Design and validation of new cavity profiles for diaphragm stress reduction in a diaphragm compressor

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Abstract.The application of diaphragm compressors is severely limited by the low reliability and short life of diaphragms. The cavity profile has considerable influence on a diaphragm's stress level and therefore on its fatigue life. This experimental and theoretical analysis of the traditional cavity profile (as defined by a single-parameter generatrix) indicated that the short life of diaphragms may be caused by the superposition of peak radial stress and additional stress due to the discharge holes in the central region that occur when the diaphragm clings to the cavity surface. Therefore, two new kinds of cavity profiles are proposed to decrease the diaphragm's radial and total stress. The new cavity profiles based on different generatrices were designed in an attempt to reduce the peak radial stress in the central region. These cavities were manufactured to experimentally validate the designs. Experiments were carried out under various design conditions to investigate the radial stress distribution in the diaphragms. The reductions of radial stress on the diaphragms in the new cavities were compared with stress levels found in the traditional design. Although their cavity volumes and radii were all the same, the radial stress levels on diaphragms in the cavities designed by the new generatrices were less than those found in the traditionally designed cavity. The greatest decline in radial stress on the diaphragms in the cavities based on new generatrices was 13.8%. With the additional stress taken into account, the decline in radial stress in the diaphragm's central region reached as high as 19.6%.

1. Introduction

A diaphragm compressor is a type of compressor that guarantees gas purity without leakage. This device is commonly applied in dealing with rare, flammable, poisonous or corrosive gases. Diaphragm compressors normally vary in discharge pressure between 0.1 MPa and 300 MPa. [1] Therefore, these compressors are generally used in the petrochemical, pharmaceutical and food industries. [2]

Failure of the diaphragm is the most common type of malfunction for diaphragm compressors. The stress caused by deflection of the diaphragm as it comes into contact with the cavity surface affects its


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fatigue life. The cavity profile also has a considerable influence on the diaphragm's stress level and therefore its fatigue life. The symmetric cavity profile is shaped by the rotation of a generatrix. The traditional generatrix equation for a cavity profile is a single parameter polynomial.

Wu et al. studied the stress distribution on a diaphragm as it clung to the cavity surface and proposed an optimisation algorithm for the traditional generatrix equation of the cavity profile. [3] Ban analysed stress variations in diaphragms during compression using an iteration method. [4] Wang and Feng verified the reliability of thin plate large deflection theory and designed a new generatrix composed of two polynomials. [5] Kalshnikov [6] et al. researched the fatigue life of diaphragms under variable loads and provided two models for calculating diaphragm reliability. Antonova et al. proposed a method to evaluate the reliability of a diaphragm compressor. Altunkhov [7] et al. found that the diameter of the discharge holes and the oil pressure were the two main factors that influenced a diaphragm's fatigue life. In recent years, however, little research has been conducted on the generatrices of the cavity profiles for diaphragm compressors.

Previous studies and cases of diaphragm failures suggest that excessive radial stress is the root cause of such failures. One of the peaks in radial stress occurs in the central region of the diaphragm's gas side, as this area endures additional stress caused by the discharge holes. Thus, the superposition of these two kinds of stress results in stress concentration, which has a significant adverse effect on diaphragm performance. This paper presents two new kinds of generatrices of the cavity profiles for diaphragm compressor and indicates their optimisation algorithms. Experimental and mathematical methods were applied to verify the diaphragm stress distributions under the proposed design theories.

2. Design theory for the cavity profile

2.1 Diaphragm deformation theory

The primary structure of a diaphragm compressor is shown in Figure (1).

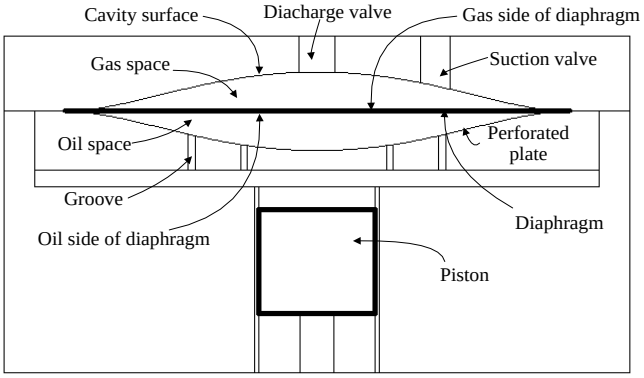


Figure 1.

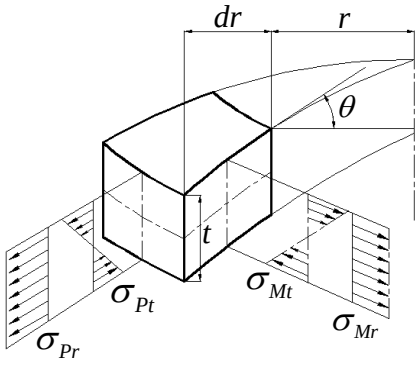


Figure 2.

The circular and symmetric deformation of the diaphragm in the compressor can be viewed as a combined effect of stretching and bending, such that the deflections and stresses are determined by the radius r. The radial tensile stress σ_{Pr}, radial bending stress σ_{Mr}, circumferential tensile stress σ_{Pt} and circumferential bending stress σ_{Mt} are shown in Figure (2).

The stress levels involved were calculated as follows, based on Von Karman's equation [8]:

$$\begin{cases} \sigma_{Pr} = \dfrac{1}{\mu-1}\left[\dfrac{d\left(\frac{1}{r}\int r\int \frac{-E\left(\frac{dH}{dr}\right)^2}{2r}dr\right)}{dr}\Big|_{r=R} - \dfrac{\mu\int r\int \frac{-E\left(\frac{dH}{dr}\right)^2}{2r}dr}{r^2}\Big|_{r=R}\right] + \\ \dfrac{1}{r^2}\int r\int \dfrac{-E\left(\frac{dH}{dr}\right)^2}{2r}dr \\ \sigma_{Mr} = \dfrac{tE}{2(1-\mu^2)}\left(\dfrac{d^2H}{dr^2} + \dfrac{\mu}{r}\dfrac{dH}{dr}\right) \\ \sigma_{Pt} = \dfrac{d\left(\frac{1}{r}\int r\int \frac{-E\left(\frac{dH}{dr}\right)^2}{2r}dr\right)}{dr} + \\ \dfrac{1}{\mu-1}\left[\dfrac{d\left(\frac{1}{r}\int r\int \frac{-E\left(\frac{dH}{dr}\right)^2}{2r}dr\right)}{dr}\Big|_{r=R} - \dfrac{\mu\int r\int \frac{-E\left(\frac{dH}{dr}\right)^2}{2r}dr}{r^2}\Big|_{r=R}\right] \\ \sigma_{Mt} = \dfrac{tE}{2(1-\mu^2)}\left(\dfrac{1}{r}\left(\dfrac{dH}{dr}\right) + \mu\dfrac{d^2H}{dr^2}\right) \end{cases},$$

Equation 1

where t is the thickness of the diaphragm, E is the Young modulus, μ is Poisson's ratio and H is the deflection of the diaphragm.

The tensile stress was uniform in the thickness direction, but the bending stress was linearly distributed. Therefore, the maximal radial and circumferential stress existed on the sides of the diaphragm. The radial stress on the gas side σ_{Gr}, the radial stress on the oil side σ_{Or}, the circumferential stress on the gas side σ_{Gt} and the circumferential stress on the oil side σ_{Ot} were calculated as having the following relations:

$$\begin{cases} \sigma_{Gr} = \sigma_{Mr} - \sigma_{Pr} \\ \sigma_{Or} = \sigma_{Mr} + \sigma_{\mathrm{Pr}} \\ \sigma_{Gt} = \sigma_{Mt} - \sigma_{Pt} \\ \sigma_{Ot} = \sigma_{Mt} + \sigma_{Pt} \end{cases}.$$

Equation 2.

$$\begin{cases} \dfrac{dH}{dr}\Big|_{r=0} = \theta\Big|_{r=0} = 0 \\ \dfrac{d^2H}{dr^2}\Big|_{r=0} = \theta'\Big|_{r=0} \le 0 \\ \dfrac{dH}{dr}\Big|_{r=R} = \theta\Big|_{r=R} = 0 \end{cases}$$

Equation 3.

The pressure on the oil side compelled the diaphragm to cling tightly to the cavity surface in the last period of the discharge process. Thus, the maximal deformation of the diaphragm was determined by the shape of the cavity surface, which was determined by the generatrix of the cavity profile. At this point in the process, the stress level of the diaphragm reached its maximum, which should be treated as the design condition for the cavity profile when evaluating the fatigue life of the diaphragm. In this paper, therefore, references to the stress of the diaphragm refer to the stress level as the diaphragm clings tightly to the cavity surface.

2.2 Design condition of the cavity profile

To avoid stress concentration, the following conditions had to be met. First, the generatrix had to be smooth in the central region of the cavity. Second, the cavity centre had to be convex. Third, the deflection angle θ had to be zero at the cavity edge. These conditions were expressed as the equation 3.

With the traditional generatrix, most diaphragmatic fractures occurred in three positions: the

diaphragm's edge, its centre and its region in contact with the outer groove of the perforated plate (see Figure 3). At the edge of the diaphragm, the crack shown in Figure 3(a) indicates that excessive radial stress (which is perpendicular to the crack) was the main reason for fractures in this region. In the centre of the diaphragm, where the discharge holes were settled (as shown in Figure 3(b)) fractures resulted from the superposition of the peak radial stress and the additional stress that occurred as the diaphragm clung tightly to the cavity surface.

These considerations indicate the paramount importance of radial stress control for effective cavity profile design. Moreover, as Moskalev suggested, reducing radial stress should be a primary design principle [9]. In view of the additional stress caused by the discharge holes, the allowable radial stress in the centre, σ_c, was lower than that for the other parts of the diaphragm, σ_a. These stress conditions were expressed as follows:

$$\begin{cases} R > r \geq R_1, \sigma_r < [\sigma_a] \\ R_1 > r \geq 0, \sigma_r < [\sigma_c] \end{cases},$$

Equation 4



(a) (b)

Figure 3

where R_1 is the radius of the discharge region's edge (in which the discharge holes are settled).

The traditional generatrix of the cavity profile was determined by Equation (5):

$$H = H_0 \frac{1}{Z-1}[2(\frac{r}{R})^{Z+1} - (Z+1)(\frac{r}{R})^2 + (H_0 - 1)].$$

Equation 5

Based on the design parameters given in Table (1), the parameters Z and H_0 of the traditional generatrix were determined by a traversal method. When $Z = 8.8$ and $H_0 = 8.3$, the cavity volume reached a maximal value of $974\ cm^3$. Under this condition, the stress distribution of the diaphragm in the cavity was as shown in Figure (4).

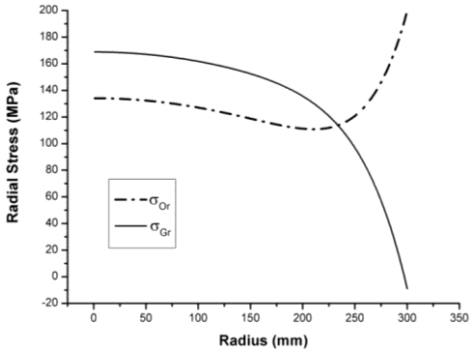


Figure 4.

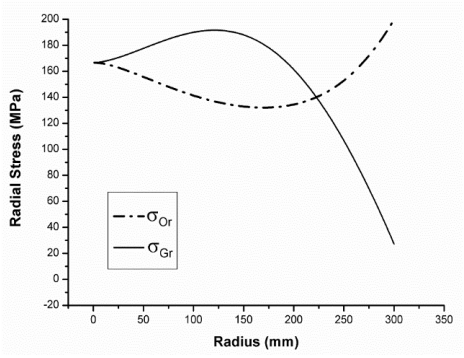


Figure 5.

Parameters	Values
R/mm	300
E/MPa	210000
μ	0.3
$[\sigma_a]$ ($20 < r \leq R$)/MPa	200
$[\sigma_c]$ ($r \leq 20$)/MPa	170
t/mm	0.5

Table 1

However, as defined by the traditional generatrix, the radial stress distribution of the diaphragm in the cavity was non-uniform, as the radial stress at the edge and in the centre was much larger. Therefore, the peak of radial stress at the centre, which was subjected to additional stress caused by the discharge holes, seriously limited the performance of the diaphragm. The radial bending stress of the diaphragm in the cavity as defined by the traditional generatrix was expressed as follows:

$$\sigma_{Mr} = \frac{tEH_0}{2(Z-1)(1-\mu^2)}\left[\frac{2(Z+\mu)(Z+1)(r^{Z-1})}{R^{Z+1}} - \frac{2(1+\mu)(Z+1)}{R^2}\right].$$

Equation 6

In light of the above considerations, two new kinds of generatrices were designed.

2.4 New generatrices of the cavity profile

2.4.1 Two-parameter generatrix

To prevent the peak value of bending stress from appearing in the centre of the diaphragm, a two-parameter generatrix was designed. The deflection angle of this generatrix was expressed by *n* and *m*, two independent parameters. In considering the boundary condition shown in Equation (3), the equation for the two-parameter generatrix was expressed using H_0 as the maximal deflection in the cavity profile. Under the design conditions shown in Table (1), the optimal parameters were found by a traversal method. The cavity volume achieved a maximum value of $1038cm^3$ when $H_0 = 8.2, n = 3.1$ and $m = 3$. The radial stress of the diaphragm was calculated with Equation (1) and the distribution is shown in Figure (5).

2.4.2 Three-parameter generatrix

As with the two-parameter generatrix described above, the deflection angle of the three-parameter generatrix was expressed with *n*, *m* and *k* as three independent parameters, and with *a* as a control factor. In consideration of the boundary condition set by Equation (3), we generated the expression as follows:

$$H(r)=\frac{H_0}{a(\frac{(n-k)r}{(k+1)(n+1)})+\frac{(m-k)r}{(k+1)(m+1)}}\left[\frac{ar}{n+1}(\frac{r}{R})^{n+1}+\frac{r}{m+1}(\frac{r}{R})^{m+1}-\frac{(a+1)r}{k+1}(\frac{r}{R})^{k+1}\right]+H_0 \cdot$$

Equation 7

The five parameters H_0, *n*, *m*, *k* and *a* had to be determined first. However, these parameters could not be obtained by the common traversal method due to the inexplicit constraints. Thus, a new optimisation algorithm was designed to find the optimum parameters of the generatrix. The boundaries of the parameters had to be determined first. According to engineering experience, H_0 ranged from 4 to 20. The control factor *a* can be derived from Equations (1), (2) and (7), and was calculated as follows:

$$a=\frac{[(m-k)-\frac{2\sigma_{MR}R^2(1-\mu^2)(m-k)}{EtH_0(k+1)(m+1)}]}{\frac{2\sigma_{MR}R^2(1-\mu^2)(n-k)}{(k+1)(n+1)}-(n-k)},$$

Equation 8

where σ_{MR} is the bending stress of the diaphragm on the edge $(r=R)$. Under the design condition shown in Table (1), σ_{MR} ranged from 60 MPa to 90 MPa.

According to the design conditions given in Table (1), when $a=-0.566$, $H_0=7.9$, $n=8.9$, $m=7.8$ and $k=2.7$, the cavity volume achieved a maximum value of $1073\,cm^3$. The stress distribution on the diaphragm in this condition is shown in Figure (6).

3 Verification of the new generatrices

3.1 Finite element method

To verify this design theory, the finite element method was used to calculate the radial stress. In performing this calculation, the cavity was defined by the generatrix mentioned in Section 2.4.1. The model for a quarter of the diaphragm and cavity was established. The results, which matched well with the theoretical estimation, are shown in Figures (7) and (8).

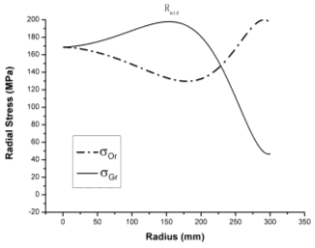


Figure 6.

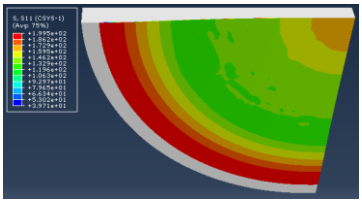


Figure 7.

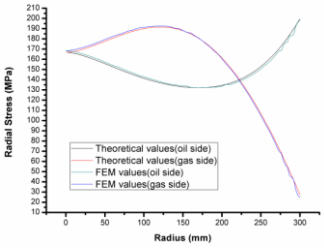


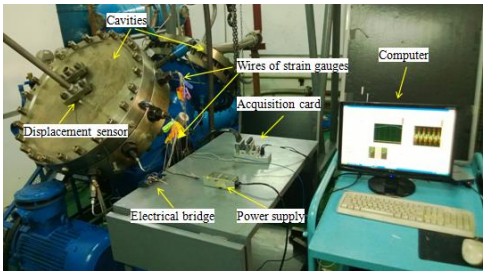
Figure 8.

3.2 Experimental method

3.2.1 Experiment on the cavity defined by the two-parameter generatrix

The relation between stress and strain in the diaphragm can be expressed as follows. Variables σ_r and σ_t are the radial and the circumferential stresses, respectively, and ε_r and ε_r are the radial and the circumferential strains, respectively. Accordingly, a test rig was built using a V-type diaphragm compressor. The two cavity profiles were separately designed with a traditional generatrix and a two-parameter generatrix.

Strain gauges were applied for the strain measurements. One displacement sensor was set near the discharge valve on each cavity to monitor the displacement of the diaphragm. A schematic diagram of the test rig is shown in Figure (9).



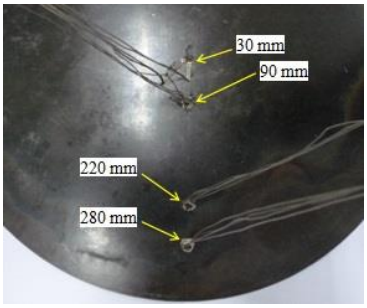
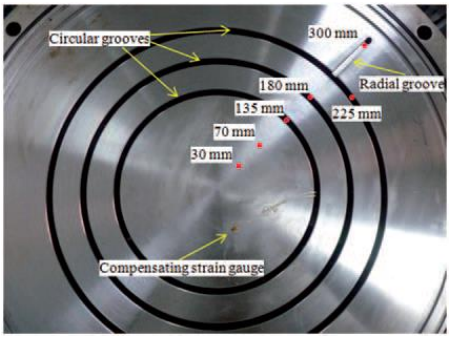


Figure 9. **Figure 10.**

The strain gauges were set equispaced along the radius for determining the radial stress distribution. [10-11] As shown in Figure (10), the strain gauges were set in both the radial and circumferential directions, at radii of 30 mm, 70 mm, 135 mm, 180 mm, 225 mm and 300 mm.

To avoid damage to the perforated plate during the working process, the strain gauges and wires were set at the radii of 30 mm, 70 mm, 135 mm, 180 mm, 225 mm and 300 mm upon the circular and radial grooves, as shown in Figure (11). Red spots indicate the positions of the strain gauges.



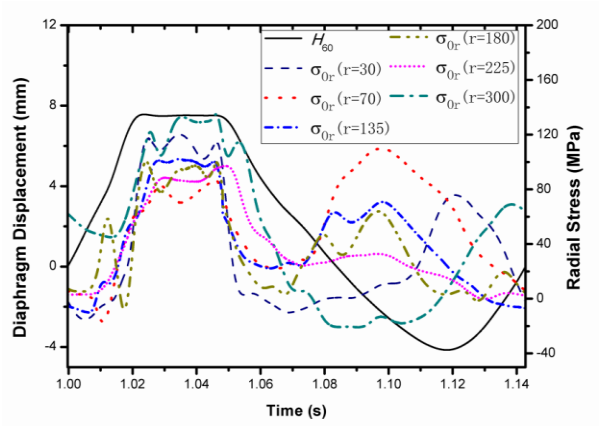
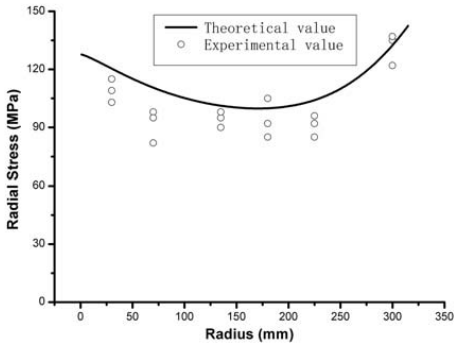


Figure 11. **Figure 12.**

Figure (12) shows the radial stress and the displacement in one operation cycle, as described above. Figure (13) shows the theoretical and experimental radial stresses of the diaphragm as it clung tightly

to the cavity surface. The trend of these theoretical stresses agreed well with the experimental stresses. However, the majority of the theoretical values were higher and the relative error was 12.3%.



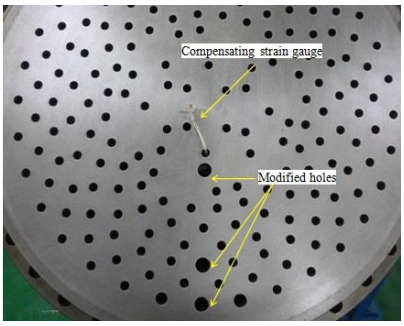


Figure 13. **Figure 14.**

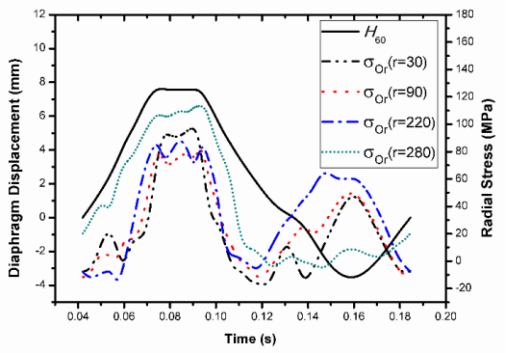
3.2.2 Experiment on the cavity as defined by the traditional generatrix

The traditional generatrix of the cavity used in this experiment can be expressed as follows:

$$H(r) = 8.3\frac{1}{8.8-1}[2(\frac{r}{300})^{8.8+1} - (8.8+1)(\frac{r}{300})^2 + (8.8-1)]\,.$$

Equation 9

Figure (15) shows the results of the experiment, including the radial stresses at different radii and the displacement of the diaphragm. The comparison of the theoretical and experimental radial stresses are shown in Figure (16). The average relative error was 11.2%.



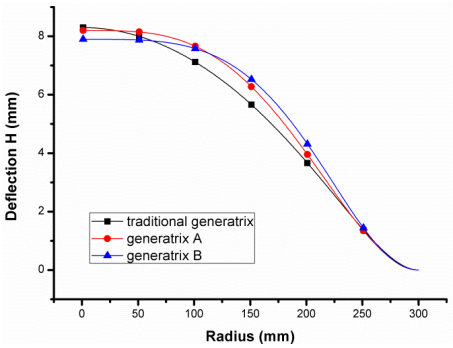


Figure 15. **Figure 16.**

4 Results and discussion

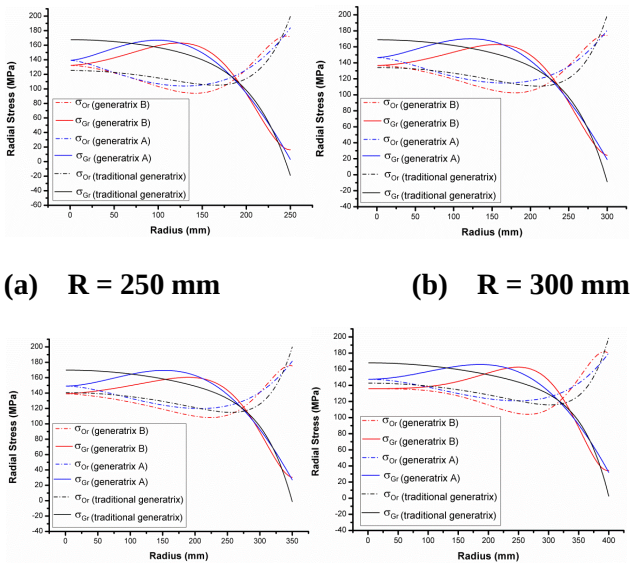
The proposed new generatrices design theory was validated by both experimental and finite element methods. The characteristics of the cavity profiles defined by the new and traditional generatrices were compared. The radial stress distributions of the diaphragms were investigated under the same set of design conditions.

The shapes of the generatrices are shown in Figure (16).

The radial stress level of the diaphragm is one of the most important evaluation criteria for a

diaphragm compressor. Thus, a comparison analysis on the radial stresses of the diaphragms in the cavities as defined by the three generatrices was conducted, with the conditions of cavity radius, cavity volume and diaphragm properties held constant.

First, the cavity radii were set at 250 mm, 300 mm, 350 mm and 400 mm. The cavity volumes defined by the two generatrices were the same at each radius and were set as 542 cm^3, 974 cm^3, 1592 cm^3 and 2417 cm^3, respectively. Figure (18) shows the radial stress distributions of the diaphragms in the cavities as defined by different generatrices at various radii. For all of the cavity radii, the peak radial stress on the diaphragm in the cavities defined by the two generatrices occurred on the edges of the diaphragms and on the oil side. Therefore, the maximum radial stress on the diaphragm in the cavities defined by the new generatrix was always lower at each radius. In the centre of the diaphragm, the radial stress of the diaphragm in the cavity defined by the new generatrix was also lower, if we consider the additional stress caused by the discharge holes. The rates of decline in radial stress on the diaphragm in the cavity defined by the new generatrix were compared with those for the traditional generatrix, as shown in Figure (17). The rates of decline in radial stress on the diaphragm in the cavity defined by the new generatrix varied slightly with the increase in cavity radius. The radial stress on the diaphragm in the cavity defined by the new generatrix also showed a decline of 10.3% in peak radial stress. The radial stress in the central region of the diaphragm showed a decline from 14.0% to 10.1% when the cavity radius changed from 250 mm to 400 mm.



(a) R = 250 mm **(b) R = 300 mm**

(c) R = 350 mm **(d) R = 400 mm**

Figure 17.

5 Conclusions

Two new kinds of generatrices for the cavity profile of a diaphragm compressor were designed to decrease the radial stress level of the diaphragm. To find the optimum parameters for the new generatrices, optimisation algorithms were developed. The design theory for the diaphragm's radial stress was verified by experimental and mathematical methods. The radial stress on the diaphragm was investigated under various design conditions. The following conclusions can be drawn.

(1) Radial stress should be the key principle in the design of the cavity profile for a diaphragm compressor. The peak radial stress should be shifted away from the central region of the diaphragm to avoid the superposition of additional stress caused by the discharge holes.

(2) The levels of radial stress on the oil side of the diaphragm were tested and the experimental results agreed with the theoretical estimation. The radial stress of the diaphragm in the cavity defined by the new generatrix was calculated by a mathematical method and showed good agreement with the theoretical estimation.

(3) Compared with the radial stress on the diaphragm in the cavity defined by the traditional generatrix, the radial stress on the diaphragm in the cavity defined by the new generatrices declined by up to 19.6%. The peaks of radial stress changed from 10.0% to 14.0% when the cavity radius shifted from 400 mm to 250 mm. When the diaphragm thickness changed from 0.3 mm to 0.5 mm, the peak of radial stress showed a decline of between 10.0% and 13.0%, and the decrease in the rate of radial stress in the central region varied between 14.1% and 18.3%.

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