IOP Conf. Series: Materials Science and Engineering **66** (2014) 012007 doi:10.1088/1757-899X/66/1/012007

Circuits for differential signal extraction in the lock-in amplifiers

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Abstract. Confirmation of the metrological characteristics of measuring transducers provides maximum accuracy of the method of comparison with the standard instrument. Practical application of this technique is impossible without accurate instruments. As a measurement instrument to ensure the resolution of the order of several nanovolts when comparing two AC signals over a wide dynamic range, widely used lock-in amplifier with a differential input. The paper presents the results of the analysis of circuits based on operational amplifiers for comparing two signals and extraction differential signal in lock-in amplifier with a differential input. Common-mode rejection ratio for each of the circuits was estimated.

1. Introduction
Upon confirmation of the metrological characteristics of measuring transducers provides maximum accuracy of the method of comparison with the standard instrument. Practical application of this technique is impossible without accurate instruments of compare, resolution which largely determines the minimum measurement error. As a measurement instrument to ensure the resolution of the order of several nanovolts when comparing two AC signals over a wide dynamic range, widely used lock-in amplifier with a differential input (LIA).

The input stage of LIA providing simultaneous comparison of the two voltages and the extraction of the differential signal is the key terms of increased the dynamic range compared voltages and maximum resolution.

The paper presents the results of the analysis of circuits based on operational amplifiers for signal comparison and measurement of the differential signal in LIA.

2. Extraction circuits of differential signal
In modern LIA with a differential input used circuits based on operational amplifiers (op-amp) to extraction the differential signal [1-3].

Output voltage U_{out} of the op-amp will considered as a function of the differential U_d and common-mode U_{cm} voltages at the inputs of op-amp.

$$U_{out} = f\left(U_d, U_{cm}\right) = K_d U_d + K_{cm} U_{cm}, \tag{1}$$

where K_d – the gain of the differential signal;

K_{cm} – common-mode signal gain.

Solving equation (1) relative to the differential signal U_d we are get:

$$U_d = \frac{U_{out}}{K_d} - \frac{U_{cm}}{K_{CMMR}} = \begin{cases} U_{out} / K_d \text{ if } U_{cm} = 0 \\ -U_{cm} / K_{CMRR} \text{ if } U_{out} = 0 \end{cases}, \tag{2}$$

where $K_{CMRR} = K_d / K_{cm}$ – common-mode rejection ratio (CMRR).

CMRR in comparison circuits based on the op-amp depends on two components: accuracy ratio of resistance in signal lines and the internal structure of the specific op-amp.

For different extraction circuits of the differential signal based on op-amps will evaluate CMRR. Since the value of CMRR determines the maximum resolution when comparing two similar signals. Analysis of the circuits will be carried out at DC with internal CMRR → ∞.

2.1 Subtraction circuit using summing op-amp

Subtraction signals can be accomplished by adding to the inverted one of the input signals [4]. The circuits carries out this operation is shown in figure 1.

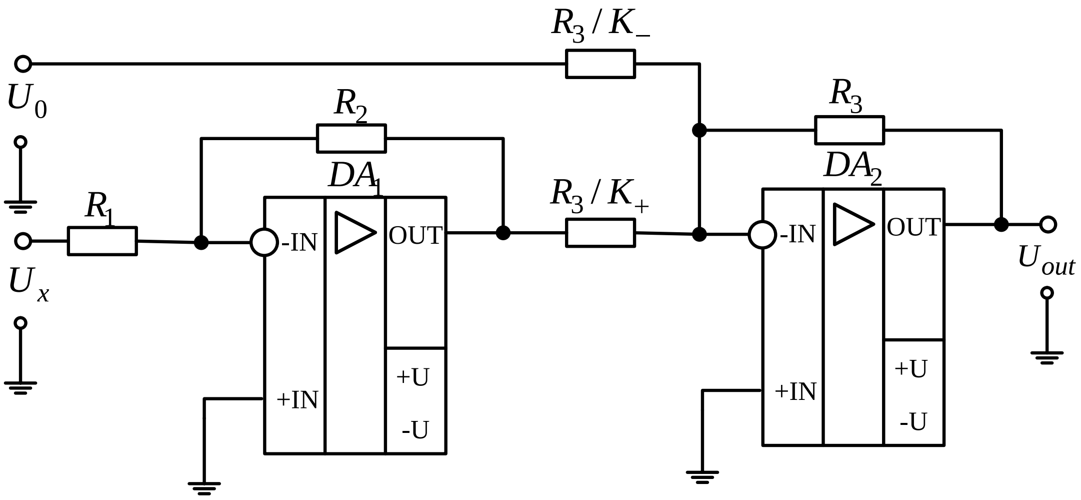

Figure 1. Subtraction circuit using summing op-amp

Op-amp DA_1 inverts the input voltage U_x, on condition of equality resistors R_1 and R_2 at the output voltage is:

$$U_{out} = K_+ U_x - K_- U_0. \tag{3}$$

Input signals represent as the sum of the common-mode and differential voltages in the inputs of op-amp DA_2 (4) and substitute in (3) to estimate the CMRR.

$$U_x = U_{cm} + \frac{U_d}{2} \text{ and } U_0 = U_{cm} - \frac{U_d}{2}. \tag{4}$$

$$\begin{gathered} U_{out} = (K_+ - K_-) U_{cm} + \frac{(K_+ + K_-)}{2} U_d; \\ K_{cm} = K_+ - K_-; \\ K_d = \frac{K_+ + K_-}{2}. \end{gathered} \tag{5}$$

From equation (5) CMRR is:

$$K_{CMRR} = \frac{1}{2} \cdot \frac{K_+ + K_-}{K_+ - K_-}. \tag{6}$$

If the resistance R_3/K_+ and R_3/K_- differ by value ΔK_d then:

$$K_+ = K_d + \frac{\Delta K_d}{2};\qquad K_- = K_d - \frac{\Delta K_d}{2}. \tag{7}$$

Then, substituting (7) in (6):

$$K_{cmrr} = \frac{K_d}{\Delta K_d}. \tag{8}$$

Consequently, the common-mode rejection ratio is inversely proportional to the difference between the resistances R_3/K_+ and R_3/K_-.

2.2 Subtraction circuit using one op-amp

Figure 2 shows the subtraction circuit using one op-amp.

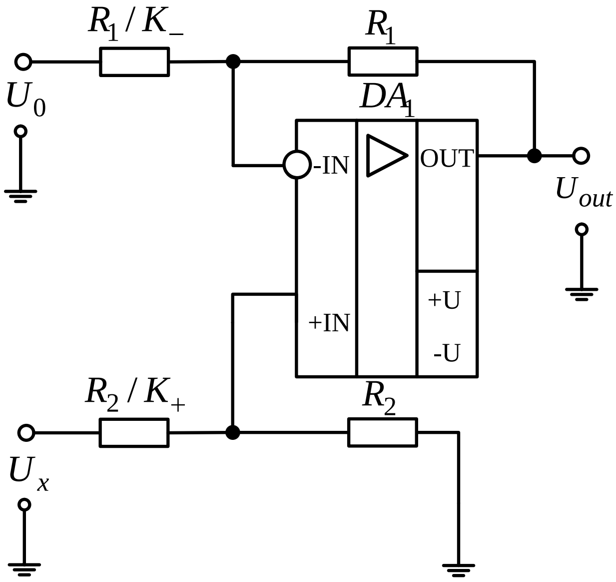

Figure 2. Subtraction circuit using one op-amp

Superposition is used to calculate the output voltage resulting from each input voltage:

$$U_{out} = \frac{(1+K_-)}{(1+K_+)} \cdot K_+ U_x - K_- U_0. \tag{9}$$

To estimate CMRR substituting (4) in (9):

$$U_{out} = \frac{(K_+ - K_-)}{(1+K_+)} U_{cm} + \frac{(K_+ + K_- + 2K_+K_-)}{2(1+K_+)} U_d;$$
$$K_{cm} = \frac{(K_+ - K_-)}{(1+K_+)}; \tag{10}$$
$$K_d = \frac{(K_+ + K_- + 2K_+K_-)}{2(1+K_+)}.$$

From equation (10) CMRR is:

$$K_{CMRR} = \frac{K_d}{K_{cm}} = \frac{1}{2} \cdot \frac{(1+K_-)K_+ + (1+K_+)K_-}{(1+K_-)K_+ - (1+K_+)K_-}. \qquad (11)$$

If the resistance R_2/K_+ and R_1/K_- differ by value ΔK_d then substituting (7) in (11):

$$K_{CMRR} \approx (1+K_d)\frac{K_d}{\Delta K_d}. \qquad (12)$$

Consequently, at constant K_d common-mode rejection is inversely proportional difference resistances R_2/K_+ and R_1/K_- and directly proportionally to the gain of the differential signal. Thus, in the above circuits CMRR will depend on the accuracy of the ratio of resistances in the signal lines and on the sources output impedance of the compared signals.

2.3 Subtraction circuit using instrumentation amplifier

The instrumentation amplifier (IS) [5-6] is a precision amplifier unit with a differential input and closed-loop. IS provides a gain of the voltages difference between the two input signals, weakening any signals that are common to both inputs.

The most widely used IS circuit consisting of three op-amps (figure 3).

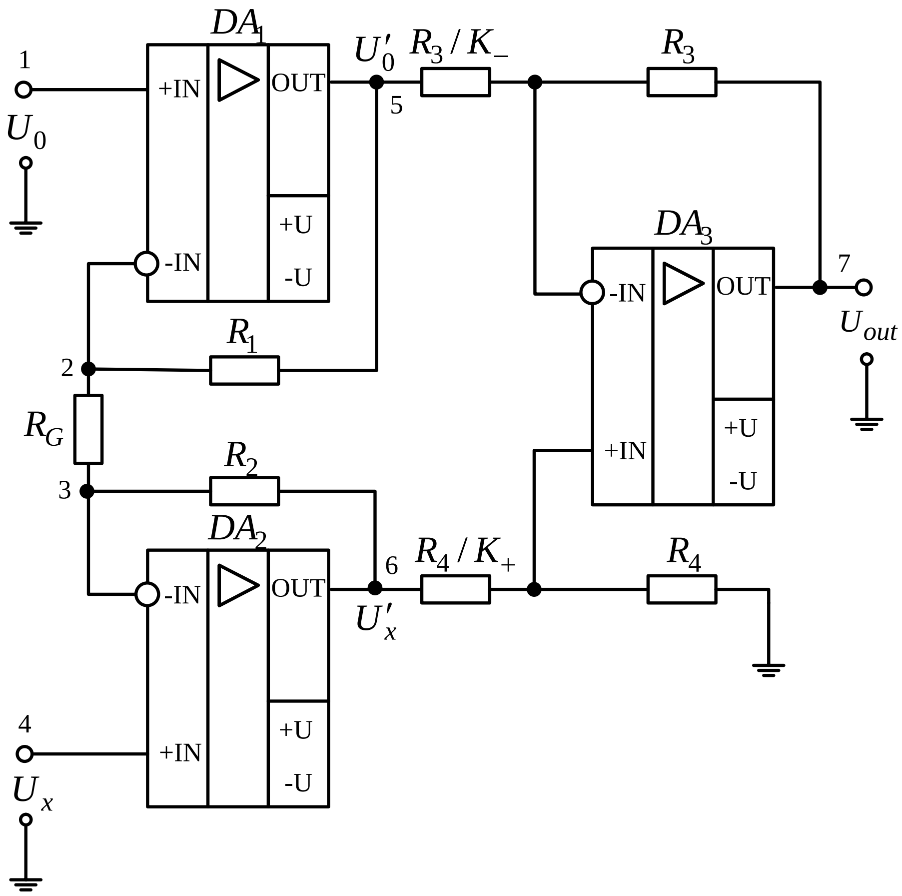

Figure 3. Instrumentation amplifier circuit consisting of three op-amps

IS constructed on a two-stage circuit, the first stage consists of op-amps DA_1, DA_2 and resistors R_1, R_2, R_G, has a the balanced input and high input impedance. The second stage consists of the op-amp

DA_3 and resistors R_3, R_4, R_5, R_6, forms the subtraction circuit on one op-amp, which is shown in figure 2. Using graph theory, we construct the signal graph for IS on the three op-amps (figure 4)

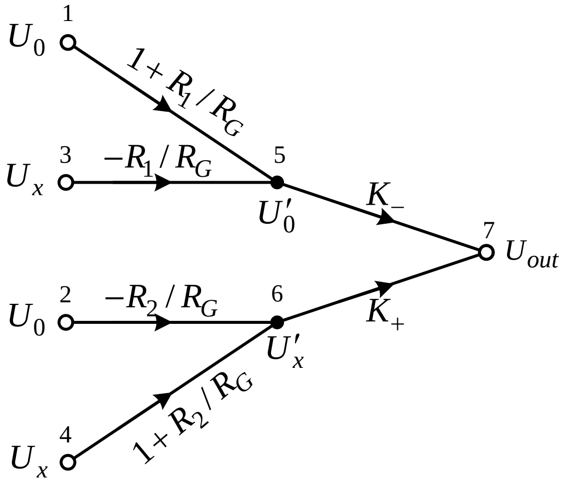

Figure 4. Signal graph for IS on the three op-amps

Amplifiers DA_1 and DA_2 we assume perfect, then on the basis of their inputs equipotential voltage U_0, simultaneously acts in the first and second nodes of the graph, and the voltage U_x, simultaneously acts in the third and fourth nodes. Using the principle of superposition, we find that the voltage U'_0 in fifth node of the graph is generated by transmission noninverting voltage U_0 of the first node and the inverting transmission voltage U_x from the third node circuit:

$$U'_0 = \left(1 + \frac{R_1}{R_G}\right)U_0 - \frac{R_1}{R_G}U_x. \tag{13}$$

Similarly, we obtain an expression for the voltage U'_x:

$$U'_x = \left(1 + \frac{R_2}{R_G}\right)U_x - \frac{R_2}{R_G}U_0. \tag{14}$$

The output voltage of the second stage and respectively output voltage of the IS:

$$U_{out} = \frac{(1+K_-)}{(1+K_+)}K_+U'_x - K_-U'_0. \tag{15}$$

Under the condition of equality K_+ and K_- output voltage of the IS:

$$U_{out} = \left(U_x - U_0\right)K_{d1} = \left(U_x - U_0\right)\left[1 + \frac{\left(R_1 + R_2\right)}{R_G}\right], \tag{16}$$

where K_{d1} - differential signal gain in first-stage.
To estimate CMRR substituting (4) in (15) considering expressions (13-14):

$$\begin{gathered} U_{out} = \frac{\left(K_+ - K_-\right)}{\left(1+K_+\right)}U_{cm} + \\ + \frac{\left(K_+ + K_- + 2K_+K_-\right)R_G + 2\left[K_+R_2 + K_-R_1 + K_+K_-\left(R_1 + R_2\right)\right]}{2R_G\left(1+K_+\right)}U_d. \end{gathered} \tag{17}$$

From equation (17) CMRR:

$$K_{CMRR} = \frac{\left(K_+ + K_- + 2K_+K_-\right)R_G + 2\left[K_+R_2 + K_-R_1 + K_+K_-\left(R_1 + R_2\right)\right]}{2R_G\left(K_+ - K_-\right)}. \tag{18}$$

If the resistance R_4/K_+ and R_3/K_- differ by ΔK_d and resistance R_1 and R_2 differ by value ΔR, substituting (7) and (19) into (18) we obtain:

$$\begin{aligned} R_1 &= R + \frac{\Delta R}{2}; \\ R_2 &= R - \frac{\Delta R}{2}. \end{aligned} \tag{19}$$

$$K_{CMRR} = \frac{\left(K_d + K_d^2 - \Delta K_d^2\right)\left(R_G + 2R\right)}{2\Delta K_d R_G} + \frac{2\Delta R}{R_G}. \tag{20}$$

From expression (17) shows that the gain of common mode signal in the first stage is 1. Consequently CMRR will increase proportional gain the differential signal of the first stage.

CMRR does not depend on the sources output impedance of the compared signals in IS. Despite the fact, what CMRR depends on internal resistances of the first and second stages, they are practically identical for IS in integrally fabricated with laser trimmed resistances. This fact is a decisive advantage when choosing IS to compare two signals and extraction the differential signal in LIA as compared to the above discussed circuits.

Gain in IS set by changing the resistor R_G (figure 3). If an external resistor used, then its accuracy and temperature coefficient is directly influence on accuracy and drift of differential gain IS, also decrease CMRR (20). It is expedient to choose IS with programmable gain, since in such IS resistor R_G matched with the other elements on the nominal resistance and temperature coefficients, for maximum accuracy and linearity of the gain in operating frequency band.

3. Conclusion

Analysis circuits based op-amps for comparing two signals and extraction differential signal was performed. Common-mode rejection ratio for each of the circuits was estimated.

Proposed use instrumentation amplifier integrally fabricated with programmable gain for comparing two signals. Since such instrumentation amplifiers common-mode rejection is practically independent of the internal resistances and the resistor determines the gain of the differential signal matched with the other elements on the nominal resistance and temperature coefficients, for maximum accuracy and linearity of the gain in the range of operating frequencies.

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