

Study on a cascade pulse tube cooler with energy recovery: new method for approaching Carnot

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Abstract: A pulse tube cryocooler (PTC) can not achieve Carnot efficiency because the expansion work must be dissipated at the warm end of the pulse tube. How to recover this amount of dissipated work is a key for improving the PTC efficiency. A cascade PTC consists of PTCs those are staged by transmission tubes in between, these can be a two-stage or even more stages, each stage is driven by the recovered work from the last stage by a well-designed long transmission tube. It is shown that the more stages it has, the closer the efficiency will approach the Carnot efficiency. A two-stage cascade pulse tube cooler consisted of a primary and a secondary stage working at 233 K is designed, fabricated and tested in our lab. Experimental results show that the efficiency is improved by 33% compared with the single stage PTC.

1. Introduction

A pulse tube cooler (PTC) has no moving parts at the cold end, thus it possesses advantages of low vibration, high reliability and long lifetime, which make it attractive for many applications^[1]. The pulse tube cooler mainly went through the basic type^[2], the orifice type^[3], the double-inlet type^[4] and the inertance tube type^[5], among which the basic type was low efficient compared with the others, while for the later three, different phase shifting devices were introduced in order to obtain a better phase relation between mass flow and pressure wave in the middle of the regenerator, thereby the cooling efficiency was improved little by little. However, all these phase shifting devices consume the acoustic power from the cold end of the pulse tube, which makes the ideal cooling efficiency of a PTC lower than the Carnot efficiency^[6]. So how to recover this amount of dissipated work is a key for improving the PTC efficiency.

In the past years, various configurations were proposed to overcome this issue, they can be divided into two different types. One is to recover this amount of work by introducing moving parts at the warm end of the pulse tube^[7-10], warm expander, for example. The other one is to recover this amount of work through a kind of loop configuration^[11-13]. Although the researches above provide some possible methods to recover the acoustic power from the cold end, these configurations either introduce moving parts thus weakening the great advantage of a PTC, or cause streaming through the looped configuration that deteriorates the cooling performance. In 2011, G. Swift et al. proposed a quarter-wavelength pulse tube cooler, in which a second pulse tube cooler was added after the quarter-wavelength pulse tube to use the expansion work^[14,15]. In this configuration, there were neither moving parts nor streaming

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brought in. Although this configuration achieved the desired result, the absence of the conventional pulse tube caused additional problems. For instance, the sudden-change in cross section between the regenerator and the pulse tube, and the viscous dissipation along this long, thin quarter-wavelength pulse tube lowered the cooling efficiency to some extent. Besides, the phase angles between the pressure waves and mass flows at both ends of this long pulse tube are fixed, which makes it inflexible in the design of a cascade PTC.

In this work, we propose a cascade pulse tube cooler capable of energy recovery. The cascade PTC consists of PTCs which are staged by transmission tubes in between. The pulse tube is reserved in each stage which makes it possible to run them in-series or independently. Each stage is driven by the recovered work from the last stage by means of the long transmission tube. It is possible to determine the length of the transmission tube according to the phase relation required. Theoretical analysis shows that the more stages it has, the closer the efficiency will approach to the Carnot efficiency. A two-stage cascade pulse tube cooler working at 233 K is optimally designed, and experimental results show that the cooling efficiency can be improved by 33%.

2. Theoretical analysis

Inspired by the quarter-wave pulse tube cooler from G. Swift, we first proposed a multi-stage cascade PTC, as shown in figure 1, it consists of n PTCs working under the same hot and cold end temperatures, they are staged one by one with transmission tubes in between (here n is a positive integer number not less than 2). Different from that of a quarter-wavelength pulse tube cooler, the pulse tube of each PTC is preserved to keep their functions of transferring PV work as well as thermally isolating the two ends. The function of the transmission tube is to reverse the phase relation between the mass flow and the pressure wave and to transfer the acoustic power as much as possible.

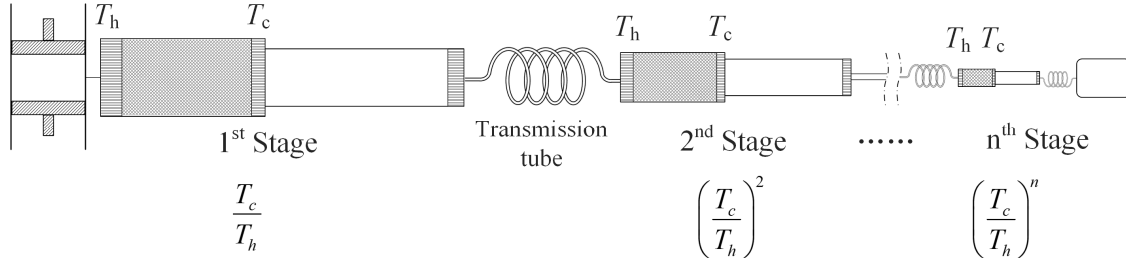


Figure 1. Diagram of a multi-stage cascade PTC

Assume the original acoustic power provided by the linear compressor is E_1 , and there is no loss in each PTC, thus the cooling efficiency of each stage is T_c/T_h . We can obtain the cooling power of the first stage:

$$Q_{c1} = E_1 \frac{T_c}{T_h} \quad (1)$$

For the secondary stage, the acoustic power inlet should be the same as the cooling power of the primary stage, that is:

$$E_2 = Q_{c1} = E_1 \frac{T_c}{T_h} \quad (2)$$

Thus the cooling power of the secondary stage is:

$$Q_{c2} = E_2 \frac{T_c}{T_h} = E_1 \left(\frac{T_c}{T_h} \right)^2 \quad (3)$$

Similarly, the cooling power of the n^{th} stage is:

$$Q_{cn} = E_{n-1} \frac{T_c}{T_h} = E_1 \left(\frac{T_c}{T_h} \right)^n \quad (4)$$

Then the total efficiency of the multi-stage cascade PTC is:

$$COP_n = \frac{Q_{c1} + Q_{c2} + \dots + Q_{cn}}{E_1} = \frac{T_c}{T_h} + \left(\frac{T_c}{T_h} \right)^2 + \left(\frac{T_c}{T_h} \right)^3 + \dots + \left(\frac{T_c}{T_h} \right)^n = \frac{T_c}{T_h - T_c} \left[1 - \left(\frac{T_c}{T_h} \right)^n \right] \quad (5)$$

This is a geometric progression, and if n goes to infinite, the COP will turn to:

$$COP_{n \rightarrow \infty} = \frac{T_c}{T_h - T_c} \quad (6)$$

That is, a cascade PTC with infinite stages could obtain Carnot efficiency. There may be many different ways to get to Carnot efficiency, equation (5) shows us one of such refrigeration methods which approaches Carnot efficiency step by step, and it is theoretically realizable learning that each term in equation (5) corresponds to a pulse tube cooler in figure 1. Figure 2 shows how the relative Carnot efficiency of the cascade PTC is improved gradually stage by stage for different working temperatures. We can see from figure 2 that in lower temperature region, the single stage PTC can almost get to the Carnot efficiency. As the cooling temperature goes up, the benefit of cascading stages becomes more and more noticeable, and more cascading stages are necessary in order to obtain Carnot efficiency. It is indicated from figure 2 that the cascade PTC can be applied to LNG (111 K) or sensors those working at higher temperatures (e.g. 150 K).

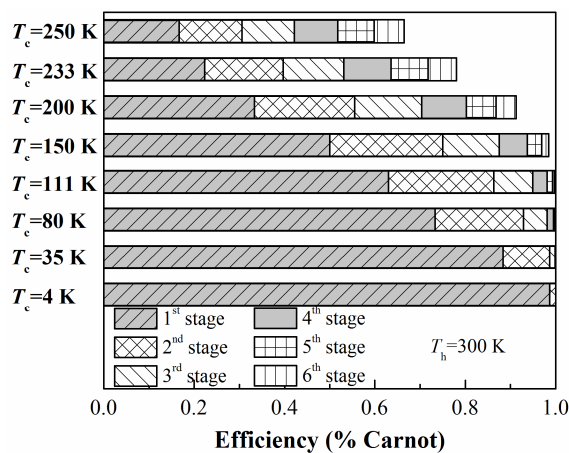


Figure 2. Percent of Carnot v.s. working temperature (multi-stage cascade PTC)

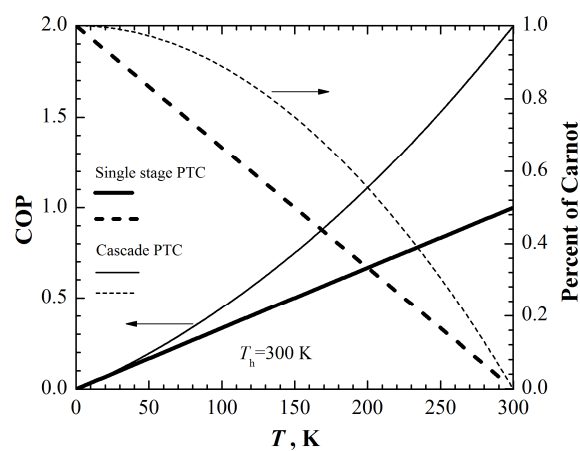


Figure 3. Efficiency v.s. working temperature (two-stage cascade PTC)

3. Design of a two-stage cascade PTC

To verify this idea, we begin from the two-stage cascade PTC. Figure 3 shows the comparison of cooling efficiency between a two-stage cascade PTC and a normal single stage PTC. It is shown that as the cooling temperature increases, the COP of single stage PTC and cascade PTC both increase, the higher the cooling temperature is, the more additional COP can be obtained by the cascade method. As for the relative Carnot efficiency, the cascade PTC also shows great potential improvement.

It should be pointed out that in practical cases, considering the dissipation of the acoustic power during the transfer process, for lower temperatures (see figure 2), or a pulse tube cooler with small cooling power, the benefit of recovering the acoustic power can be neglected. However, in higher temperature region, particularly for a pulse tube cooler with relatively high cooling power, the acoustic power at the cold end becomes worthwhile to be recovered. In addition, we used an existing linear compressor of model CFIC 2s132 (the rated power is 500 W at 60 Hz, 2.5 MPa charging pressure) in

our lab, so as try to obtain a relatively high cooling power. Therefore, a cooling temperature of 233 K (-40°C) was chosen for the first step. Based on software REGEN^[16] and Sage^[17], a two stage cascade pulse tube cooler was optimally designed.

The detailed design process of the primary and secondary stage PTC could be found in references[18,19]. It is known that the key to design a good PTC is to ensure a good phase distribution inside, that is, at the warm end of the regenerator, the mass flow leads the pressure wave, while at the cold end of the regenerator, the pressure wave leads the mass flow. Besides, reasonable amplitudes of both mass flow and pressure wave are necessary to obtain a good cooling performance. For a two-stage cascade PTC, it is even more difficult to meet these requirements for both primary and secondary PTCs respectively at the same time. Figure 4 shows the distributions of phases and amplitudes of mass flow and pressure wave as well as the energy flows along the x direction calculated from Sage. As both the transmission tube (7 m) and inertance tube (1.4 m) are much longer than the two PTCs (0.261 m of the primary stage and 0.289 m of the secondary stage), here the x -axis is divided into four parts.

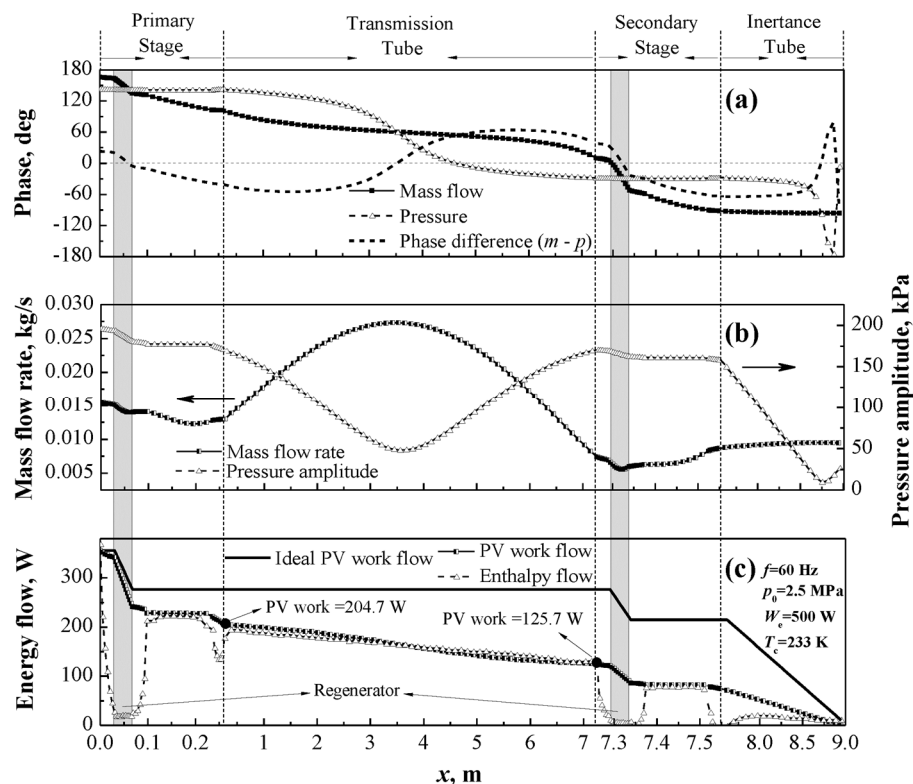


Figure 4. Distributions of phases (a), amplitudes (b) and energy flows (c)

It is shown from figure 4(a) that, for the mass flow, its phase always changes as long as there is void volume, and it changes more sharply inside regenerators where the temperature gradient enhances this effect. As for the pressure wave, its phase almost keeps constant in both stages of PTC, the variation of pressure wave phase happens inside both the transmission tube and inertance tube whose lengths are in the same order of magnitude with that of wavelength. Because the change of pressure wave phase is larger than that of the mass flow phase in the transmission tube, the phase relation that the mass flow lags the pressure wave at its inlet is turned to that the mass flow leads the pressure wave at the outlet, this makes it possible that good phase distributions in both stages of PTC are satisfied at the same time. Figure 5 illustrates the phasor diagram between mass flow and pressure wave in the whole cascade PTC. Here the two black vectors represent the pressure waves in the primary stage and the secondary stage, respectively, the light gray vectors with closed arrows represent the mass flow in the primary stage while the gray vectors with open arrows represent that in the secondary stage. It is shown that the pressure

wave of the primary stage is turned by 171.3 degrees while entering the secondary stage, at the same time the mass flow is turned by 97.7 degrees. This is how the phase difference ($m-p$) of -39.4 degrees at the inlet is turned to that of 34.2 degrees at the outlet.

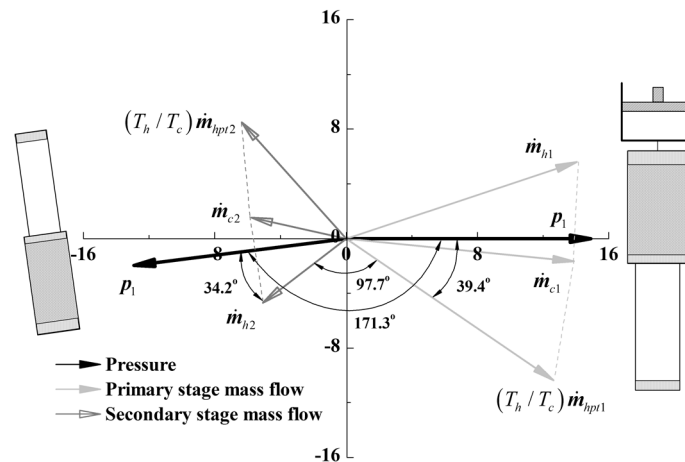


Figure 5. Phasor diagram of the cascade PTC

Figure 4(b) shows the amplitude distributions of mass flow and pressure wave. For the mass flow, its amplitude varies sharply along the transmission tube, first increases and then decreases in a wide range. It is known that the losses in a regenerator are proportional to the mass flow magnitude, thus relatively small mass flow amplitude means high efficiency. It is indicated from figure 4(b) that the two regenerators are located near the two antinodes of the mass flow wave. As for the pressure wave, its amplitude decreases progressively in both PTCs mainly due to the frictional loss. While it decreases first and then increases to a large extent along the transmission tube. Generally higher pressure amplitude results in better performance, and it is indicated from figure 4(b) that the two regenerators are located near the two peaks of the pressure wave. From this point of view, the cascade PTC is quite well-designed.

From figure 4(c) we can see that, both the PV work flow and enthalpy flow in the two stages of PTC are similar as that of a normal single-stage PTC. It is shown that the transmission tube is playing such an important role that 204.7 W PV work which should be dissipated at the warm end of the pulse tube of the primary PTC, is now recovered and transferred backwards, among which 79 W is consumed in this long tube while the other 125.7 W arrives the secondary PTC. Even so, if compared with the ideal PV work flow shown as the red line in figure 4(c), this cascade pulse tube cooler is still far from 'ideal' so far due to some inherent losses.

Table 1 lists the calculation results of both cascade PTC and single-stage PTC. It is shown that with the same 500 W electric power input, the cascade PTC is able to obtain a cooling power of 249.6 W, which is 27% higher than that of the single stage PTC.

Table 1. Simulation performance comparison of the single stage cooler and the cascade cooler

	Single stage PTC	Cascade PTC		
		Primary stage	Secondary stage	cascade
PV work input / W	389.9	379.1	125.7	379.1
Cooling power at 233 K / W	196.3	184.5	65.1	249.6
COP	0.3926	0.369	—	0.4992

4. Experimental verification

To demonstrate the feasibility of the basic idea, experiments were carried out. Based on the existing primary stage PTC (with 500 W electric power input, 181.3 W cooling power was obtained at 233 K)^[18], a cascade PTC setup is shown in figure 6, it consists of a linear compressor, a primary stage PTC, a

transmission tube and a secondary stage PTC.

The linear compressor is CFIC model 2s132. Non-vacuum expanded pearlite is used for thermal insulation. Three rhodium-iron resistance thermometers are mounted on the cold end of the primary stage PTC, while two platinum resistance thermometers are mounted on the cold end of the secondary stage PTC, all with accuracy of ± 0.1 K. Four pressure sensors are employed to measure the pressures at the compressor back space, the compression space, the inlet of the secondary stage and the entrance of the inertance tube, shown as P_1 , P_2 , P_3 and P_4 in Fig. 7, for P_1 , P_3 and P_4 , KISTLER 601B1 piezoelectric pressure transducers are used to measure the amplitude and phase of the oscillating pressure, while for P_2 , Entran EPX piezoresistive pressure transducer is used to measure the dynamic as well as the mean pressure. For the measurement of cooling power, four $50\ \Omega$ resistors capable of providing 200 W heating power are installed on the cold end of the primary stage, while another three $50\ \Omega$ resistors capable of providing 150 W heating power were installed on the cold end of the secondary stage.

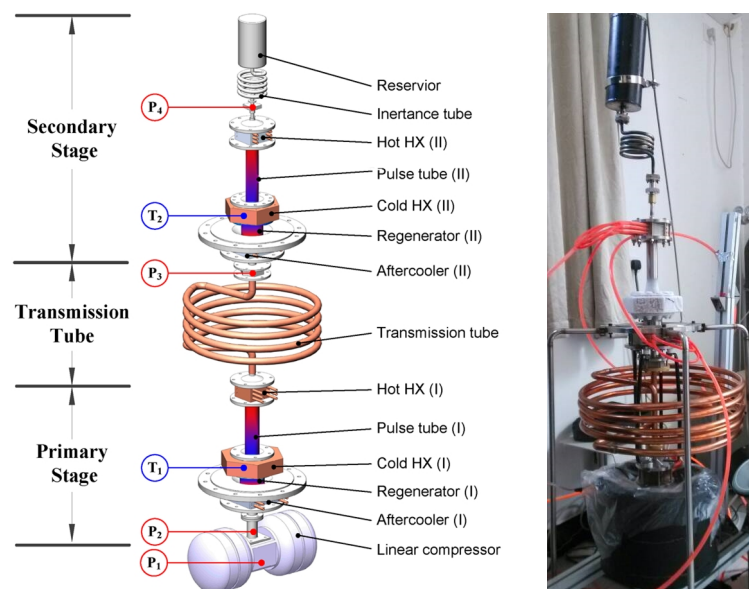


Figure 6. 3-D drawing (left) and picture (right) of the two-stage cascade PTC

Table 2 lists the main parameters of the cascade PTC, in which 200 mesh stainless steel matrix with a porosity of 0.6704 is applied in the two regenerators. Also, in order to compare the cooling performance of single-stage and cascade PTC, a special inertance tube and reservoir are designed for the primary stage.

Table 2. Main parameters of the cascade PTC

	Part Name	Dimension
Primary Stage	Regenerator	53.7 mm i.d., 37.7 mm long
	Pulse tube	30.5 mm i.d., 134.3 mm long
	Inertance tube (only in single-stage operation)	8 mm i.d., 2.67 m long
	Reservoir (only in single-stage operation)	450 cm ³
Transmission tube		14.2 mm i.d., 7 m long
Secondary Stage	Regenerator	47.6 mm i.d., 48 mm long
	Pulse tube	27 mm i.d., 150 mm long
	Inertance tube	6 mm i.d., 1.4 m long
	Reservoir	1000 cm ³

Figure 7 shows the cool-down curves of the PTC in single-stage and cascade operations. It is shown that the lowest temperature of the single-stage is 99.7 K, while during cascade operation, the primary

stage finally reaches 120.8 K and the secondary stage 127.2 K.

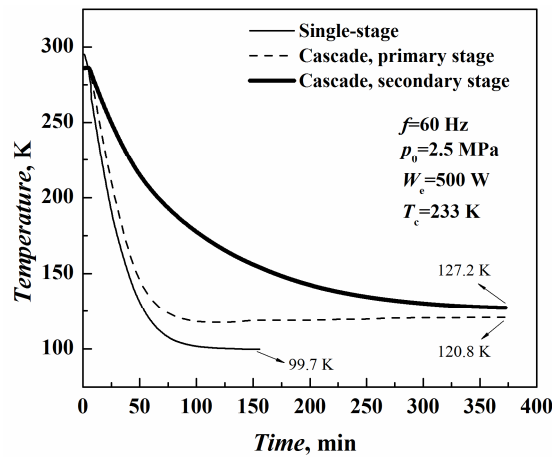


Figure 7. Cool-down curves

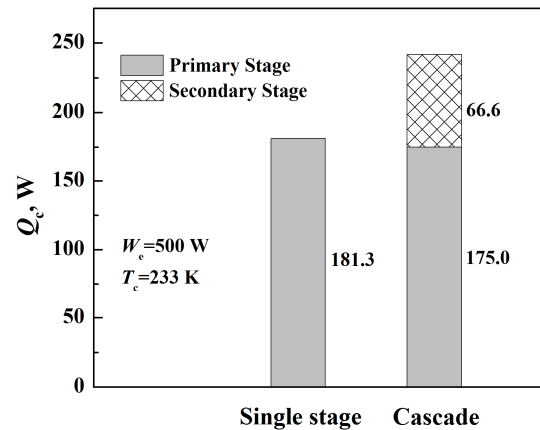


Figure 8. Comparison of cooling power

Figure 8 shows the cooling power at 233 K of the single-stage and cascade PTC. Here the electric power input to the linear compressor is fixed at 500 W. It is shown that the single-stage PTC can obtain a cooling power of 181.3 W, and for cascade PTC, the cooling powers of the primary and secondary stages are 175.0 W and 66.6 W, respectively, making a total cooling capacity of 241.6 W. The cooling efficiency is improved by 33%, which positively demonstrates the feasibility of the cascade concept. In this work, the cooling temperature is higher than the cryogenic temperature (≤ 120 K) reflecting the existing conditions in our lab, but for most cryogenic applications such as HTS and LNG, lower temperatures and even higher cooling capacities are required, in such cases, the cascade PTC can be attractive.

The pressure waves at different positions shown in figure 6 are measured at 233 K under cascade operation, as shown in figure 9. It is shown that the phase angle between P_1 and P_2 is 131.9 degrees, according to P_1 and P_2 , an acoustic power of around 396 W delivered from the linear compressor can be calculated^[20], thus the efficiency of the linear compressor is about 79% (converting electric power into acoustic power), indicating a quite good impedance match between the compressor and the cascade PTC. What is more important, it is also shown in figure 9 that the measured phase difference between P_2 and P_3 , that is between the primary and secondary stages is around 175.1 degrees, which accords well with the calculation result (see figure 5). This verifies the phase reversion function of the transmission tube, which is a key design in the whole system.

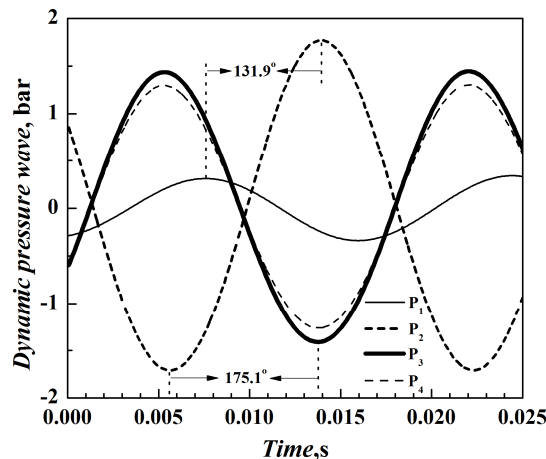


Figure 9. Measured pressure waves

5. Conclusion

A multi-stage cascade PTC is proposed, theoretical analysis shows that the more stages it has, the closer its efficiency will approach to the Carnot efficiency. A two-stage cascade PTC is designed and tested, simulation results show that 125.7 W of the 204.7 W PV work at the warm end of the pulse tube could be recovered for driving the secondary PTC, which brings an extra cooling efficiency of 27%. And experimental results show that, with 500 W electric power input, the cascade PTC obtains a total cooling power of 241.6 W at 233 K, the cooling efficiency is increased by 33%. This work may be extended to high power pulse tube cryocoolers working at much lower temperatures.

Acknowledgments

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