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# Study on Design of Rotor Profile for the Twin Screw Vacuum Pump with Single Thread Tooth

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**Abstract**. The trapezoidal profile is always used as part of tooth profile in the screw vacuum pump. However, interferential phenomenon occurs when using trapezoidal tooth profile. The arc is added to the trapezoidal profile to obtain the self-conjugate curve. The geometric characters of new profiles are compared with the old ones from view point of area of leakage triangle, length of contacting line, area utility coefficient, and etc. The rotor with the self-conjugate curve has characteristics of the small axial leakage area. Radial leakage from the carryover was within the same volume and would not influence the mass flow rate of the vacuum pump. High ultimate vacuum can be reached by utilizing this profile.

## 1. Introduction

The dry vacuum pump is no mixing of the working fluid with oil and contact between the rotors is prevented by timing gears which mesh outside the working chamber. The tooth profile of the rotors is most important for a design of a vacuum screw rotor pump. Many studies on the theory and technology of the screw compressor could be now referred to D Dingguo and S Pengcheng on the generation of rotor profiles by the analytical method [1]; Stosic N, on the development of N type line [2]; Dmytro Z, on the modification and generation of meshing contour line [3]. Rotor profile is obtained through traditional trapezoid tooth and line correction using analytical method.

An efficient dry vacuum pump needs a rotor profile with a large flow cross section area, a short sealing line and a small blow-hole area. Stosic N describes details of a new lobe profile, which yields a larger cross-sectional area and shorter sealing lines resulting in higher delivery rates for the same tip speed [4]. In this paper, we proposed correction method of cycloidal meshing for a design of single screw pump using the traditional trapezoid tooth and designed a conjugate rotor for a screw vacuum pump. At the same time, we put forward a new profile and the shape of two rotors are exactly the same thus it could reduce rotor machining costs. We analysed and compared pump performance using the rotor profile improved from view point of contact line length, leakage triangle area and the area of utilization coefficient and carryover area of three profiles.

**Nomenclature**

A center distance between male and female rotor
$A_1$ rotor area of male
$A_2$ rotor area of female
$C_n$ Area of utilization coefficient
D rotor diameter of male and female rotor
$\tau$ rotational angle
t rotational angle
$t_1$ rotation angle of ab
$t_2$ rotation angle of bc
d diameter of dedendum arc
i transmission ratio, equal to 1
$\theta$ parameter of the generating profile
P Spiral characteristic number

## 2. Design and modification of profile

The specific requirements of profile design are as follows:

(1) In order to ensure there are no interferes with the rotor, rotor profiles are supposed to meet the theory of gearing [5].

(2) Tooth profiles are supposed to have good air tightness in the axis and horizontal direction in order to sealing between tooth volumes. Rotor contact line is a smooth continuous curve formed from addendum circle to dedendum one and minimize the length of contact line.

(3) To reduce leakage area of the triangle, the vertex of the contact line should reach the intersection of the female and male rotor's cylinder.

### *2.1. A subsection Mathematical model of the trapezoid tooth*

Traditional Archimedes curve equation is obtained from rectangular (trapezoid) profile via the interference correction. This article selects the trapezoidal tooth. Rectangle tooth can be seen as a special case of the trapezoidal tooth. Axial cross section of Traditional trapezoid tooth is a trapezoid. The coordinate systems of the conjugate tooth are created as shown in figure 1.

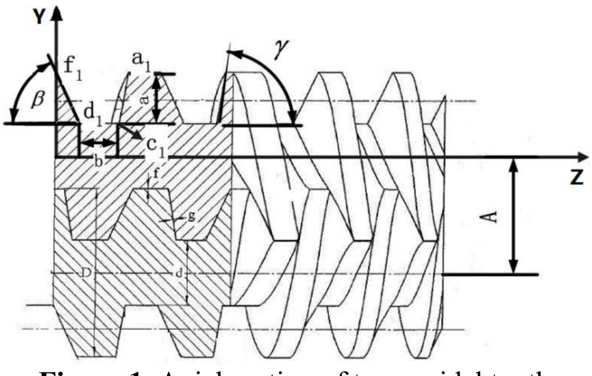



**Figure 1.** Axial section of trapezoidal tooth.

As shown in figure 1, Trapezoidal tooth can be regarded as the rotor helical motion under the trapezoidal tool cutting $a_1c_1d_1f_1$ [6].Therefore, tooth profile can be achieved through the spiral curve of four curve $a_1c_1d_1f_1$. Take $c_1d_1$ as example, calculating the corresponding tooth profile equation. Linear equation of $c_1d_1$ can be represented in the coordinate system as:

$$\begin{cases} X_u = 0 \\ Y_u = \dfrac{D}{2} - a \\ Z_u = t \end{cases} \qquad t \in [\frac{a}{\tan\beta}, \frac{a}{\tan\beta} + b] \tag{1}$$

Substituting equation (1) into equation (2);

$$\begin{cases} X = X_0(t)\cos(\tau)\, m\, Y_0(t)\sin(\tau) \\ Y = mX_0(t)\sin(\tau) - Y_0(t)\cos(\tau) \\ Z = Z_0(t) + \dfrac{P}{2\pi}\tau \end{cases} \tag{2}$$

The equation of the helix tooth space surface is obtained as following:

$$\vec{r}(u,\tau) = X \cdot \vec{i} + Y \cdot \vec{j} + Z \cdot \vec{k} \tag{3}$$

where

$$\begin{cases} X = -(\dfrac{D}{2} - a)\cos\tau \\ Y = (\dfrac{D}{2} - a)\sin\tau \\ Z = t + \dfrac{P}{2\pi}\tau \end{cases} \qquad \tau \in [\frac{a}{\tan\beta}, \frac{a}{\tan\beta} + b]$$

The screw tooth profile expresses the helix tooth space surface in the arbitrary OXY coordinate. Here, set Z=0 and we can obtain the axial curve segment $c_1d_1$. Substituting $t = -\frac{P}{2\pi}\tau$ into equation (3), it yields the radial equation as follows：

$$\vec{r} = -\frac{d}{2}\cos t \cdot \vec{i} + \frac{d}{2}\sin t \cdot \vec{j} \qquad t \in [\frac{-2\pi a}{P\tan\beta}, -\frac{2\pi}{P}(\frac{a}{\tan\beta} + b)] \tag{4}$$

As a result, radial equation is given by the axial cross section equation. Similarly, the remaining three sectional curves are calculated by the radial equations.

$f_1d_1$ section equation is:

$$\vec{r} = (-\tan\beta\frac{P}{2\pi}t - \frac{D}{2})\cos t \cdot \vec{i} + (-\tan\beta\frac{P}{2\pi}t + \frac{D}{2})\sin t \cdot \vec{j} \qquad t \in [\frac{-2\pi a}{P\tan\beta}, 0] \tag{5}$$

$a_1f_1$ section equation is

$$\vec{r} = -\frac{D}{2}\cos t \cdot \vec{i} + \frac{D}{2}\sin t \cdot \vec{j} \qquad t \in [-\frac{2\pi}{P}(\frac{a}{\tan\beta} + b + \frac{a}{\tan\gamma}), 0] \tag{6}$$

$a_1c_1$ section equation is

$$\vec{r} = -(\frac{d}{2} + \tan\gamma(-\frac{P}{2\pi}t - \frac{a}{\tan\beta} - b))\cos t \cdot \vec{i} + (\frac{d}{2} + \tan\gamma(-\frac{P}{2\pi}t - \frac{a}{\tan\beta} - b))\sin t \cdot \vec{j}$$
$$t \in [-\frac{2\pi}{P}(\frac{a}{\tan\beta} + b + \frac{a}{\tan\gamma}), -\frac{2\pi}{P}(\frac{a}{\tan\beta} + b)] \tag{7}$$

Basically, through substitution of β, γ, the common Archimedes curve and cycloid curves are obtained by the trapezoidal tooth correction.

### *2.2. Conjugate correction of profile*

The equation of the trapezoidal tooth profile was obtained using method above. Generally β is given as 45 °. However, this kind of tooth profile not only does not meet the theory of gearing but also interfere may occur in segment $a_1c_1$. The theory of gearing was thus introduced for designing the vacuum pump rotor profile and reducing the design problem as shown in figure 2. The profile consists of cycloid (a b), epicycloid (b c), dedendum arc(c d), pseudo Archimedes curve (d e), Pseudo Archimedes conjugate envelope curve (e f), addendum arc (f a). Female rotor profile is composed of conjugate curves of male rotor profile and thus is the same with that of the male rotor and we can make the shape of the female rotor same with that of the male rotor. Through this correction, we can manufacture female and male rotors by the same processing, thus greatly reducing the cost of manufacture.

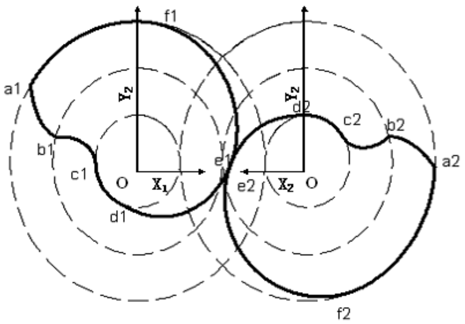



**Figure 2.** Conjugate correction of line.

The curve segment equations in the Cartesian coordinate are as follows:

Cycloid segment ab equation:

$$\begin{cases} x = A*\sin(2*t+t_1) - 1/2*d*\sin(2*t+t_1) + A*\cos(t) \\ y = A*\cos(2*t+t_1) - 1/2*d*\cos(2*t+t_1) - A*\sin(t) \end{cases} \tag{8}$$

Epicycloidal segment bc equation:

$$\begin{cases} x = A*\sin(2*t+t_2) - 1/2*d*\sin(2*t+t_2) + A*\cos(t) \\ y = A*\cos(2*t+t_2) - 1/2*d*\cos(2*t+t_2) - A*\sin(t) \end{cases} \tag{9}$$

Dedendum arc segment cd equation:

$$\begin{cases} x = -d*\cos(t)/2 \\ y = \ \ d*\sin(t)/2 \end{cases} \tag{10}$$

Equation for pseudo Archimedes curve segment de:

$$\begin{cases} x = -(a*\rho(t)+b)*\cos(t) \\ y = \ \ (a*\rho(t)+b)*\sin(t) \end{cases} \tag{11}$$

Archimedes curve ef is expressed by the rotation and translation of the conjugate curve segment de. The conjugate curve $e_2f_2$ is calculated from de segment equation:

$$\begin{cases} X_2 = -x\cos(k\varphi) \mp y\sin(k\varphi) + A\cos(i\varphi) \\ Y_2 = \mp x\sin(k\varphi) + y\cos(k\varphi) + A\sin(i\varphi) \\ \dfrac{\partial x}{\partial t} \times \dfrac{\partial y}{\partial \varphi} - \dfrac{\partial x}{\partial \varphi} \times \dfrac{\partial y}{\partial t} = 0 \end{cases} \tag{12}$$

Their coordinate transformations are as follows, where $\mp$ is determined from the rotational directions of the two rotors:

$$\begin{cases} x=X_2*\cos\theta-Y_2*\sin\theta \\ y=X_2*\sin\theta+Y_2*\cos\theta \end{cases} \quad (13)$$

Addendum arc segment fa equation:

$$\begin{cases} x=-D*\cos(t)/2 \\ y= D*\sin(t)/2 \end{cases} \quad (14)$$

Female rotor profile is composed of its conjugate curves obtained by the envelope equation.

### *2.3. Smooth correction of Archimedes spiral*

As shown in figure 3, Archimedes curve, the cusps between big arc and small arc, c, c', d, d' are not connected smoothly and it causes the abnormal junctions between Archimedes segment and arc segments.

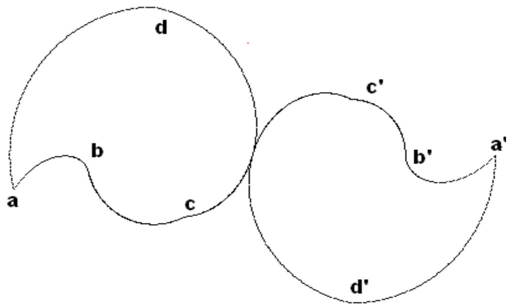



**Figure 3.** The traditional cycloid - Archimedes line.

Most of rotor profiles in the references and patents currently published have such as cusps and such a rotor profile is not conjugate each other. If the function f (x) in the interval (a, b) is of first order continuous differentiable, the graphic becomes a smooth tangent curve and it rotates continuously with the movement of the tangency points. The curve is called a smooth curve. However, this curve is not smooth, due to the first order discontinuous differentiable between the Archimedes curve and arc, as shown in figure 4 (from fig. 4 we could not read its cause, due to first order discontinuous differentiable or other.). We select a sine curve [7] for generating the spiral moving according to the spiral tooth surface model. Finally curve cd can be gotten as is shown in figure 5. In this way, the curves cd, ab, and bc, are connected smoothly and the cd curve, its conjugate curve $c'd'$, curve $a'd'$ and curve $b'c'$ form a continuous line. Not only their joints became smooth, but also two rotors satisfied the conjugate conditions.

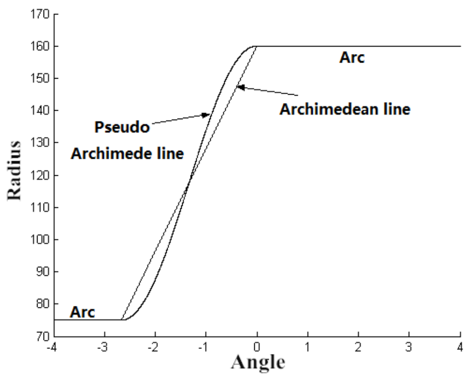



**Figure 4.** Archimedes angle-radius.

The Archimedes spiral equation is given by:

$$\rho = a\theta + b \quad (15)$$

Revised as following [7]:

$$\rho = a\sin(\theta) + b \tag{16}$$

The curve is corrected from Archimedes curve, so we call it "pseudo Archimedes curve". After correcting, the cusps in the line are eliminated and we can easily find out the conjugate curve as shown in figure 5.

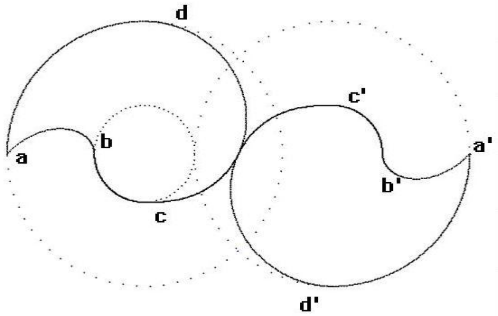



**Figure 5.** Smooth Archimedes spiral.

*2.4. Profile of male rotor same with female one:*

Even segments expressed by Archimedes conjugate curve are not the same each other, i.e., female rotor is not exactly the same with that of male. Two types of tools for hobbing, milling and grinding must be prepared in manufacturing screw vacuum pump, resulting in the cost increase. Therefore the Archimedes curve should be modified so as to be in coincidence with the conjugate curves.

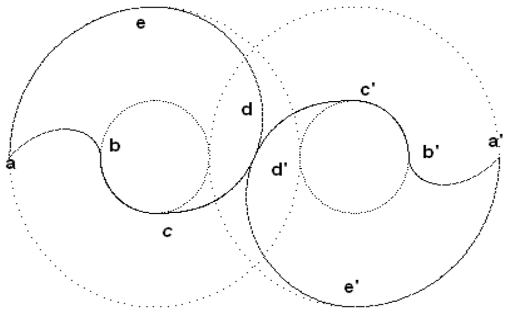



**Figure 6.** Rotor profile modified by line correction.

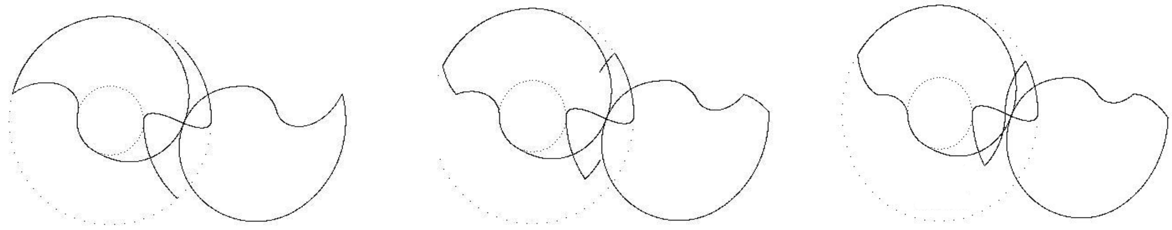
The original curve can be divided into two segments. One segment is the original Archimedes curve and the other segment is obtained from Pseudo Archimedes curve by the rotational and translational transformation. And the same rotors are obtained being conjugated with each other. The original curves are divided at point d and the conjugate curve d'e' is transformed by rotation and translation to the male rotor and replaced by de curve. In the same way, female rotor curve can be calculated. The line correction is validated as shown in figure 6. The female rotor given by its conjugate curve is the same as male rotor. Specific relation between curve and its conjugate can be seen in table 1.

**Table 1.** Corresponding relation between elements of the modified line.

| Curve | Curve type | Conjugate | Conjugate curve type |
|---|---|---|---|
| $bc$ | Arc | $a'e'$ | Arc |
| $cd$ | Pseudo Archimedes | $d'e'$ | Pseudo Archimedes line envelope |
| $de$ | Pseudo Archimedes line envelope | $d'c'$ | Pseudo Archimedes |
| $ea$ | Arc | $c'b'$ | Arc |
| $a$ | Point | $a'b'$ | cycloid |
| $ab$ | cycloid | $a'$ | Point |

### 2.5. *Continuous correction of contact line*

The conventional twin screw compressor is designed to form a sealing line between mated rotors. However, the sealing line is usually incomplete. We select a point in cycloidal curve and calculate its cycloid. Female and male rotors are calculated by the meshing line of the modified profile. The meshing line is shown in figure 7(a). As we can see from figure 7(a), meshing line is not closed so that vacuum tightness is broken [8]. But the vertex of the contact line reaches the intersection of the female and male rotor's cylinder. There is no leaking triangle between the contact line vertex and cylinder. By shifting the fixed point at the origin of meshing cycloid to the dedendum circle, the closed extent of meshing line is changeable. Eventually we can achieve the closed meshing line. The resulting meshing line is shown in figure 7(c).



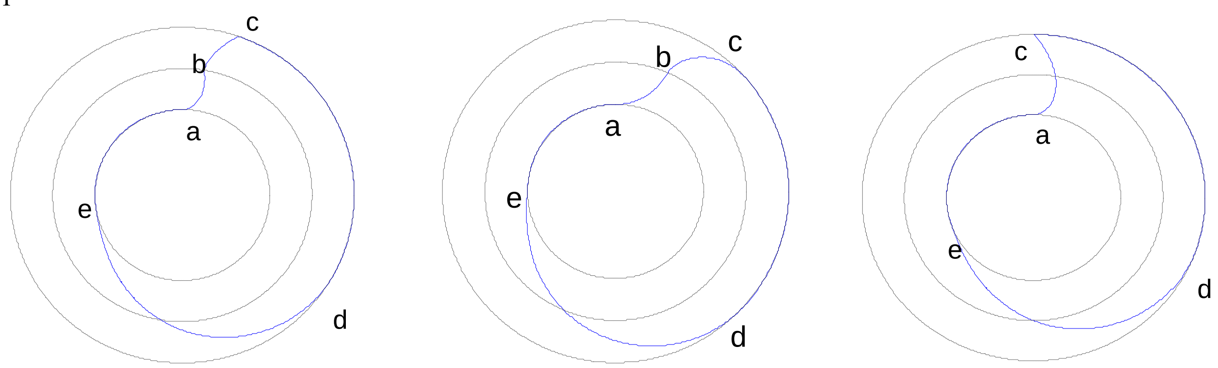
(a) (b) (c)

**Figure 7.** Rotor meshing line.

The meshed diagram of twin screw vacuum pump rotor is shown in figure 6. Male and Female rotors are single head screw. Two rotor tooth profiles are the same and mutually conjugated. The profile consists of cycloid, arc, Archimedes curve and its envelope curve.

## 3. Design examples and discussions of pump performance

As shown in figure 8, three types of rotor curve segments are modified according to the methods mentioned above. For comparing geometric features, let's take the diameter of the circle cd as 130 mm, the diameter of the circle ea 66 mm, and the polar Angle of curve de 0.8*pi. In figure 8(a), the cycloid ac consists of two different cycloids ab and bc. The corresponding meshing point of cycloid ab is point c and the corresponding meshing point of cycloid bc is point b; In figure 8(2), arc ac consists of ab and its conjugate curve bc; In figure 8(c), ac is a length of cycloid and corresponding meshing points is point c.




(a)Example 1 (b) Example 2 (c) Example 3

**Figure 8.** Rotor profiles.

### 3.1. *Area of utilization coefficient*

Let's select the lead as 72 mm for calculating the geometric parameters such as the rotor area, area between teeth and the area of utilization coefficient and the result is shown in table 2. The space area of utilization coefficient calculated by equation $C_n = \frac{(A_1 + A_2)}{D^2}$.

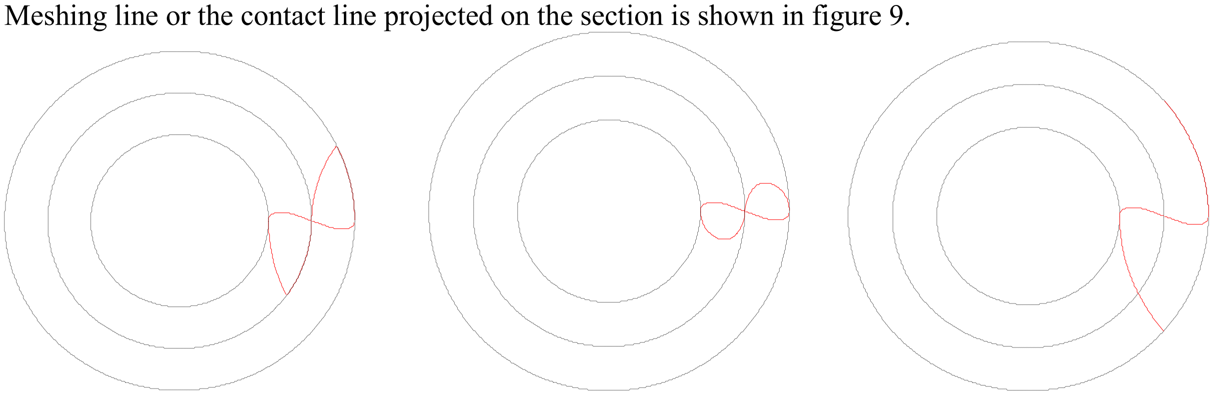
**Table 2.** Area of utilization coefficient.

| | Example 1 | Example 2 | Example 3 |
|---|---|---|---|
| rotor area /mm$^2$ | 8108.99 | 8118.31 | 7981.10 |
| Tooth area/mm$^2$ | 5164.24 | 5154.92 | 5292.13 |
| Area of utilization coefficient /% | 61.12 | 61.00 | 62.63 |

As we can see from table 2, all kinds of rotor profiles are no major difference in the rotor area, tooth space area and the area of utilization coefficient, in which the maximum difference is within 2%.

### *3.2. Contact line*

Meshing line or the contact line projected on the section is shown in figure 9.



(a) Example 1 (b) Example 2 (c) Example 3

**Figure 9.** Contact line.

The contact line, L, is composed of three parts: the contact line length of cycloidal segment L1, the contact line length of pseudo Archimedes segment L2 and the contact line length of arc segment L3 and the contact line length and their partial contact line lengths are listed in table 3.

**Table 3.** Length of contact line.

| | Example 1 | Example 2 | Example 3 |
|---|---|---|---|
| L1/mm | 123.80 | 58.12 | 94.52 |
| L2/mm | 46.32 | 46.32 | 46.32 |
| L3/mm | 39.34 | 33.79 | 43.20 |
| L/mm | 208.46 | 138.23 | 184.04 |

The result shows the epicycloid shortened can produce a shorter line of action. Thus, the pump efficiency could be increased.

### *3.3. Leakage triangle and carryover*

As shown in figure 10, there is a carryover between two mated rotors. This means a larger gas may be carried from high–pressure port back to low-pressure port. This phenomenon will lead to larger leakage and reduce the pump performance [10].

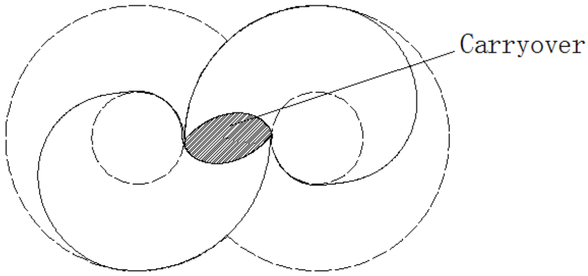



**Figure 10.** Carryover between two mated rotors.

Leaking triangle area S1 and carryover area S2 of three profiles are listed in table 4. The sealing performance of the rotor profiles, Example 3, is better than Example 2. In this paper, leaking triangle area of Example 3 is smaller than Example 1 by 27.26mm and by 87.18mm than Example 2. Further study of Example 3 founds that carryover connects the contiguous volume. Therefore, the rotor radial path of leakage does not affect sealing.

**Table 4.** Leakage triangle and carryover.

| | Example 1 | Example 2 | Example 3 |
|---|---|---|---|
| S1/mm$^2$ | 27.26 | 87.18 | 0 |
| S2/mm$^2$ | 0 | 0 | 386.7 |

From view point of the air tightness, the contact line length of Example 3 is between Example 1 and Example 2 and no leakage triangle. The carryover of Example 3 connects contiguous volume and does not influence the air tightness. So the volumes of Example 3 is completely sealed. The profile of Example 3 can reach the higher vacuum and reduce the gas reflux.

## 4. Conclusion

The conjugate mathematical model was introduced to design three types of rotor profiles from which the conjugate tooth profile and its design constraints were derived. The following are our conclusions:

(1) We proposed a conjugated mathematical model for design of screw rotors for the single screw vacuum pump, by which it is enough to process two rotors with same shape in manufacturing the pump. This leads to the cost performance of the screw vacuum pump.

(2) We proposed a mathematical method to generate a conjugate curve to the original one to design two rotors conjugated each other using the rotational and translational transformation. a method of compositing new profile: curve-conjugate curve rotation and translation.

(3) We compared the pump performances depending on the different rotor profiles in terms of contact line length, leakage triangle area, the area of utilization coefficient and carryover. The sealing performance in case of Example 3 is better than Example 2.

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