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Transient analysis of single stage GM type double inlet pulse tube cryocooler

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Abstract. Transient analysis of single stage GM type double inlet pulse tube cryocooler is carried out using a one dimensional numerical model based on real gas properties of helium. The model solves continuity, momentum and energy equation for gas and solid to analyse the physical process occurring inside of the pulse tube cryocooler. Finite volume method is applied to discretize the governing equations with realistic initial and boundary conditions. Input data required for solving the model are the design data and operating parameters viz. pressure waveform from the compressor, regenerator matrix data, and system geometry including pulse tube, regenerator size and operating frequency for pulse tube cryocooler. The model investigates the effect of orifice opening, double inlet opening, pressure ratio, system geometry on no load temperature and refrigeration power at various temperatures for different charging pressure. The results are compared with experimental data and reasonable agreement is observed. The model can further be extended for designing two stage pulse tube cryocooler.

1. Introduction

A pulse tube refrigeration phenomenon was first discovered by Gifford and Longsworth [1] in early 1960s which was then called the basic pulse tube cryocooler and followed by this discovery, in 1984 Mikulin et al. [2] suggested an improved version of pulse tube cryocooler (PTC) by placing passive phase shifter or orifice between hot end of pulse tube and reservoir which is named as orifice pulse tube cryocooler. Orifice pulse tube cryocooler was further improved in early 1990s by Zhu et al. [3] by adding second phase shifter between hot end of pulse tube and hot end of regenerator. This version of pulse tube cryocooler is known as double inlet pulse tube cryocooler. The working mechanism of pulse tube refrigeration is quite complex due to oscillating nature of compressible flow of working gas. Since the invention of the basic pulse tube cryocooler, various analysis methods giving explanation to the working process of pulse tube have been proposed. The list includes Phasor analysis, Thermodynamic analysis, Isothermal model and numerical analysis. Earlier work regarding numerical analysis of pulse tube refrigeration has been reported by Wang et al. [4] in 1992 which was mainly focused on Stirling type pulse tube cryocooler with no axial heat conduction. In 1994, the same model was extended to double inlet version [5]. Other researcher Ju et al. [6] also reported the numerical simulation of Stirling pulse tube cryocooler considering the axial heat conduction. The present numerical simulation model follows the numerical model given by Wang [7].

This paper gives one dimensional transient analysis of single stage GM type double inlet pulse tube cryocooler using governing equations like continuity equation, momentum equation and energy equation for gas & solids. The results include cool down history of cold end temperature with zero loading condition, effect of operating frequency, effect of orifice opening, double inlet opening on no

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load temperature, effect of geometry change on no load temperature and effect of pressure ratio on refrigeration temperature for a given value of refrigeration power.

2. Numerical model

The transient analysis considers the following assumptions:

- One dimensional compressible flow of helium gas
- Axial heat conduction in regenerator matrix and pulse tube wall is neglected
- Reservoir pressure is considered as constant average pressure
- Pressure drop in pulse tube is neglected

The present model considers the simulation of working process in regenerator and pulse tube domain as shown in figure 1.

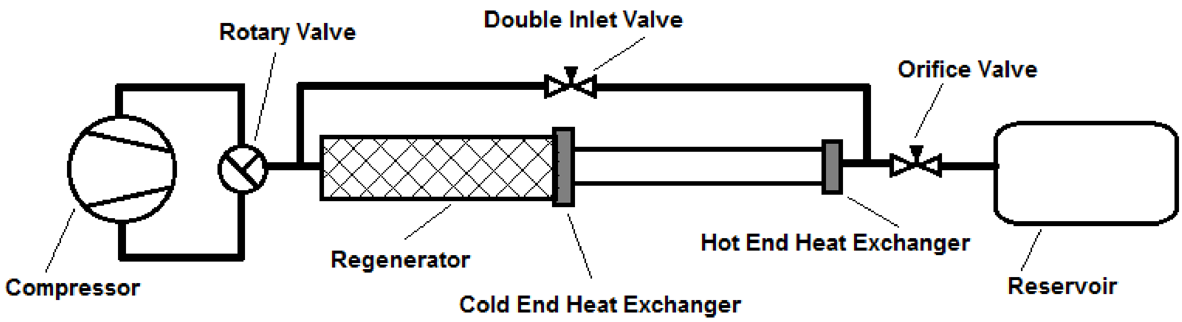

Figure 1. Schematic of single stage GM type double inlet pulse tube cryocooler.

2.1 Governing equations

Continuity equation for gas:

$$\frac{\partial m}{\partial t} = \dot{m}_{in} - \dot{m}_{out} \tag{1}$$

Momentum equation for gas:

$$\frac{\partial P}{\partial x} = -f \frac{1}{2D_h} \rho u^2 \tag{2}$$

The energy equation for gas

$$\frac{\partial (mC_pT)}{\partial t} + \frac{\partial (\dot{m}C_pT)}{\partial x} + \alpha A(T - T_w) - V\frac{dP}{dt} = 0 \tag{3}$$

Energy equation for Solid

$$\frac{\partial (C_wT_w)}{\partial t} = \alpha A(T - T_w) \tag{4}$$

Where m is mass of gas in a cell, ṁ is mass flow rate, u is velocity in x-direction, T is temperature of gas, P represents the pressure, D_h is hydraulic diameter, ρ is density, C_p is specific heat at constant pressure, α is heat transfer coefficient, V is volume of gas in a cell, *f* is friction factor, T_w is

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temperature of matrix for regenerator & wall temperature for pulse tube domain. C_w is heat capacity of regenerative material & pulse tube material. The simulation uses real gas properties of helium gas [8].

The above sets of governing equations are discretized by Finite Volume Method (FVM). Resulting sets of linear equations are solved at each discretized cell or node of domain. The domain consists of regenerator and pulse tube. Gauss Seidel iterative scheme (with under relaxation) is used to solve the resulting linear equations after discretization. Time derivative terms in above governing equations are discretized by first order Euler's method and Second Order Upwind (SOU) is used to discretize the advection terms. Pressure, temperature of gas and solid, density are calculated at node or cell centre while mass flow rate or velocity is calculated at face of cell.

2.2 Boundary Conditions

The numerical model requires boundary condition to define physical process occurring inside the pulse tube cryocooler. As the model is applied to GM type pulse tube cryocooler, pressure waveform generated by rotary valve is input to the model. This model considers the trapezoidal waveform to represent the pressure oscillations in this pulse tube cryocooler. The model requires the use of three boundary conditions viz. gas temperature, pressure and mass flow rate.

Hot end of regenerator is considered as left boundary of the domain and hot end of pulse tube is termed as right boundary of the domain. The boundary values of governing variables at these ends are considered as boundary conditions. Because of different cross sectional area of different parts of PTC, interface mass flow rate should be same. This is mathematically taken care of while calculating mass flow rate by continuity equation (1). In order to solve continuity equation for mass flow rate calculation, it necessary to calculate rate of change of mass in a given cell between two faces during a time step. The mass is calculated based on real gas property i.e. density (which is function of pressure and temperature) & corresponding volume of cell. If a cell belongs to regenerator, volume of cell should be based on regenerator parameters and the same applies to pulse tube and different part of PTC also.

Because of oscillating nature of flow, gas direction reverses during a cycle and due to this mass flow rate convention is considered as per the direction of flow. The mass flow rate from regenerator to pulse tube is termed as positive mass flow rate and for reverse direction; mass flow rate is taken negative.

2.2.1 Boundary value of temperature.

Temperature at left boundary (T_L) is given as follows:

T_L = Room temperature (for positive mass flow rate at left end)

T_L = Gas temperature of 1st cell of regenerator (for negative mass flow rate at left end)

The right boundary temperature (T_R) is given as follows:

T_R = Gas temperature of N_{th} cell of pulse tube (for positive mass flow rate at right end)

T_R = Temperature of reservoir (for negative mass flow rate at right end)

2.2.2 Pressure boundary condition. In GM type pulse tube cryocooler, rotary valve generates pressure waveform from a given valve timing diagram. Pressure boundary at left end is given by pressure waveform generated by rotary valve. The current work takes trapezoidal nature of waveform in consideration for pressure boundary.

2.2.3 Mass flow rate boundary condition. The right boundary of the domain is hot end of pulse tube and the amount of mass flow rate at last face of pulse tube domain is calculated from mass flow rate equation as given below [9]:

$$\dot{m}_o = C_d A_o \left[\frac{2P_N \left(P_N - P_{avg}\right)}{RT_N} \right]^{\frac{1}{2}} \quad \text{for } P_N > P_{avg} \tag{5}$$

$$\dot{m}_o = -C_d A_o \left[\frac{2P_{avg}\left(P_{Pavg} - P_N\right)}{RT_{res}} \right]^{\frac{1}{2}} \quad \text{for } P_N < P_{avg} \tag{6}$$

Where, C_d is coefficient of discharge, A_o is orifice area; P_N & T_N are pressure and temperature at N^{th} cell, P_{avg} is pressure inside the reservoir which is termed as constant pressure due to large volume compared to system volume. T_{res} is temperature of gas in reservoir or buffer. The mass flow rate through the double inlet valve is given by the same equations (using equations (5) and (6) which are used for orifice mass flow rate) by changing the corresponding end pressure values. The net mass flow rate through the hot end of pulse tube is algebraic difference of mass flow rate through orifice valve and double inlet valve.

2.3 Initial conditions

The model is of transient nature and thus, before start of numerical calculations, initial gas & wall temperature in pulse tube and gas & matrix temperature in regenerator domain is taken as room temperature. Initial pressure in the complete system is considered as average pressure. The initial mass flow rate at each face of cell can be solved from initial temperature and pressure data using continuity equation.

2.4 Solution methodology

Gauss Siedel iterative method is utilized to solve the linear equations generated by discretization of governing equations. The mass flow rate is obtained by solution of continuity equation and the latest value of mass flow rate is fed to momentum equation to get pressure value at a given cell. The energy equation for gas uses the latest updated value of mass flow rate and pressure to calculate the gas temperature. The energy equation for solid is coupled to energy equation for gas to calculate the solid wall temperature in pulse tube and matrix temperature in regenerator. The iterations are performed till one time step gets completed and the process is repeated till a cycle is finished. The repetition of such cycles is performed till the cycle steady state of temperature of gas is achieved.

3. Results and Discussion

The current transient model is capable of investigating the effects on no load temperature by effect of change in orifice & double inlet opening, operating frequency, geometry and pressure ratio. Using present transient numerical model, various numerical investigations on no load temperature are carried out with change in frequency, orifice opening and double inlet opening. With optimum frequency of 3.25 Hz and 0.477 mm optimum orifice opening, investigations are carried out for different double inlet opening. The figure 2, as representative data, shows the reasonable agreement between experimental results [10] & theoretical results. The input data for the results shown in figure 2 are taken from table 1.

Table 1. Input data for investigations.

Sr. No	Particulars	Value
1	Regenerator Length	210 mm
2	Regenerator ID	16.5 mm
3	Matrix material	#200 SS wire mesh
4	Pulse Tube Length	230 mm
5	Pulse Tube ID	15 mm
6	High pressure	20 bar
7	Low pressure	8 bar

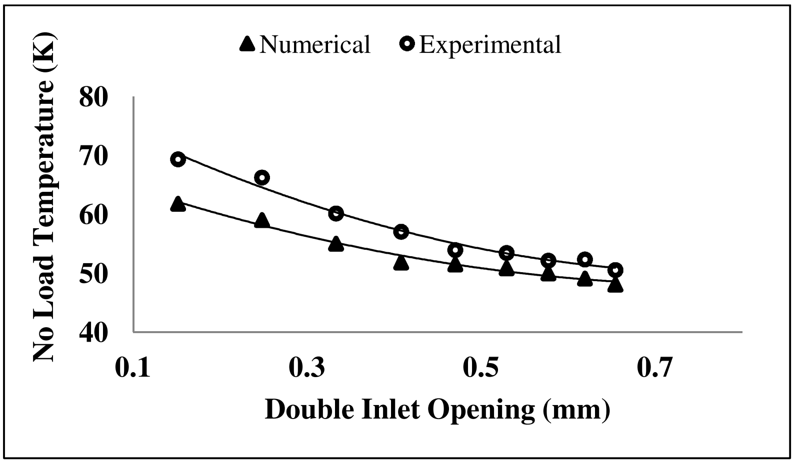

Figure 2. Comparison between experimental and theoretical results.

Typical cool down curve is shown in figure 3 which gives cooling down behaviour of cold end with no load condition. Theoretical investigations by present model are carried out and steady state results are plotted in subsequent results.

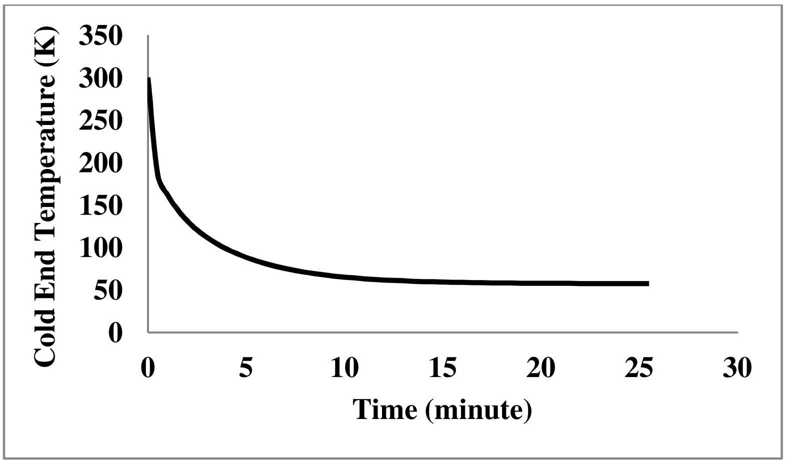

Figure 3. Cooling down history of cold end.

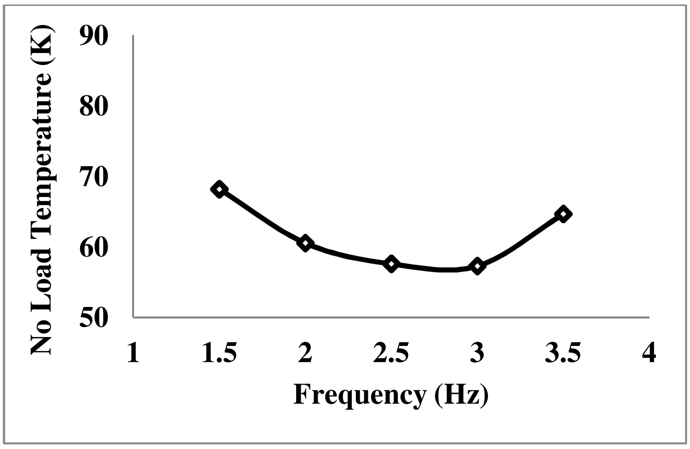

Figure 4. Effect of change in operating frequency on no load temperature.

The figure 4 shows the effect of change in operating frequency on cold end temperature under no load condition. The trend shows there is an optimum frequency corresponds to the minimum no load temperature. The figure 4 shows 3 Hz is an optimum operating frequency corresponds to the 57.24 K no load temperature. With this optimum frequency subsequent investigations are carried out.

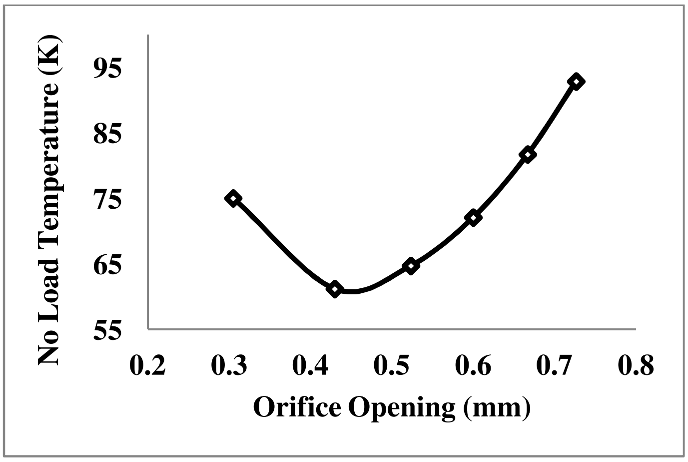

Figure 5. Effect of change in orifice opening on no load temperature.

Figure 5 shows the change in no load temperature corresponds to a given orifice opening/diameter (mm) keeping double inlet valve closed. It is found that 0.4298 mm orifice diameter gives the lowest temperature of 61.24 K. Once the operating frequency and orifice valve is set at optimum value, one can investigate the effect of double inlet valve opening. Such investigation is given in figure 6 which shows the optimum value of double inlet opening 0.5233 mm corresponding to minimum no load temperature.

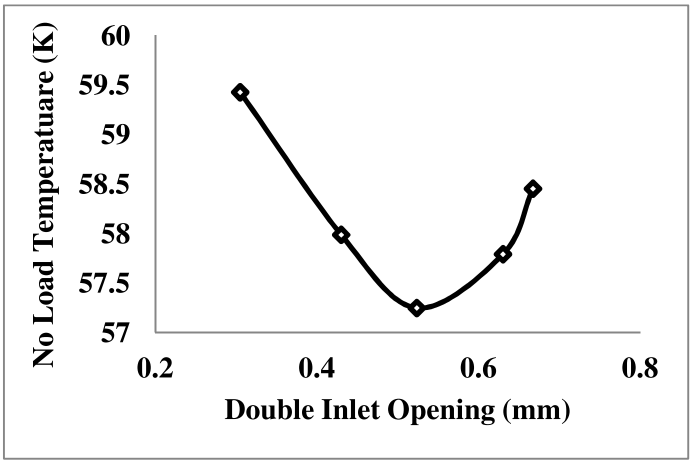

Figure 6. Effect of change in double inlet opening on no load temperature.

The figure 7 shows the effect of change in pressure ratio on cold end temperature. It is found that there exists an optimum pressure ratio for which no load temperature has lowest value.

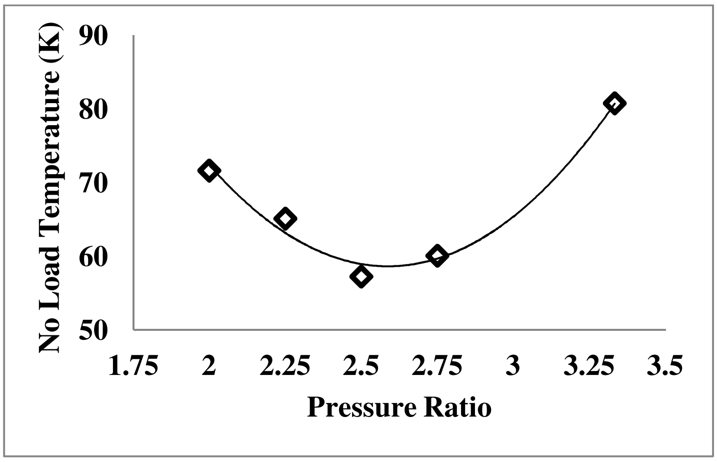

Figure 7. Effect of change in pressure ratio on no load temperature.

Table 2. Effect of change in pulse tube geometry.

Sr. No	Length, (mm)	Inner Diameter, (mm)	Cold end Temperature (K)
1	230	15	57.25
2	200	15	59.24
3	250	15	56.28
4	230	13	63.20
5	230	16	64.18

Table 2 shows the effect of change in length and internal diameter of pulse tube on no load temperature. By increase in length of pulse tube, cold end temperature with no load decreases while decrement in length gives increased cold end temperature. It is also found that change in inner diameter of pulse tube has corresponding change on no load temperature. Table 2 shows the variation of no load temperature corresponding to the change in inner diameter of pulse tube, which reveals that there exits an optimum value of inner diameter of pulse tube for a given no load temperature. Table 3 depicts the variation in geometry of regenerator and its effects on no load temperature of PTC. Change in geometry gives an optimum value of length and inner diameter for which there is a minimum temperature of cold end under zero loading condition.

Table 3. Effect of change in regenerator geometry.

Sr. No	Length, (mm)	Inner Diameter, (mm)	No load Temperature (K)
1	210	16.5	57.25
2	190	16.5	64.70
3	230	16.5	59.76
4	210	15	66.60
5	210	18	63.49

Table 4. Effect of change in pressure ratio on refrigeration temperature.

Sr. No	Pressure Ratio	Charging Pressure (bar)	Refrigeration Temperature at 1W (K)
1	2	15	75.63
2	2.25	13	68.90
3	2.5	14	61.07

Table 4 shows the effect of pressure ratio on refrigeration temperature for a given refrigeration power of 1 Watt and at different charging pressure. Pressure ratio and charging pressure are significant parameters for investigating the refrigeration power. It shows higher pressure ratio of 2.5 and optimum value of charging pressure 14 bar provides a given refrigeration power of 1 W at lower temperature i.e. 61.07 K

4. Conclusion

A transient numerical analysis for single stage GM type double inlet pulse tube cryocooler is presented here. The model can be used as design tool to investigate effect of geometrical parameters as well as capable of doing theoretical investigations for no load temperature by effect of change in pressure ratio, openings of orifice and double inlet valve. The model can also calculate the refrigeration power at given value of refrigeration temperature. With the proper boundary conditions, the present model can be extended to simulate working process in two stage GM type double inlet pulse tube cryocooler.

5. Acknowledgement

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