

# Analysis of the screw compressor rotors' non-uniform thermal field effect on transmission error

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**Abstract.** The vibrational state of the screw compressor is largely dependent on the gearing of the rotors and on the possibility of angular backlash in the gears. The presence of the latter leads to a transmission error and is caused by the need for the downward bias of the actual profile in relation to the theoretical. The loss of contact between rotors and, as a consequence, the current value of the quantity, characterizing the transmission error, is affected by a large number of different factors. In particular, a major influence on the amount of possible movement in the gearing will be exerted by thermal deformations of the rotor and the housing parts in the working mode of the machine. The present work is devoted to the analysis of the thermal state in the operation of the screw oil-flooded compressor and its impact on the transmission error and the possibility of losing contact between them during the operating cycle.

## 1. Introduction

The technique that allows us to analyze the correctness of engagement during the compressor operation has been presented in the literature [1, 2]. Researchers have demonstrated the possibility of the use thereof during the engagement pre-analysis at the operating conditions of the compressor. At the same time, it is also important to note the fact that it is also suitable for a more detailed analysis of the influence of various factors on the engagement work of the screw compressor rotors in the event of some adaptation and refinement. Thermal deformations in these studies were calculated based on the assumption of uniformity of temperature fields in any cross section of the rotors, which was calculated on the basis of an empirical relationship generalizing the results of experimental studies. This approach is quite acceptable for preliminary analysis; however, it should be noted that, for a more detailed engagement analysis, it is also necessary to assess the influence of both operating and design features of the test compressor (rotor speed, design and size of the pressure and suction ports) that does not allow to making of the empirical formula in these studies. All of the above factors make it possible to formulate the problem of this study on the synthesis and improvement of methods of calculation of temperature fields of the rotors with subsequent analysis of their impact on the rotor engagement.

## 2. Dynamics of free rotors and their ability to lose direct contact with driving rotors

Changing the free rotor rotational correction angle causes it to rotate with a certain acceleration. This can be caused both by direct contact of the rotors and by gas forces torque. The gas forces torque



is typically common for screw compressors operating at nominal conditions: rotational correction of the female rotor into contact with the male rotor in the direction of rotation of the rotors in the implementation of the male rotor drive. The value of this rotational correction angle can be calculated by the method described in the studies [1 and 2]. Therefore, for the continuation of the rotors being engaged, the possible acceleration of the rotor from gas forces should exceed the required acceleration associated with a change in the rotational correction angle.

Assuming the uniform rotation of the drive rotor in the first approximation that can be justified by its significantly higher moment of inertia thanks to the shells and partial compensation by the driving torque, while taking into account all the factors stated above, the condition of continuation being engaged can be written as follows:

$$\frac{d^2\theta_2}{d\tau^2} \geq \frac{d^2\beta_2}{d\theta_1^2} \cdot \omega_1^2, \quad (1)$$

where  $\beta_2$  is a rotational correction angle of the female rotor into contact with the male rotor,  $\omega_1$  is the drive rotor angular velocity,  $\theta_2$  is the female rotor angle of rotation,  $\tau$  is a time. The “>” sign in the inequality (1) would indicate that the free rotor can “try to twist” the drive rotor.

Approach to the definition of  $\frac{d^2\theta_2}{d\tau^2}$  is considered in the studies [1, 2 and 3]. From the dynamic equation it can be written that:

$$\frac{d^2\theta_2}{d\tau^2} = \frac{M_{GF} + M_{AF} - \alpha_R \cdot M_R - \alpha_{F1} \cdot M_{F1} - \alpha_{F2} \cdot M_{F2} - M_{MEC}}{J_2}, \quad (2)$$

where  $J_2$  is the free rotor moment of inertia;  $M_{GF}$  is the moment created by the gas forces, which is determined based on the indicator (P-V) diagram according to the sub-tense method [4];  $M_{MEC}$  is the moment of resistance of mechanical compressor components, such as seals, dummy pistons and bearings (its value is determined largely by the design of mechanical components of the rotor);  $M_R$ ,  $M_{F1,2}$  are the moments of friction on the medium being compressed in radial and axial clearances, respectively;  $M_{AF}$  is the moment produced by the forces of adhesion;  $\alpha_R$ ,  $\alpha_{F1}$  and  $\alpha_{F2}$  are the correction factors that take into account the absence of radial and axial surfaces of the housing in the areas of suction and discharge ports execution (They were taken as the ratio of the surface areas on which there is a clearance, excluding the suction and discharge ports, to the area of the corresponding surface, including the suction and discharge ports area, i.e. to the complete cylindrical surface of the complete annular surface). It is not hard to show that the term “adhesion” in this case may be replaced with the more accurate term “cohesion”. In this case, this moment will be proportional to the surface tension coefficient, the contact area and the contact angle; it should be noted at the same time that this moment will be considerably smaller than the other force moments, which in turn makes it possible to ignore it in subsequent calculations:  $M_{AF} \approx 0$ .

The moments of friction on the medium being compressed in end clearances depend on the operating mode [1, 2 and 4] according to the design diagram (Figure 1):

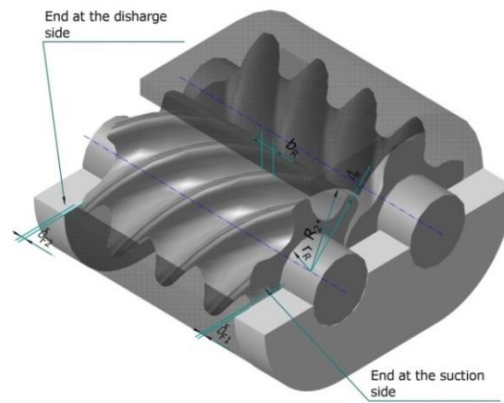
$$M_{F1,2} = \begin{cases} \frac{\pi \cdot \omega_2 \cdot \eta_{MIX} \cdot R_{eq}^4}{2 \cdot \delta_{F1,2}} \cdot \left(1 - \frac{r_R^4}{R_{eq}^4}\right), & \text{if } Re_{F1,2} < 10^4 \\ \frac{\rho_{MIX} \cdot \omega_2^2}{4} \cdot (c'_{M1,2} \cdot R_{eq}^5 - c''_{M1,2} \cdot r_R^5), & \text{if } 10^6 \geq Re_{F1,2} \geq 10^4 \end{cases}, \quad (3)$$

where  $c_{M1,2} = 0,0277 \cdot Re_F^{-0,2} \cdot \left(\frac{\delta_{F1,2}}{R}\right)^{-0,2}$  is a coefficient of friction torque;  $r_R$  ( $R = r_R$ ) is used as a determining

size to ascertain  $c''_M$ , and  $R_{eq}$  ( $R = R_{eq}$ ) is used in all other cases;  $Re_F = \frac{R^2 \cdot \omega_2 \cdot \rho_{MIX}}{\eta_{MIX}}$  is the Reynolds

number;  $\omega_2 = \omega_1 \cdot \frac{z_1}{z_2}$  is the nominal angular velocity of the free rotor;  $\rho_{MIX}$ ,  $\eta_{MIX}$  are, respectively, the density and kinematic viscosity of the gas-oil mixture in the working chamber of the compressor;  $\delta_{F1}$ ,  $\delta_{F2}$  are the end clearances between the rotor and the housing on the suction and discharge side, respectively,  $z_1$  и  $z_2$  are the number of teeth of the male and female rotors.

Equivalent radius is defined as  $R_{eq} = \sqrt[4]{\frac{1}{2 \cdot \pi} \int_0^{2 \cdot \pi} (r(t))^4 \cdot dt}$ , where  $r(t)$  is the function describing the radius vector of the rotor profile in the end section;  $t$  is the most convenient formal parameter selected by trial and error method (it is more convenient to use an angle in the polar coordinate system, but it is not always acceptable).



**Figure 1:** Design diagram of clearances to determine the moments of resistance

By analogy, the moments of friction on the medium being compressed in the radial clearance can be written as [1, 2 and 4]:

$$M_R = \begin{cases} \frac{2 \cdot \pi \cdot \xi \cdot \omega_2 \cdot \eta_{MIX} \cdot R_{2*}^3 \cdot \tilde{z}_2 \cdot b_R}{\delta_R}, & \text{if } Re_R < 2500 \\ \pi \cdot c_R \cdot \rho_{MIX} \cdot \omega_2^2 \cdot z_2 \cdot b_R \cdot R_{2*}^4, & \text{if } 10^5 \geq Re_R \geq 2500 \end{cases}, \quad (4)$$

where  $c_R \approx 0,0076 \cdot Re_R^{-0,25}$  is a friction coefficient [1, 2, 4],  $Re_R = \frac{\omega_2 \cdot R_{2*} \cdot \delta_R \cdot \rho_{MIX}}{\eta_{MIX}}$  is the Reynolds

number,  $b_R$  is the rotor tooth thickness in the Z axis direction,  $R_{2*}$  is the outer radius of the rotor,  $\delta_R$  is a radial clearance between the rotor and the compressor housing,  $\xi = \frac{m_{OIL}}{m_{GAS}}$  is a gas-oil ratio,

$\tilde{z}_2 = \frac{l_R}{l_S} \cdot z_2$  is the reduced number of the female screw teeth in the axial direction,  $l_R$  is the rotor length,  $l_S$  is the screw pitch in the axial direction.

Thermodynamic parameters of the mixture (kinematic viscosity, density, coefficient of cubical expansion and thermal conductivity, respectively) are determined by the following relationships [1, 2, 4 and 5]:

$$\eta_{MIX} = \left( \frac{1}{1+\xi} \right) \cdot \eta_{GAS} + \left( \frac{\xi}{1+\xi} \right) \cdot \eta_{OIL}, \quad \rho_{MIX} = (1+\xi) \cdot \rho_{GAS}, \quad \beta_{MIX} = \left( \frac{1}{1+\xi} \right) \cdot \beta_{GAS} + \left( \frac{\xi}{1+\xi} \right) \cdot \beta_{OIL} \quad (5) \quad (6) \quad (7)$$

$$\lambda_{MIX} = \lambda_{GAS} + \left[ 0.72 \cdot \left( \frac{\xi}{1+\xi} \right)^2 + 0.28 \cdot \left( \frac{\xi}{1+\xi} \right) \right] \cdot (\lambda_{OIL} - \lambda_{GAS}), \quad (8)$$

In this case, the suction temperature for the clearance on the suction side and the discharge temperature for the clearance on the discharge side can be taken as a determining temperature in the preliminary calculations for the equation (3). The arithmetic mean temperature between the suction and discharge can be taken as a determining temperature for the equation (4). In addition, the  $\xi$  value can be taken as constant for all areas of the rotor in order to simplify the calculations.

### 3. Calculation of temperature fields of rotors and thermal deformation thereof

A raft of studies [6, 7, 8 and 9] is currently concerned with calculating the temperature fields of screw compressor rotors, which in varying degrees reveal the influence of various factors on the compressor rotors temperature field. The most comprehensive approach described for temperature field determination is presented in the studies [5 and 6]. However, at the same time, it should be noted that the methodologies in these studies require additional critical analysis and methodological justification.

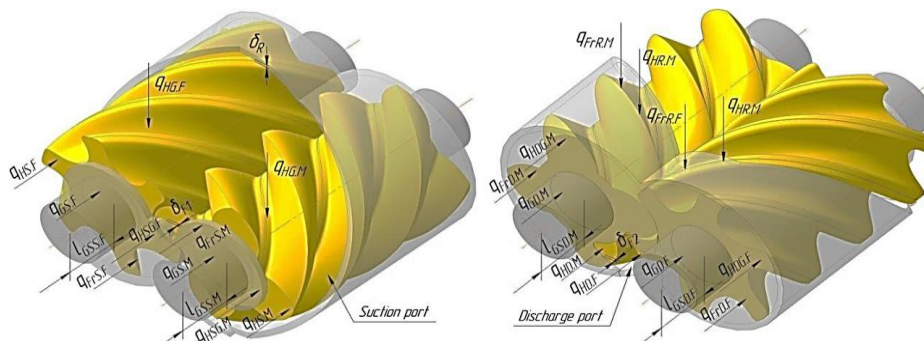
The thermal condition of the rotors is determined by the heat conduction equation:

$$\frac{\partial T}{\partial \tau} = \alpha \cdot \nabla^2 T, \quad (9)$$

where  $\alpha$  is the rotor material thermal diffusivity,  $T$  is a rotor temperature,  $\nabla$  is the nabla operator. The heat load on the compressor rotors is cyclical. However, the period of one cycle is short enough so that the time temperature field of the rotor remains practically unchanged. The latter thesis is confirmed by the research presented in the study [6]. This gives us the opportunity to assume that the pseudostationarity of thermal processes occurs in the rotor. Therefore, the equation (9) can be simplified to:

$$\nabla^2 T = 0. \quad (10)$$

Boundary conditions for the equation (10) are determined by the design diagram (Figure 2), on the basis of equality  $\lambda_M \frac{\partial T}{\partial n} = q$ ,  $\lambda_M$  is the rotor material thermal conductivity, where  $\frac{\partial T}{\partial n}$  is the normal derivative to the rotor surfaces upon which the heat flow  $q$  is acting. Thus, it is necessary to separate the heat gains from the gas-oil medium heat transfer with the rotor surfaces in the working chamber from the heat exchange of the rotor end surfaces with the gas-oil medium in the increasing spaces through the end surfaces of the suction and discharge ports, the heat leakage from the friction of rotors on the gas-oil medium at the end and radial clearances as well as the heat leakage throughout the shaft areas from the bearings.



**Figure 2:** Design diagram for the determination of boundary conditions

The heat flow characterizing the heat transfer of the the gas-oil medium with the rotor surface in the working chamber is described by criteria equations. Thus, the equations [10, 11] were used to describe the heat transfer of the gas-oil medium with the rotor surface:

$$Nu_{GL, Re_{GZ}} = \left[ 49.371 + \left( 1.685 \cdot (Re_{GZ} \cdot Pr_G)^{\frac{1}{3}} - 0.7 \right)^3 + \left( \frac{2}{1 + 22 \cdot Pr_G} \right)^{\frac{1}{6}} \cdot (Re_{GZ} \cdot Pr_G)^{\frac{1}{2}} \right]^{\frac{1}{3}}, \quad Nu_{GT, Re_{GZ}} = \frac{(Re_{GZ} - 1000) \cdot \left( \frac{f}{2} \right) \cdot Pr_G}{\left( 1 + 12.7 \cdot (Pr_G^{2/3} - 1) \cdot \left( \frac{f}{2} \right)^{0.5} \right)}, \quad (11) \quad (12)$$

$$Nu_{G0} = \begin{cases} Nu_{GL, Re_{GZ}}, & \text{if } Re_{GZ} \leq 2300 \\ Nu_{GT, Re_{GZ}}, & \text{if } Re_{GZ} \geq 10000 \\ Nu_{GL, 2300} + [Nu_{GT, 10000} - Nu_{GL, 2300}] \cdot \frac{Re_{GZ} - 2300}{2700}, & \text{if } 10000 > Re_{GZ} > 2300 \end{cases}, \quad (13)$$

where  $f = (1.58 \cdot \ln(Re_{GZ}) - 3.28)^{0.5}$  is a friction coefficient;  $Pr_G$  is the Prandtl number (a mean gas temperature in the working space in the given rotor section is taken as the determining temperature to calculate the Prandtl number);  $Re_{GZ} = \frac{D_H \cdot \left( \frac{w_c}{2} \right) \cdot \rho_{MIX}}{\eta_{MIX}}$  is the Reynolds number;  $D_H$  is the hydraulic

diameter of the corresponding rotor intertooth space end section;  $w_c$  is the determining velocity of the capacity from which a rotor's contact point velocity divided by 2 was taken. The equation (13) was obtained for the fixed channel, while the cavity of the screws is rotated around their axes. i.e. the heat exchange occurs in the gravitational (centrifugal) force field that intensifies the heat exchange somewhat. Correction for this fact is suggested in the study [12] and is slightly modified in the study [13]. According to the latter, the real Nusselt number is determined from the following relation:

$$\frac{Nu_G}{Nu_{G0}} = 0.262 \cdot \left( Gr_* \cdot \frac{Pr_G}{4} \right), \quad (14)$$

where  $Gr_* = Gr \cdot Nu_{G0}$  is the modified Grashof number;  $Gr = \frac{\omega^2 \cdot R_{MC} \cdot D_H^3 \cdot \rho_{MIX}^2}{\eta_{MIX}^2} \cdot \beta_{MIX} \cdot \Delta T$  is the rotating

Grashof number;  $\omega$  is an angular velocity of the corresponding rotor;  $R_{MC}$  is the corresponding rotor's intertooth space end section mass center radius vector;  $\Delta T$  is the temperature difference between the rotor surface and gas in a working chamber. Then, the heat flows to the male and female rotors are respectively determined as follows (the second index in brackets refers to the female rotor):

$$q_{HG.M}^{[HG.F]} = \frac{\alpha_{R1(M)} \cdot Nu_{G(M)} \cdot \lambda_{MIX}}{\frac{D_H^{(M)}}{[H(F)]}} \cdot (T_g - T), \quad (15)$$

where the “(M)” index means that parameters of the male rotor are used as the geometrical parameters, while the “(F)” index means that geometrical parameters of the female rotor are used;  $T_g$  is the mean temperature of the gas-oil medium, with which the heat exchange takes place;  $T$  is the surface temperature, with which the heat exchange takes place;  $\alpha_{R1}$  is a correction factor that takes into account the geometrical characteristics of the compressor (the ratio of the diameters of the rotor housing bores, the presence and size of ports on the radial part of the housing); these coefficients are equal to 1 for areas of rotors that do not form a radial clearance, and are equal to the relationship of the arc length whereon a clearance is formed to the circumference of the rotor housing bore for the sections forming a radial clearance.

Flowing of gas-oil fluid through the radial clearance is fairly complex. For the purposes of approximate description, this clearance may be represented as the portion of the rotary cylinder and the flow in it may be represented as the flow in the axial direction within the clearance between the rotary cylinder and the stationary housing. In this case, the flow is accompanied by the formation of Taylor vortices; the heat exchange in those conditions in the widest range of flow and rotation velocities as well as in the wide range of transport properties of the flowing medium is presented in the study [14]. It is proposed to use the following equation in order to describe the heat exchange therein:

$$Nu_R = 0.21 \cdot (Ta \cdot Pr_R)^{0.25}, \quad (16)$$

where  $Ta = \frac{\left(\frac{w_c}{2}\right)^2 \cdot \delta_R^2 \cdot \rho_{MIX}^2}{\eta_{MIX}^2} \cdot \frac{\delta_R}{R_*}$  is the Taylor number;  $Pr_R$  is the Prandtl number (the mean temperature between the mean temperature of the gas-oil medium in the working chamber in the given rotor section and the rotor heat transfer surface temperature is taken as the determining temperature to calculate the thermal and transport properties of the medium being compressed);  $\delta_R$  is a radial clearance;  $R_*$  is a rotor housing bore nominal radius. Then, the heat flows to the male and female rotor will be respectively defined as:

$$q_{HR,M}^{[HR,F]} = \frac{\left(1 - \alpha_{R1(M)}^{[R1(F)]}\right) \cdot Nu_{R(M)}^{[R(F)]} \cdot \lambda_{MIX}}{\delta_{R(M)}^{[R(F)]}} \cdot (T_g - T), \quad (17)$$

The end surfaces heat exchange is determined by the following equation [15]:

$$Nu_{FG,S}^{[FG,D]} = \begin{cases} 0.616 \cdot Re_w^{0.5} \cdot Pr^{0.435}, & \text{if } Re_w \leq 2300 \\ 0.0267 \cdot Re_w^{0.8} \cdot Pr^{0.6}, & \text{if } Re_w \geq 10000 \\ 0.616 \cdot (2300)^{0.5} \cdot Pr^{0.435} + \left(0.0267 \cdot (10000)^{0.6} \cdot Pr^{0.6} - 0.616 \cdot (2300)^{0.5} \cdot Pr^{0.435}\right) \cdot \frac{Re_w - 2300}{7700}, & \text{if } 10000 > Re_w > 2300 \end{cases}, \quad (18)$$

where  $Re_w = \frac{R^2 \cdot \omega \cdot \rho_{MIX}^2}{\eta_{MIX}}$  is the rotating Reynolds number,  $R$  is a heat exchange surface point radius

vector. The temperature in the port is the determining temperature here. The “<sub>s</sub>” index refers to the suction side, while the “<sub>d</sub>” index refers to the discharge side. Then, the heat flow to the rotor from the direction of the suction and discharge end face is respectively defined as:

$$q_{HS,M}^{[HS,F]} = \frac{\alpha_{FS1(M)}^{[FS1(F)]} \cdot Nu_{FG,S(M)}^{[FG,S(F)]} \cdot \lambda_{MIX}}{R} \cdot (T_g - T), \quad q_{HD,M}^{[HD,F]} = \frac{\alpha_{FD1(M)}^{[FD1(F)]} \cdot Nu_{FG,D(M)}^{[FG,D(F)]} \cdot \lambda_{MIX}}{R} \cdot (T_g - T), \quad (19) \quad (20)$$

$\alpha_{FS}$  and  $\alpha_{FD}$  are correction factors equal to the ratio of the port sectors area, which is intersected by the rotor during rotation on the end surface of the housing, to the area of the end section of the cylinder by the rotor housing bore radius.

The heat exchange at the end clearance can be defined in a similar fashion [15]:

$$Nu_{FGG,S}^{[FGG,D]} = \begin{cases} 0.922 \cdot Re_w^{0.5} \cdot Pr^{0.435}, & Re_w \leq 2300 \\ 0.0251 \cdot Re_w^{0.8} \cdot Pr^{0.6}, & Re_w \geq 10000 \\ 0.922 \cdot (2300)^{0.5} \cdot Pr^{0.435} + \left(0.0251 \cdot (10000)^{0.6} \cdot Pr^{0.6} - 0.922 \cdot (2300)^{0.5} \cdot Pr^{0.435}\right) \cdot \frac{Re_w - 2300}{7700}, & 10000 > Re_w > 2300 \end{cases}, \quad (31)$$

$$q_{HSG,M}^{[HSG,F]} = \frac{\left(1 - \alpha_{FS1(M)}^{[FS1(F)]}\right) \cdot Nu_{FGG,S(M)}^{[FGG,S(F)]} \cdot \lambda_{MIX}}{R} \cdot (T_g - T), \quad q_{HDG,M}^{[HDG,F]} = \frac{\alpha_{FD1(M)}^{[FD1(F)]} \cdot Nu_{FG,D(M)}^{[FG,D(F)]} \cdot \lambda_{MIX}}{R} \cdot (T_g - T), \quad (22) \quad (23)$$

In this case, the mean temperature between the gas-oil medium temperature in the port and the rotor heat exchange surface temperature is taken as the determining temperature to calculate the thermal and transport properties of the medium being compressed.

The equations describing the friction in the clearances [4, 6] under the assumption of a uniform distribution of heat generated between the housing and the rotor takes the following form:

$$q_{FR,M}^{[FR,F]} = \frac{\left(1 - \alpha_{R1(M)}^{[R1(F)]}\right) \cdot \eta_{MIX} \cdot R^2 \cdot \omega_1^2}{2 \cdot \delta_{R(M)}^{[R(F)]}} \cdot (T_g - T), \quad (24)$$

$$q_{FrG,M}^{[FrG,F]} = \frac{\left(1 - \alpha_{FS1(M)}^{[FS1(F)]}\right) \cdot \eta_{MIX} \cdot R^2 \cdot \omega_1^2}{2 \cdot \delta_{F1(M)}^{[F1(F)]}} \cdot (T_g - T), \quad q_{FrG,M}^{[FrG,F]} = \frac{\left(1 - \alpha_{FD1(M)}^{[FD1(F)]}\right) \cdot \eta_{MIX} \cdot R^2 \cdot \omega_1^2}{2 \cdot \delta_{F2(M)}^{[F2(F)]}} \cdot (T_g - T) \cdot \quad (25) (26)$$

The parameters of the gas-oil medium in this case were selected using a method similar to that used in the calculation of the heat exchange in the relevant section. Calculation of the heat gain from the bearings through the rotor spike can be carried out according to the following dependences:

$$q_{GS,M}^{[GS,F]} = \frac{\lambda_R}{l_{GSS,M}^{[GSS,F]}} \cdot \left(T_{S,M}^{[S,F]} - T\right), \quad q_{GD,M}^{[GD,F]} = \frac{\lambda_R}{l_{GSD,M}^{[GSD,F]}} \cdot \left(T_{D,M}^{[D,F]} - T\right), \quad (27) (28)$$

where  $\lambda_R$  is the thermal conductivity of the rotor material;  $l_{GSS}$ ,  $l_{GSD}$  are the length of the rotor spikes on the suction and discharge side,  $T_D$  is the temperature in the section at the spike end; it may with a reasonable degree of reliability be assumed to be equal to the temperature of the oil discharged from the bearing.

The computational grid was structured and built by a method similar to that described in the study [16]. The gas temperature as well as the forces and moments acting on the rotors were determined based on the temperature and the indicator diagrams obtained using a mathematical model [4]. The gas-oil medium temperature was time-averaged for each section.

The equations describing the displacements and stresses ratio [17] were used to calculate the thermal deformations:

$$(\varepsilon_x - \beta_R \cdot (T - T_{St})) = \frac{\sigma_x - \nu \cdot (\sigma_y + \sigma_z)}{E}, \quad (\varepsilon_x - \beta_R \cdot (T - T_{St})) = \frac{\sigma_x - \nu \cdot (\sigma_y + \sigma_z)}{E}, \quad (29) (30)$$

$$(\varepsilon_x - \beta_R \cdot (T - T_{St})) = \frac{\sigma_x - \nu \cdot (\sigma_y + \sigma_z)}{E}, \quad \gamma_{xy} = \frac{\tau_{xy}}{G}, \quad \gamma_{xz} = \frac{\tau_{xz}}{G}, \quad \gamma_{yz} = \frac{\tau_{yz}}{G}, \quad (31) (32) (33) (34)$$

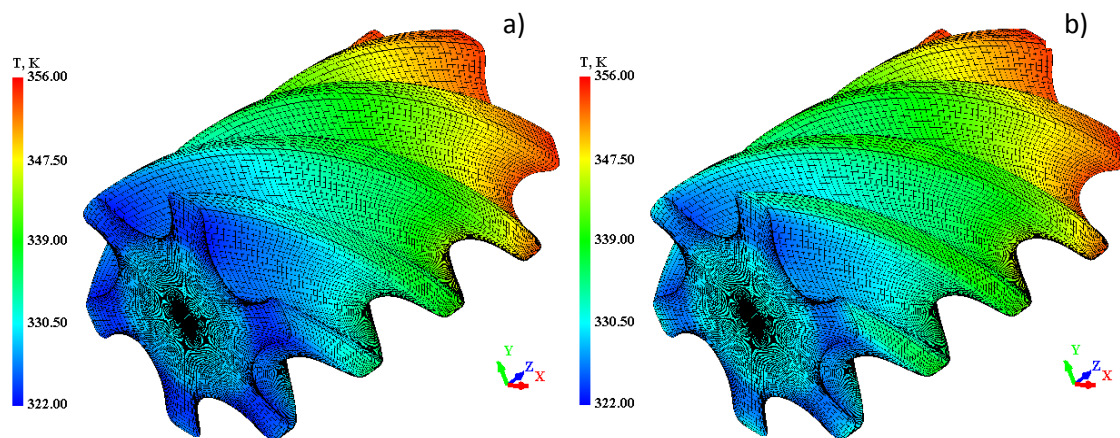
where  $x, y, z$  are the arrangement of axes;  $\beta_R$  is the rotors material coefficient of thermal expansion,  $G$  is the rotors material modulus of elasticity in shear;  $E$  is the rotors material modulus of elasticity under tension;  $\nu$  is the Poisson's ratio;  $\sigma$ ,  $\tau$  are the normal and shear stresses, respectively;  $\varepsilon$ ,  $\gamma$  are the relative elongation and relative shear deformation,  $T_{St}$  is the standard rotor temperature that was assumed to be 293 K. In this case, the absence of axial movement of the mid-section of the rotor, and the absence of deformation of the rotors from external forces were taken as the boundary conditions (they were neglected as being small quantities [4]).

#### 4. Calculation data

A standard screw compressor developed by CJSC "NIIturbokompressor" named after V.B. Shneppe" (successor of the Special Design Bureau for Compressor Engineering) was selected as the object of research with a rotor diameter of 250 mm, a length of the rotor profile section of 340 mm, a screw pitch of 440 mm and 660 mm for the male and female rotor, respectively, and a gear ratio of 4/6 similar to the profiles described in the studies [1, 2 and 4]. The following quantities were taken for the end clearance between the rotor and the housing: 0.5 mm on the suction side and 0.06 mm on the discharge side. The value of 0.2 mm was taken as the radial clearance. The following condition was selected as the study condition: the compressor's outdoor operating condition with compression from 1 to 9 at (pressure ratio of  $\Pi = 9$ ), suction temperature of 25°C, discharge temperature of 90 °C, injected oil temperature of 60 °C and geometric compression ratio of 4.5. The gas-oil-ratio was assumed to be equal to 3.8. Rotor bearings center distance was taken as 200.11 mm on the suction side and 200.132 mm on the discharge side, respectively.

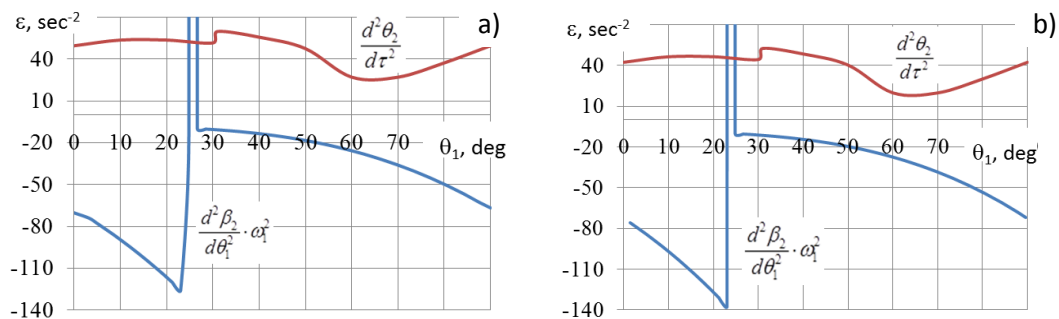
The results of the female rotor temperature fields' calculations, as well as the plots characterizing the engagement in the compressor operation process are shown in Figures 3 and 4. The calculations are given for two cases: when the compressor design provides for a suction port on the radial housing bore, and when it is not provided.

The results of temperature fields' calculations show that the radial rotor vertices are exposed to maximum warming. This is especially noticeable at the vertex of the female rotor, and can be explained by the significant frictional heat in the radial clearance. It should be noted here that the gas-oil medium temperature at the suction is lower, and hence its viscosity is considerably higher, and, as a consequence, the frictional heat in the clearance is also higher. This explains the higher rotor temperature in the housing without a suction port on the radial housing bore, as well as lower resistance of the latter to loss of contact in engagement in the course of operation because of the large frictional losses. Analysis of the results shows correctness of engagement during compressor operations in both considered variants. The gearing disconnection occurs only at the moment of the contact rerun between the rotor teeth. This moment is characterized by the discontinuity of the  $\frac{d^2\beta_2}{d\theta_1^2} \cdot \omega_1^2$  function in the Figure 4. (At this moment the  $\frac{d^2\beta_2}{d\theta_1^2} \cdot \omega_1^2$  function increases to infinity).



**Figure 3:** Temperature profiles of rotors:

- a) with a suction port on the radial housing bore;
- b) without a suction port on the radial housing bore



**Figure 4:** Acceleration of female rotor

(where  $\theta_1$  is the male rotor angle of rotation,  $\varepsilon$  is the female rotor angular acceleration):

- a) with a suction port on the radial housing bore;
- b) without a suction port on the radial housing bore



## 5. Conclusions

The developed computational model makes it possible to calculate the temperature fields of the rotors. At the same time, it allows us to estimate the impact of all factors, both individually and in their entirety, including not only the operating condition, but also the design features of the compressor.

The obtained results will also allow us to improve the accuracy of a screw compressor mathematical model, which can be used for the calculation of the gas parameters during the working process. In this case, the possibility exists to predict the sizes of the real clearances more accurately, which means the separation of the working chambers during the compressor working process. It should be noted that the additional heating of rotor teeth tips in the case of the absent ports on the radial surface of the housing may cause an increasing amount of back leakage gas temperature through the gaps, which are formed by the rotor teeth tips surface and the housing surface. Also, the compressing gas temperature may increase during the suction process due to the contact of the suction gas with hotter surfaces. These factors explain the reduction of the compressor capacity and increase of the power consumption in the case of the absent ports on the radial surface of the housing. The power consumption increases due to the increasing of the compressor indicated power and friction loss. This additional heating also exerts its influence on the rotor thermal field only in the boundary zones of the rotor teeth tips; therefore, the influence of this heating on the rotor thermal deformations of the contact zones and in consequence on the rotor meshing is marginal. In view of the above factors, it is recommended to manufacture the compressor ports or undercutting that repeat the port contours on the compressor housing radial surfaces.

The analysis of engagement during the operating conditions of the compressor was conducted on the basis of the obtained temperature field. This technique complements and builds upon the previously presented engagement analysis technique and can be the basis for further research of rotor engagement and vibration in the future.

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