steel tubes

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Abstract. The purpose of this study was to evaluate the shear strength and structural behavior of reinforced-concrete beams with web openings (hereinafter, “HPRWO”), where the web openings are reinforced with circular steel tubes. The experiments were conducted under the monotonic loading condition. Based on the structural experiments involving HPRWO, ultimate load ratio ($d_0/d_0/h$, etc.), ductility, load-deflection curve, and failure mode comparisons were made for evaluation purposes. This study utilized the design formula for predicting the shear strength proposed by the previous studies and formulas to determine the appropriate shear strength for HPRWO. The results of the experiments confirmed that the rigidity, ductility, and other properties of the HPRWO specimens reinforced with circular steel tubes, fiber, and admixtures were superior to those of the unreinforced HPRWOs. With Mansur’s formula, a noticeable tendency for the increase in d_0 and the sectional area of the web openings to lead to the overestimation of $V_u/V_{u,cal}$ was found. The $V_u/V_{u,cal}$ value was found to be more in line with the experiment results based on the AIJ formula compared with the results obtained using other formulas.

1. Introduction

In the field of architectural and civil engineering, there has been a tendency of late for buildings to be built in such a way as to avoid the conventional standardized forms, thus incorporating non-standardized forms, various new styles, and state-of-the-art equipment and systems. The end result is ultra-high-rise buildings. That said, the taller the buildings are, the greater the dead space on each of the floors. Such increase in the dead space is playing a major role in increasing the total construction costs. The possible solutions to the said problems include the use of HPRWO, where perforation is introduced to the beam members along their vertical axes, wherein various facility-ducting services (e.g., HVAC systems, electrical equipment, and hygienic systems) and piping are installed in the depths of the beams [1]. At present, the HPRWO system is roughly divided into (a) HPRWO with steel frames and (b) HPRWO with steel-concrete reinforcement. The strength formula from the previous decades frequently used the conditions subject to the application of both bending based on balance and the shearing force. Since 1990, two suggestions have been made: (a) the design formula for nominal strength, which calculates the maximum shearing force in the unreinforced rectangular openings while plastic collapse is achieved due to the Vierendeel phenomenon [2] in the openings and the maximum

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moment from bending; and (b) the three-dimensional correlative equation for the maximum strength while both shearing force and moment are being applied [3]. In South Korea, the related research on the dynamic behavior of H-beams with web openings was started in 1978. Since then until 2003, a number of papers have been published, though lacking specified codes, relying too much on the structural designer's experience while designing or constructing reinforced stiffeners at the sites, and showing insufficient research on the structures subject to repeated loading [4-5]. As regards the HPRWO with steel-concrete reinforcement, American researchers have published an extensive body of research outcomes, starting with the research in HPRWO focusing on RC bending, shearing, and torsion. In 1984, the study on rectangular HPRWOs subject to a combination of bending and torsion gave impetus to the research in this area, followed by the publication of many studies and proposed formulas until the early 2000s [6-8]. In South Korea, the outcomes of the researches on reinforced HPRWOs started to be published in 1985. Since then, the experiments on and analysis results of the structural performance of web openings have been unsatisfactory [9].

Garnet is simply used for physical filtering to remove minute turbid and floating matters. It has been reported that experiments using garnet contributed to improving the strength and brittleness of polymer mortar [10]. Based on the outcomes of previous studies [11], this study aimed to examine topics such as analysis of crack development in specimens and investigation of failure mechanisms using parameters affecting the structural performance of HPRWOs, including web opening area ratio, width-thickness ratio of steel tubes, and mixing proportions of fiber and admixtures. In addition, this study examined the web openings' load-deflection curve, ultimate load ratio ($V_u/V_{u,hprc}$), and ductility, and evaluated the results of the previous studies and compared them with the results of the current studies in terms of the applicability of the formula for calculating the shear strength proposed by previous researchers, and their design formulas regarding the shear strength of HPRWO bending members.

2. Test sample overview of high-performance RC beams with web openings

Table 1 and Figure 1 show the experimental parameters of the eight specimens that were used for the bending tests under monotonic loading. A total of eight specimens were produced (two HPRWOs without reinforcing circular steel tubes and six HPRWOs with circular steel tubes), along with detailed drawings of such specimens. The principal parameters that were used in the tests were fiber, admixtures, web opening area ratio, web opening configuration, and width-thickness ratio. Testing was conducted to identify the structural performance of the new reinforcement method with circular steel tubing used to reinforce the openings, instead of the conventional reinforcement method using rebar. The proportioning strength of the concrete that was used for the openings was 44.7 MPa for high-strength concrete. The test set-up and LVDT location are listed in Figure 1.

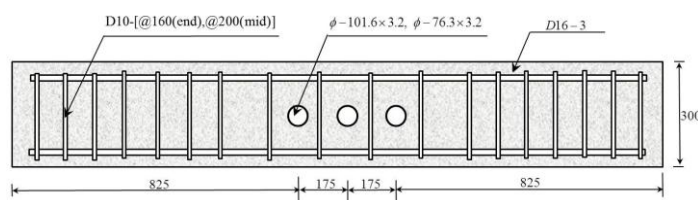




Figure 1. Test specimen and loading system.

Table 1. Details of HPR specimens.

Specimens	b	h	Main -bar	Additive	Fiber	Circle Steel Tube	Web Opening			
							Area	Number	Location	D/t
HPRC3	150 mm	300 mm	D16	None	None	None	0.333	3	M	None
HPRGNC3				Garnet	Ny	76.3×3.2				
HPRGPC3					PP					
HPRGSFC3					SF					
HPRC4				None	None	None				0.444

HPRFNC4				Fly-ash	Ny	101.6× 3.2				
HPRFPC4					PP					
HPRFSFC4					SF					

Note: HPRGNC: HPR(High Performance RC Beams with Web Opening), G(Garnet), F(Fly-ash), N{Fiber; N(Nylon), P(Polypropylene), SF(Steel Fiber)}, C(Circle Steel Tube), 3(0.333), 4(0.444)

3. HPRWO test results and crack development

Figure 2 confirms that in general, the specimens whose HPRWO sectional area was 0.333 showed a more extensive crack development than the specimens whose HPRWO sectional area was 0.444. Specimens HPRGNC3, HPRGPC3, and HPRGSFC3, with different fibers incorporated, exhibited the initial crack development pattern (i.e., bending cracks) that intensively progressed from the lower edges of the HPRWO in the center to the areas surrounding the openings as the load increased. Furthermore, since the occurrence of crack development on the loading points due to the concrete crush, the cracks had gradually expanded. With the HPRGSFC3 specimen including steel fiber, shear cracks were observed to be developing at a location 32 cm away from the opening, which was by far the closest; the range of failure was found to be the smallest. Before and after the yield, shear cracks comprised the majority of the cracks observed; towards the end of the testing, however, a combination of various cracking patterns was visible, as confirmed. Specimen HPRGSFC3 exhibited vertical fractures in the lower edges of the HPRWO in the center, but it showed no buckling or rupture of the steel tubes. Specimens HPRGPC3 and HPRGNC3 showed a composite mode around the openings, which was the dominant pattern, while specimen HPRGSFC3 showed a mixture of diagonal and tangential modes. As for the specimens whose opening size was larger than the depth of the openings (i.e., HPRFNC4, HPRFGC4, and HPRFSFC4), the initial cracks were found to be similar to those of the previous specimens; their shear cracks were found within the 30, 20, and 25 cm range on each side of the openings, respectively. Little cracking or failure was observed in the $\ell/4$ area (edge) of the total span, although extensive bending cracks were found around the openings in the center. Additionally, fracture was observed on the surface of the concrete at the loading locations, due to crush. The range of such fracture tended to be focused on the distributions smaller than the steel tube's $D/t = 21.8$. Specimen HPRFNC4 showed shear cracks before and after the yield, as well as the composite mode failure around the openings. Specimens HPRFPC4 and HPRFSFC4 exhibited crack and fracture patterns that include a combination of diagonal and tangential modes.

Based on the above findings, the circular steel tubing provided as a reinforcement on the web openings showed crack control effects for the specimens, and offered expected merits as an alternative material for the conventional reinforcement (i.e., steel reinforcement and steel bars) used for the openings.

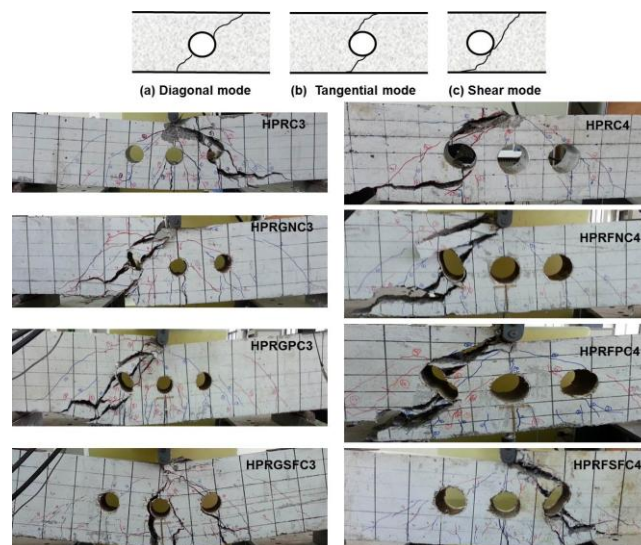


Figure 2. Failure mode types and failure modes of specimens.

4. Load-deflection curve

Table 2 and Figure 3(a) and (b) list the maximum loads for the specimens for testing the web openings, and the load-deflection curve measured from the specimens' center LVDTs. The overall behavior exhibited by the specimens was the typical bilinear one, and the figures and table confirmed that the C4 series showed somewhat lower values than the C3 series in terms of the rigidity of the members. Regarding their behavior after the achievement of the maximum strength, the unreinforced openings exhibited a rapid drop in strength after the achievement of the maximum strength, whereas the reinforced openings had a somewhat gradual decrease in strength. No significant difference was observed from the center part's deflection to the yield load at each stage of loading, but from the yield load to the maximum strength, the deflection in the reinforced openings, compared with that in the unreinforced openings, showed a relatively large increase. For the C3 series, except for HPRGPC3, the increase was between 11.72 and 55.21%; the C4 series showed increases of 13.27 to 25.44%. In terms of the maximum strength, specimen HPRGSFC3, compared with the unreinforced specimens, exhibited a 4.57% increase in strength, but the rest of the C3 series specimens showed somewhat small strength values. The C4 series, compared with specimen HPRC4, had 14.59-23.48% increases in strength (Figure 3(b)).

5. Ultimate load ratio ($V_u/V_{u,hprc}$) and the relationship between d_0 and d_0/h

Table 3 and Figure 4 (a), (b) show the relationship between the web openings' ultimate load (V_u) and ultimate load ratio ($V_u/V_{u,hprc}$). Overall, as the depth of the openings (d_0) and the d_0/h value increased, V_u decreased while $V_u/V_{u,hprc}$ showed an increase. In particular, specimen HPRFSFC4 showed a significant increase. Compared with the unreinforced openings, specimens HPRFSFC3 and HPRFSFC4 showed 4.8 and 30.6% increases in the $V_u/V_{u,hprc}$ value, respectively. The presumed causes of such increase include the following: (a) contrary to the expectation that an increase in the area ratio of the steel tubes used as reinforcing materials would naturally decrease the ultimate load value, the yield (V_y) and ultimate load (V_u) values, after the initial loading, were found to be far greater; and (b) compared with the ultimate load value of the specimens produced with nylon and polypropylene, the specimens produced with steel fiber supposedly underwent a split in strength among their components (i.e., reinforcing steel tubes, steel fiber, and stirrups) from the point where the yield load was achieved until the point where the ultimate load was reached.

6. Ductility

The specimens' ductility performances can be expressed as:

$$\Delta = \Delta_u / \Delta_y \quad (1)$$

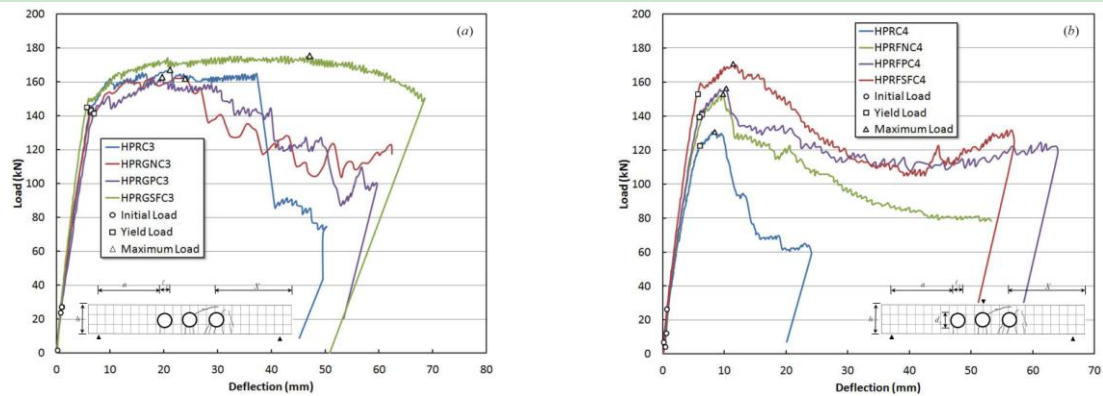
In the above formula, Δ_y refers to the yield displacement, and Δ_u to the displacement in the limit strength. Their values were determined when the member strength was approximately 80% less than the maximum strength. The criteria reflect the member's non-elastic deformation yield strength. Figure 4 (c) shows the comparison of the circular steel tubes' width-thickness ratios (D/t) and ductility rates. With the unreinforced specimen HPRC3, the ductility was found to be 6.349, while the specimens with reinforced openings ($D/t = 21.8$) exhibited ductility values of between 4.683 and 11.579. It was found that the ductility of the beams that had web openings did not show a significant difference from the other specimens' if reinforced with steel tubes and fiber. Additionally, the results of this testing were found to be similar to those from previous studies; it was understood that greater effects could be achieved by providing web openings in the moment area rather than in the composite or shear mode area.

Compared with the unreinforced specimens with web openings, specimens HPRGNC3 and HPRGPC3 showed a tendency for their ductility to be lower (by 26.3 and 10.1%, respectively), although most specimens were found to excel in ductility compared with the unreinforced beams with web openings.

Table 2. Test results of HPR.

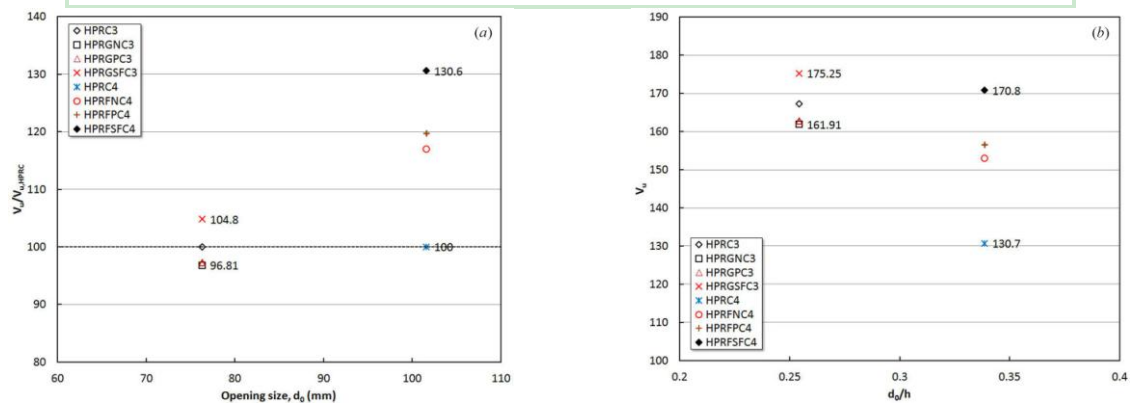
Specimen	V_i		V_y		V_u		$V_u/V_{u,HPRC}$	Ductility
	Load (kN)	Deflection (mm)	Load (kN)	Deflection (mm)	Load (kN)	Deflection (mm)		
HPRC3	27.58	1	142.34	6.3	167.25	21.1	100*	6.349
HPRGNC3	24.02	0.7	143.23	6.4	161.91	23.9	96.81	4.683
HPRGPC3	27.58	0.9	141.45	7.0	162.79	19.7	97.3	5.714
HPRGSFC3	1.78	0.2	145.00	5.7	175.25	47.1	104.8	11.579
HPRC4	7.12	0.2	122.7	6.0	130.7	8.5	100**	1.667
HPRFNC4	12.45	0.6	141.0	6.3	153.01	9.8	117.0	2.698
HPRFPC4	4.45	0.4	139.6	5.9	156.57	10.3	119.7	3.729
HPRFSFC4	26.68	0.7	153.0	5.7	170.80	11.4	130.6	3.947

Note: *-HPRC3, **HPRC4, V_i -Initial load, V_y -Yield load, V_u -Maximum load

**Figure 3.** (a), (b) Experimental curves of load versus deflection.**Table 3.** Comparisons between tests and shear strength.

Specimen	V_u (kN)	$V_{u,cal}$ (kN)			$V_u/V_{u,cal}$		
		①	②	③	$V_u/①$	$V_u/②$	$V_u/③$
RCB	221.5	152.4/15%	183.4/12%	152.4/15%	1.453	1.207	1.453
HPRC3	167.3	167.9/9.9%	198.4/8.4%	101.7/16%	0.996	0.843	1.645
HPRGNC3	161.9	167.9/9.6%	198.4/8.2%	101.7/16%	0.964	0.816	1.592
HPRGPC3	162.8	167.9/9.6%	198.4/8.2%	101.7/16%	0.969	0.821	1.601
HPRGSFC3	175.3	167.9/10%	198.4/8.8%	101.7/17%	1.044	0.884	1.724
HPRC4	130.7	167.9/7.8%	188.2/6.9%	86.3/15%	0.778	0.694	1.514
HPRFNC4	153.0	167.9/9.1%	188.2/8.1%	86.3/18%	0.911	0.813	1.773
HPRFPC4	156.6	167.9/9.3%	188.2/8.3%	86.3/18%	0.933	0.832	1.815
HPRFSFC4	170.8	167.9/10%	188.2/9.1%	86.3/20%	1.017	0.908	1.979

① ACI; ② AIJ; ③ Mansur



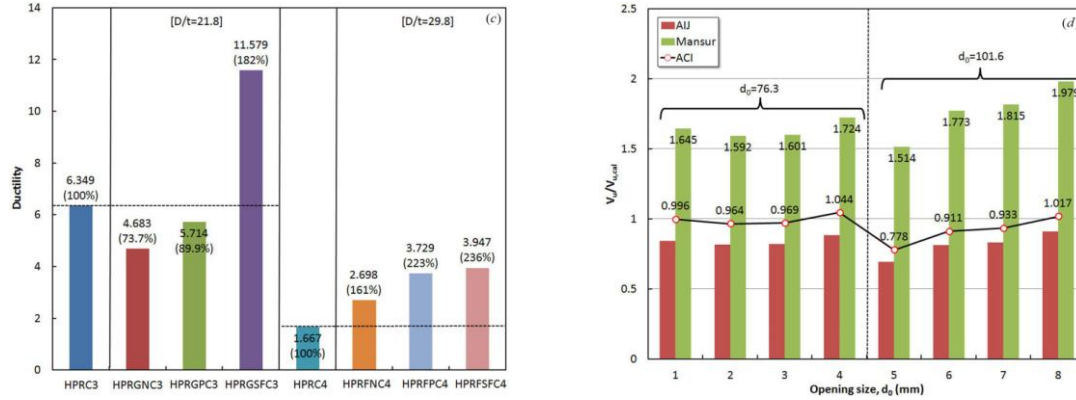


Figure 4. Test results of HPRWOE: (a) relation between d_0 and $V_u/V_{u,hprc}$; (b) relation between d_0/h and V_u ; (c) displacement ductility; (d) relation between d_0 and $V_u/V_{u,cal}$.

In particular, the steel-fiber-infused HPRGSFC3 and HPRFSFC4 exhibited a considerably superior ductility of 82 and 136%, respectively, compared with the unreinforced web openings, with particularly superior ductility found in the C4 series, where the sectional area of openings and D/t had increased. Overall, an average of 25.7% increase in ductility was found in the reinforced openings compared with the unreinforced openings. The related observations include the following: (a) the fiber (i.e., nylon, polypropylene, steel fiber) and admixtures (i.e., garnet, fly ash) incorporated into the production of the specimens affected the ductility performance of the specimens; and (b) compared with the previous researchers' use of steel reinforcement or rebar as reinforcing materials to control the crack and increase the rigidity and ductility around the web openings, the use of steel tubing as reinforcing material for the openings was equally effective.

7. Design items and analysis of HPRWO

Regarding the design practices for the typical steel-framed beams with web openings, the formula proposed by Darwin [3], as described in Chapter 1, a three-dimensional formula for the maximum strength under conditions subject to both shear and moment, has been frequently used. That said, the most commonly and frequently used design practice for HPRWOs at present is the additional strength theorem. This method is based on the conventional design method ($V_c + V_s$) of the ACI [12] shear design. The formula of AIJ (Architectural Institute of Japan) [13] is also an additional strength theorem addressing the stress withstood by concrete and steel, respectively. In addition to these theorems, the formula proposed by Mansur [14], which applies various parameters for web openings, was used to compare, analyze, and examine the shear strength compared with the test values.

7.1. ACI 318

Two categories were employed: (a) the concrete-distributed load (V_c) for the concrete resisting against shearing prior to crack development; and (b) the steel-distributed load as the web reinforcement resists against shearing (V_s). The nominal shear strength [12] is provided in formula (2).

$$V_{u,cal} = V_c + V_s, \quad (2)$$

$$\text{where } V_c = \left(\frac{\sqrt{f_{ck}}}{6} \right) b_w d, \quad V_s = \left(\frac{A_v f_{yt} d}{S} \right) \leq 0.67 \sqrt{f_{ck}} b_w d.$$

7.2. AIJ

The next formula is the one that is specified in AIJ's Standard for Structural Calculation of Reinforced Concrete Structures (1988) [13], which was proposed and standardized by Hirosawa. This formula specifies the evaluation of the shear strength ($V_{u,cal}$) of beams, including the fine perforation, and is similar to the conventional method mentioned earlier. With this formula, as shown below, the total shear resistance is achieved by adding the values for the steel reinforcement horizontally crossing the

45° angle failure section that passes the concrete and the center line of the web openings. K_u is the value falling between 0.72 and 1, which addresses the sizing effects within the shearing. It is the function of the effective depth d .

$$V_{u,cal} = \left\{ \frac{0.092K_p(f_{ck} + 17.7)}{\frac{M}{Vd} + 0.12} \times \left(1 - \frac{1.61d_0}{h} \right) + 0.846\sqrt{\rho_w f_{yv}} \right\} bd_v \quad (3)$$

where K_p is $K_p = 0.82 \left(\frac{100A_s}{bd} \right)^{0.23}$ and d_0 is the diameter of the circular web openings or of the circular circumference. With rectangular web openings, it is either $h/3$ or a value smaller than $h/3$. In addition, h is the total depth of the beams and is expressed as $M/(Vd) \leq 3$. In formula (3), ρ_w refers to the ratio of the web reinforcement placed within the vertical distance $d_v/2$ from the center of the web openings. ρ_w is defined as follows:

$$\rho_w = \frac{A_v(\sin \alpha + \cos \alpha)}{bd_v} \quad (4)$$

Here, d_v refers to the distance between the upper and lower reinforcements; A_v to the sectional area of the web reinforcement (vertical stirrups and diagonal reinforcement); α to the angle of inclination for the web reinforcement; and f_{yv} to the yield strength of the web reinforcement. The first part of formula (3) refers to the shear resistance of concrete; it is assumed to decrease in proportion to the depth of the web openings. The second part of the formula refers to the shear resistance by web reinforcement.

7.3. Mansur

The formula proposed by Mansur [14] was based on ACI's shear design method, as follows:

$$V_{u,cal} = V_c + V_s \quad (5)$$

$$V_c = \left(\frac{\sqrt{f_{ck}}}{6} \right) b_w (d - d_0) \quad (6)$$

$$V_s = V_{sv} + V_{sd} = \left(\frac{A_v f_{yv}}{s} \right) (d - d_0) + A_d f_{yd} \sin \alpha \quad (7)$$

Formula (6) refers to the shear resistance in the concrete while V_{sv} in formula (7) refers to the case where shear reinforcement was provided with vertical stirrups, and V_{sd} to the shear reinforcement using diagonal reinforcement. In addition, A_d refers to the total sectional area of the diagonal reinforcement through the fracture section; α to the angle of inclination of the diagonal reinforcement; and f_{yd} to the yield strength of the diagonal reinforcement.

8. Influence of the web opening depth (d_0) vs $V_u/V_{u,cal}$

Overall, as the d_0 value increased, the $V_u/V_{u,cal}$ value tended to increase as well in the design formulas, except for the ACI formula. The $V_u/V_{u,cal}$ value, in particular, in the Mansur-proposed formula, increased to the maximum value (23.5%) in specimen HPRFSFC4 compared with the unreinforced specimens (Figure 4 (d)). There was a noticeable tendency of overestimation with Mansur's formula, whereas the AIJ and ACI specified formulas tended to underestimate or approximate the value of 1. The comparison of the test results for the unreinforced specimens and the values $d_0 = 76.3$ and $d_0 = 101.6$ confirmed that the design strength values increased to 0.37 and 18.41%, respectively. With the ACI formula, as shown in Table 3, $V_u/V_{u,cal}$ showed a lower value

(0.973) on average. This result was based on the fact that in the design formula for concrete (V_c) and steel reinforcement (V_s), the effects on d_0 were not incorporated. Also, as shown in the calculated strength ($V_{u,cal}$), there was little change in the strength value, which could indicate that the multi-parameter conditions of the specimens that were used in this study had not been reflected properly. Thus, similar results were found in the strength comparison data. The AIJ formula exhibited a tendency in which an increase in d_0 led to a moderate increase in $V_u/V_{u,cal}$, as shown in the analysis, with no significant change. This is thought to be attributable to the fact that the effects of the shear resistance design area of the HPRWOs $\{1 - (1.61d_0/h)\}$ were properly incorporated into the concrete. As for $d_0 = 101.6$ (i.e., a 24.9% increase from the d_0 value), the $V_u/V_{u,cal}$ value was found to have increased by approximately 1.3%.

9. Conclusions

- The results of the comprehensive examination of the data from the tests conducted on the HPRWOs showed that when providing facility systems in a specimen that combines a steel tubing width-thickness ratio (D/t) of 21.8 and a mixture of steel fiber and garnet, benefits, including increased strength and crack-controlling effects, can be expected.
- Compared with the unreinforced specimens, the steel-tube-reinforced HPRWPOs did not underperform in terms of ductility; rather, all the specimens in the category of the C4 series were found to excel in performance. It was confirmed that the smaller the d_0 value was, and with garnet and steel fiber incorporated into the make-up of the specimens, the better the performance became.
- The ACI formula is unable to reflect the parameters of the tests conducted for this study, thus making it difficult to achieve accurate comparative results. In the case of the shear strength evaluation based on Mansur's formula, the increases in the d_0 value led to noticeable decreases in shear strength. As for the AIJ standardized formula, the results were by far the most satisfactory, with an insufficient incorporation of the effects resulting from the increased D/t of the steel tubing and those resulting from the reinforcement of the web openings with steel tubing. Such insufficiency makes it difficult to apply the AIJ formula to the specimens for this study as it is.

Acknowledgement

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