Damping Properties of Sandwich Truss Core Structures by Strain Energy Method

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Abstract.   Sandwich panel structures with stiff face sheets and cellular cores are widely used to support dynamic loads. Combining face sheets made of carbon fibre reinforced plastics (CFRPs) with an aluminium pyramidal truss improves the damping performance of the structure due to viscoelastic character of CRFP composites. To predict the damping characteristics of the pyramidal truss core sandwich panel the strain energy method is adopted. The procedure for evaluating the damping of the sandwich panel was performed using commercial finite element software NASTRAN and MATLAB. Non-contact vibration tests were performed on the real sandwich panels in order to extract the modal characteristics and compare them with the numerical predictions.

1. Introduction

Sandwich structures with a pyramidal truss core are widely used in the form of plates, beams, or cylindrical shells. Their principal advantage is the high specific stiffness and strength ratio with respect to mass. The strength and stiffness characteristics of the sandwich structures have been already studied exclusively [1, 2]. The exceptional stiffness properties of the sandwich structures are due to high bending stiffness of the face sheets and high shear stiffness of the core. Little work, however, was done on the dynamic response of truss core sandwich structures which includes both resonant and damping characteristics [3, 4]. The present work aims to predict the modal loss factors of the sandwich panels made of two parent materials: carbon fibre reinforced plastics (CFRPs) and an aluminium alloy. To predict the damping characteristics the strain energy method was used [5, 6, 7] and combined with the finite element solutions. To validate the numerical predictions the non-contact vibration tests were performed on the real sandwich panels in order to extract their dynamic characteristics.

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2. Materials and samples

Damping properties of a pyramidal truss core sandwich panel was investigated in the present study. Three sandwich panels were manufactured for the experimental purposes. The panels had length l, width a, and hight h (Table 1). A global Cartesian co-ordinate system (x, y, z) was located in the corner of the panel, with z axis upward and normal to the face layer (Figure 1). Each sandwich panel was composed of two parent components: (1) a laminated carbon fibre reinforced plastic (CFRP) was used for the face sheets; (2) the aluminium alloy PA6 was used for the pyramidal truss core. The face sheets were cut out of a long unidirectional CRFP tape, which were proven to be good candidates for structural stiffness improvement [8]. Additionally, the CFRP composites improves the damping performance of the whole structure due to viscoelastic character of the matrix. The face sheets were composed of 8 layers having material properties as listed in Table 2 and the total thickness t. For each layer the lamina co-ordinate system $(1, 2, 3)$ was defined with the direction 1 along the lamina fibres, 2 transverse to this direction, and 3 - through the thickness direction. A lamination angle (Φ) of a single layer was defined between the x-axis and the 1-axis. The pyramidal truss core was assembled from separate pieces which were cut out of an aluminium plate by means of water-jet cutting. The pieces were bonded together with a thermoset epoxy adhesive to form a continuous pyramidal truss core. The material properties of PA6 alloy are given in Table 2. The parent components were bonded together to form a sandwich panel with the same thermoset epoxy adhesive as used for the core manufacture. The epoxy adhesive was applied locally on the face sheets where the contact with the core existed.

3. Damping model

The damping prediction of truss core sandwich panel is evaluated using the modal strain energy method [5, 7]. According to the method, the specific damping capacity (SDC) ψ_n is defined as the ratio of the total dissipated energy ΔU_n to the maximum strain energy U_n stored in a structure during a stress cycle at n^{th} mode of vibration:

$$\psi_n = \frac{\Delta U_n}{U_n} = 4\pi\zeta_n = 2\pi\eta_n \tag{1}$$

where η is the modal loss factor and ζ is the damping ratio.

3.1. Strain energy

It is proposed to express the total modal damping of the sandwich panel as the sum of the damping fractions form the individual parent components. The energy stored in the structure is invariant of the coordinate system, therefore the strain energy of each parent component is considered in a different coordinate system. The total strain energy stored in a sandwich panel may be written as (the mode

Table 1. Sandwich panels used in the experimental investigation.

Sample	l [mm]	a [mm]	h [mm]	t [mm]	Φ [deg]
Sandwich-1	380.3	50.0	27.8	1.4	0
Sandwich-2	380.1	50.0	27.8	1.4	0
Sandwich-3	380.2	50.0	27.8	1.4	0
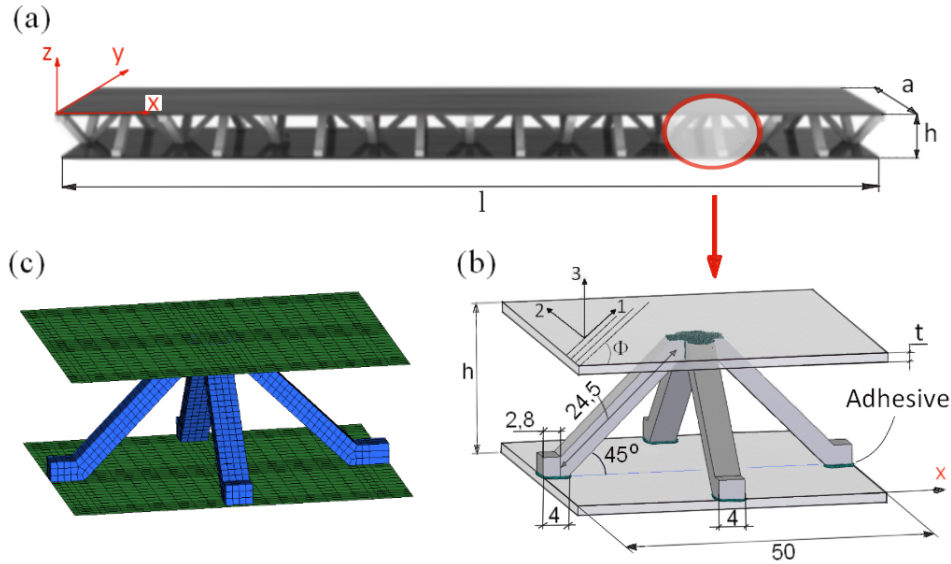
Figure 1. Sandwich panel with pyramidal truss core: (a) Manufactured sample; (b) Unit cell of the truss core sandwich panel; (c) Finite element model of the unit cell - dimensions in [mm].

number has been dropped for a notation convenience):

$$^sU = {}^lU + {}^cU \tag{2}$$

where ^lU and ^cU are the strain energy stored in the laminated face sheets and the aluminium core, respectively. The strain energy stored in a single finite element e of the thin laminated face sheets is the sum of the energies stored in the given lamina directions:

$$^lU^e = {}^lU_{11}^e + {}^lU_{22}^e + {}^lU_{12}^e \tag{3}$$

For a finite element composed of i layers, the element's energies are:

$$^lU_{11}^e = \sum_{k=1}^{i} {}^lU_{11,k}^e; \quad {}^lU_{22}^e = \sum_{k=1}^{i} {}^lU_{22,k}^e; \quad {}^lU_{12}^e = \sum_{k=1}^{i} {}^lU_{12,k}^e \tag{4}$$

where $^lU_{ij,k}^e \, (ij = 11, 22, 12)$ is the strain energy stored in the kth layer of element e, and is related to the strain and stress components as:

$$^lU_{ij,k}^e = \int_{V_k} \{\varepsilon_{ij}\}_k^T \{\sigma_{ij}\}_k dV_k \tag{5}$$

where V_k is the layer volume. Thus the total strain energy stored in the laminated face sheets made of M elements is:

$$^lU = {}^lU_{11} + {}^lU_{22} + {}^lU_{12} \tag{6}$$

where:

$$^lU_{11} = \sum_{e=1}^{M} {}^lU_{11}^e; \quad {}^lU_{22} = \sum_{e=1}^{M} {}^lU_{22}^e; \quad {}^lU_{12} = \sum_{e=1}^{M} {}^lU_{12}^e \tag{7}$$

The aluminium truss core is mostly subject to the transverse shear stress [4]. Therefore the strain energy stored in the finite element p of the core, in the global coordinate system, is given as:

$$^cU^p = {}^cU^p_{xz} + {}^cU^p_{yz} \tag{8}$$

And the total transverse shear strain energy stored in the core made of J elements is:

$$^cU = {}^cU_{xz} + {}^cU_{yz} \tag{9}$$

where

$$^cU_{xz} = \sum_{p=1}^{J} {}^cU^p_{xz}; \quad {}^cU_{yz} = \sum_{p=1}^{J} {}^cU^p_{yz} \tag{10}$$

The total strain energy of the sandwich panel is given as:

$$^sU = {}^lU_{11} + {}^lU_{22} + {}^lU_{12} + {}^cU_{xz} + {}^cU_{yz} \tag{11}$$

3.2. Dissipated energy

The energy dissipated by k^{th} layer of the element e is given as:

$$\Delta^l U^e_k = \psi_{11,k} {}^lU^e_{11,k} + \psi_{22,k} {}^lU^e_{22,k} + \psi_{12,k} {}^lU^e_{12,k} \tag{12}$$

where $\psi_{ij,k}$ $(ij = 11, 22, 12)$ are the specific damping capacities of the material used for layer k. The energy dissipated by element e is given as:

$$\Delta^l U^e = \sum_{k=1}^{i} \Delta^l U^e_k \tag{13}$$

And the total dissipated energy by the laminated face sheets is:

$$\Delta^l U = \sum_{e=1}^{M} \Delta^l U^e \tag{14}$$

Accordingly, the energy dissipated by element p of the aluminium truss core is given as:

$$\Delta^c U^p = \psi_{xz} {}^cU^p_{xz} + \psi_{yz} {}^cU^p_{yz} \tag{15}$$

For the isotropic material $\psi_{xy} = \psi_{xz} = \psi_{yz} = \psi_G$, and therefore Eq. 15 simplifies to:

$$\Delta^c U^p = \psi_G \left({}^cU^p_{xz} + {}^cU^p_{yz} \right) \tag{16}$$

The total dissipated energy by the aluminium truss core is:

$$\Delta^c U = \sum_{p=1}^{J} \Delta^c U^p \tag{17}$$

Thus, the total dissipated energy by the sandwich panel is given as:

$$\Delta^s U = \Delta^c U + \Delta^l U \tag{18}$$

The specific damping capacity of the sandwich panel which deforms at the nth mode of vibration, is as follows:

$$^s\psi_n = \frac{\Delta^s U_n}{{}^sU_n} \tag{19}$$

4. Numerical model

The finite element (FE) method was used to develop the numerical model of the sandwich panel. The FE model of the laminated face layers based on the First Order Shear Deformation Theory (FSDT). Shell layered linear elastic elements CQUAD4 were used to model the laminated face sheets. 3D CHEXA linear elastic elements were used to develop the FE model of the aluminium truss core. The face layers and truss core elements were connected together by node fit at the locations where the contact spots between the elements existed (Figure 1). The epoxy adhesive was not modelled in the present study. The eigenvalue problem for undamped free vibrations of the sandwich panel was represented by $\left(\mathbf{K} - \omega_n^2 \mathbf{M}\right) \Psi_n = 0$, where $\mathbf{K}$ and $\mathbf{M}$ are the global stiffness and mass matrices of sandwich panel, respectively; Ψ_n are the eigenvectors (mode shapes) corresponding to eigenvalues $\omega_n = 2\pi f_n$, with f_n being eigenfrequencies. The commercial finite element code NASTRAN was used to solve the eigenvalue problem. The Real Eigenvalue Analysis using Lanczos eigensolver was applied to determine eigenvalues and corresponding eigenvectors of the undamped vibrations of the FFFF sandwich panel (all edges free).
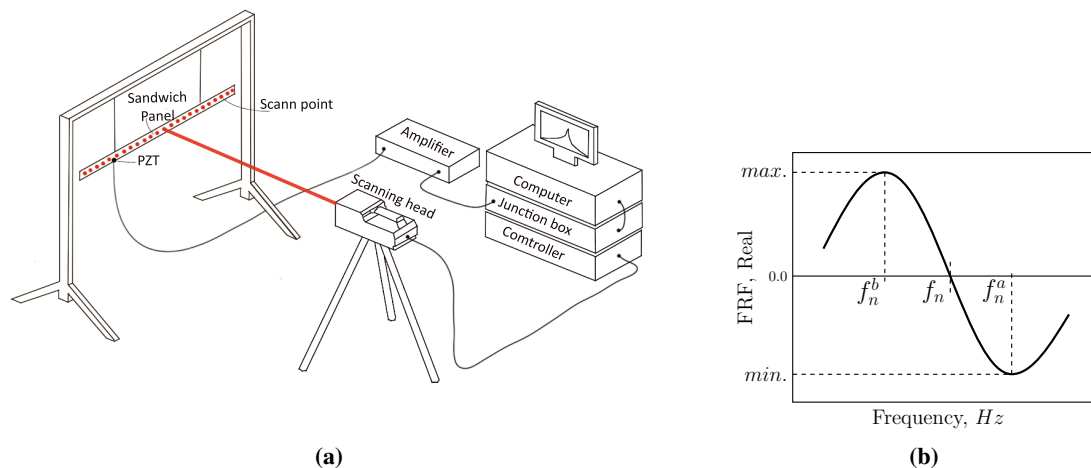
5. Experimental procedure

In the present investigation a non-contact method for vibration sensing is applied using a scanning laser vibrometer POLYTEC PSV-400-B (Figure 2a). The PSV-400-B system consists of: the scanning head PSV-I-400 LR; the controller OFV-5000 (with internal signal generator), the junction box PSV-E-400, PC with data acquisition board and PSV Software 8.8. Additionally the power amplifier Bruel&Kjaer type 2732 is employed and a piezoelectric disc actuator (PZT) or loudspeaker is used to drive a structure into vibrations. The required data from the vibration test are frequency response functions (FRFs) which serve for the estimation of modal loss factors (Figure 2b). The comprehensive description of the operating procedure of the PSV-400-B system and its proper assemblage has been given elsewhere [9, 10, 11]. The estimation of the loss factor is based on the real part of the measured mobility FRF. The sandwich panels' loss factor ($^s\eta_n$) for the mode with resonance frequency (f_n) are calculated as:

$$^s\eta_n = \frac{1 - \left(\frac{f_n^b}{f_n^a}\right)^2}{1 + \left(\frac{f_n^b}{f_n^a}\right)^2} \tag{20}$$

where n stands for a mode number, f_n^a and f_n^b are frequencies estimated from the real part of the FRF (Figure 4). Prior to the damping prediction of the sandwich panel, the constitutive stiffness and damping data of the parent components were estimated. To do so, the resonant method [12] was used to estimate the stiffness properties. Laser vibrometer was used for vibration sensing and loudspeaker for the excitation purposes. The dynamic results from the resonant method were then combined with

Table 2. Properties of the constituent materials.

Material	E_1[GPa]	E_2[GPa]	G_{12}[GPa]	ν_{12}	ν	ρ[kg/m^3]	ψ_{11}[%]	ψ_{22}[%]	ψ_{12}[%]	ψ_{xy}[%]
CFRP	142.2	10.5	7.8	0.38	-	1540.0	0.43	5.95	6.62	-
PA6	66.2	-	-	-	0.25	2700.0	-	-	-	0.31

<center>(a)   (b)</center>

Figure 2. Experimental procedure: (a) Experiment set-up for vibration testing; (b) Data acquisition for loss factor estimation.

energy method as proposed in [3] to find the specific damping capacities of the parent components. Seven beams made of CFRP with unidirectional alignment of fibres as $[0]_8$ and 4 aluminum beams were used to estimate the constitutive properties (Table 2).

6. Damping prediction

The finite element solution on the FE model of the sandwich panel is performed at first. The selected eigenfrequencies and eigenvectors are then extracted - first 3 bending modes. The strain and stress components corresponding to selected eigenvectors are stored in a text file. To do so, each shell element has been decomposed to separate layers and ε_{ij} and σ_{ij} of the layers were extracted. In addition, the element dimensions were preserved. Since the layer thickness was known, the volume of the layer was calculated and multiplied by the strain and stress components giving the strain energy of the layer. Summing up all the layers of all the elements gave the strain energy stored in the face layers of the panel. The procedure was repeated for the core, with the exception that no element decomposition was required and only shear strain and stress components were stored. The total strain energy of the panel is the sum of the energies stored in the face sheets and truss core. Then, the specific damping capacities of each layer and core material are used to calculate the dissipated energy components. The specific damping capacity of the sandwich panel is then calculated according to Eq.19 and the modal loss factor according to Eq.1. The postcalculations of the finite element data were performed using MATLAB software.

7. Results and conclusions

Three sandwich panels were subjected to vibration tests in order to extract the real part of the FRFs. The excitation was provided by the PZT disc. The necessary frequency information was then stored and the loss factors of the first three bending modes (Figure 3) were calculated according to Eq.20. The mean value of the loss factor was calculated and compared with the corresponding values predicted numerically (Table 3). A good coincidence of the results can be read from Table 3, although the predicted eigenfrequencies are overestimated in most cases and all the loss factors are underestimated. The major source of the results discrepancy is the lack of the epoxy adhesive included in the model. The

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energy dissipated by the adhesive makes the experimental loss factors higher than the predicted ones. Additionally, the lack of adhesive makes the model stiffer and increases the predicted eigenfrequencies. From the experimental point of view, the major drawback arises from the lack of vacuum chamber during vibration tests. In such case the loss factors increase due to air - structure interaction. Finally, the proper estimation of both stiffness and damping properties of the constituents encloses the sources of results discrepancy. Summing up, the numerical model improvement for the adhesive material and the tests done in a vacuum chamber should bring better results agreement.
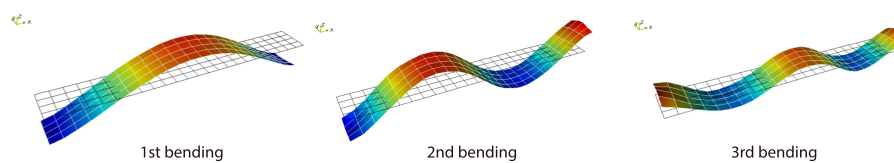
Figure 3. Experimental mode shapes of the truss core sandwich panel - Sandwich-2.
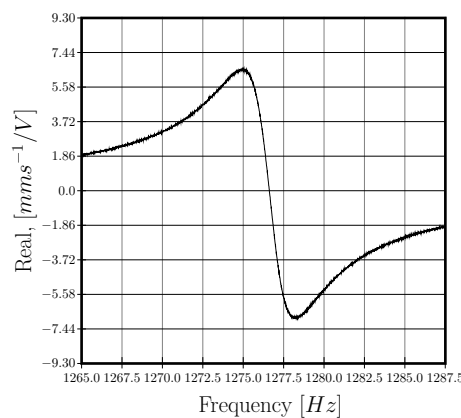
Figure 4. Zoomed fraction of the real FRF for the first bending mode of the Sandwich-1 panel.

Table 3. Numerical and Experimental results.

| | Experiment | | | | | | | Numerical prediction | |
| | Sandwich-1 | | Sandwich-2 | | Sandwich-3 | | Mean | FEM model | |
Mode	$f_n[Hz]$	$^s\eta_n[\%]$	$f_n[Hz]$	$^s\eta_n[\%]$	$f_n[Hz]$	$^s\eta_n[\%]$	$^s\eta_n[\%]$	$f_n[Hz]$	$^s\eta_n[\%]$
1	1296.89	0.32	1277.55	0.36	1274.50	0.47	0.38	1299.50	0.35
2	2172.52	0.70	2132.80	0.69	2117.35	0.75	0.71	2166.30	0.60
3	2927.66	0.88	2861.88	0.87	2838.75	0.77	0.84	2764.30	0.75

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