

A two-level model of rise time in quantum cascade laser materials applied to 5 micron, 9 micron and terahertz-range wavelengths

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Abstract. An equivalent circuit simulation of a two-level rate equation model for quantum cascade laser (QCL) materials is used to study the turn on delay and rise time for three QCLs with 5 micron, 9 micron and terahertz-range wavelengths. In order to do this it is necessary that the model can deal with large signal responses and not be restricted to small signal responses; the model used here is capable of this. The effect of varying some of the characteristic times in the model is also investigated. The comparison of the terahertz wave QCL with the others is particularly important given the increased interest in terahertz sources which have a large range of important applications, such as in medical imaging.

1. Introduction

Quantum cascade lasers (QCLs) [1] have many applications. Of particular relevance to the work here are applications in communication systems. For free space communications[2] measurements have shown that there are two windows, 3 to 5 μm and 8 to 13 μm , in which the atmosphere is relatively transparent compared to near infrared wavelengths. At high altitudes terahertz-range waves can be used for communications. This range has many other applications, such as in medical imaging and spectroscopy. Therefore in this paper, we consider three QCLs with the emission wavelengths of 5 μm [3], 9 μm [4] and, for the terahertz-range, 103 μm [5].

A lot of numerical[6–9] and analytical[3, 4, 10] work has been published on the modulation behaviour of QCLs. But this work has been concerned with steady-state and small-signal responses. Alone this is not enough to adequately describe the range of the modulation behaviour; the large signal response for pulse code modulation is also important. In order to deal with this we employ an equivalent circuit model[6] of a QCL based on improved two level rate equations[3]. The equivalent circuit (Fig. 1) is relatively straightforward can easily deal with both small and large signal responses which is convenient for studying the turn-on delay and rise time of the QCLs considered here.

Under the following assumptions [3]:

- (i) all quantum wells and injectors are identical and hence the gains of the wells are the same,
- (ii) confinement factors are identical,



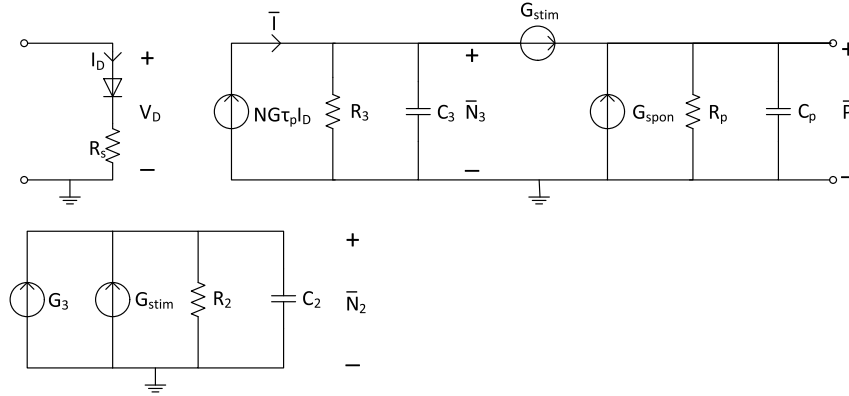


Figure 1. Equivalent circuit for a QCL modelled from normalized two-level rate equations. The normalized quantities are $\bar{I} = NG\tau_P I$, $\bar{N}_3 = NG\tau_P N_3$, $\bar{N}_2 = NG\tau_P N_2$ and $\bar{P} = P/K$, with $K = 1/[G\tau_3(1 + \tau_{21}/\tau_{31})]$. Also, $V_D = nV_T \ln(I/I_S + 1) + I_D R_S$, n is a diode ideality factor, $V_T = kT/e$, k is Boltzmann's constant, e is the electronic charge magnitude, $G_{stim} = eKG(\bar{N}_3 - \bar{N}_2)\bar{P}$, $G_{spon} = e\beta\bar{N}_3/\tau_{sp}$, $G_3 = e(1/\tau_{32} + 1/\tau_{sp})\bar{N}_3$, $R_3 = \tau_3/e$, $R_2 = \tau_{21}/e$, $C_3 = C_2 = e$, and $C_P = eKG\tau_P$, $R_P = 1/(eKG)$.

(iii) removal rates for electrons are fast,

(iv) the coupling of the spontaneous emission to the lasing mode can be ignored,

the two level rate equations of a QCL including the coupling of the spontaneous emission to the lasing mode can be written as [3]:

$$\frac{dN_3}{dt} = \frac{I}{e} - \frac{N_3}{\tau_3} - G(N_3 - N_2)P \quad (1)$$

$$\frac{dN_2}{dt} = \left(\frac{1}{\tau_{32}} + \frac{1}{\tau_{sp}} \right) N_3 - \frac{N_2}{\tau_{21}} + G(N_3 - N_2)P \quad (2)$$

$$\frac{dP}{dt} = NG(N_3 - N_2)P + N\beta\frac{N_3}{\tau_{sp}} - \frac{P}{\tau_p} \quad (3)$$

where N_2 and N_3 are the electron numbers in levels 2 and 3 respectively; I is the injected current; G is the gain coefficient per stage, which is the product of the confinement factor and the gain due to stimulated emission; τ_{31} , τ_{32} and τ_{21} are the lifetimes representing the transitions from levels 3 to 1, 3 to 2 and 2 to 1 respectively; τ_{sp} is the spontaneous lifetime between level 3 and 2; τ_P is the photon lifetime; P is the photon number in the cavity; N is the number of stages; β is the fraction of spontaneous emission entering the lasing mode; and t is time. τ_3 is given by $1/\tau_3 = 1/\tau_{31} + 1/\tau_{32} + 1/\tau_{sp}$. As discussed in Ref. [6], if Eqs. (1) to (3) are used to derive the circuit simulator it will fail to converge. This problem is solved by employing a systematic normalization procedure given in Ref.[6].

2. Results and Discussion

Turn-on delay and rise time of the three QCLs at the wavelengths 5 μm , 9 μm , and 103 μm are obtained through numerical simulation using the equivalent circuit model outlined above[6]. The parameter value for each QCL is given in Table 1.

Turn-on delay is defined as the time taken for the photon number to reach 10% of its steady value upon switching on. Rise time is defined as the time taken for the photon number to rise

Parameters			
Wavelength (μm)	5	9	103
Number of gain stages, N	25	48	30
Total confinement factor, Γ_N	0.5	0.32	0.27
Cavity width, W (μm)	11.7	34	80
Cavity length, L (mm)	3	1	3
Facet reflectivity, R	0.27	0.29	0.29
Mode-group index, n_g	3.4	3.27	3.3
Cavity internal loss, α_i (cm^{-1})	11	20	24
Differential gain, a (cm)	4×10^{-9}	4×10^{-9}	5×10^{-8}
Gain coefficient per stage, $G = a \frac{\Gamma_N}{N} \frac{c}{n_g L W}$ (s^{-1})	2×10^3	0.7×10^3	1.7×10^3
Spontaneous emission factor, β	2×10^{-3}	2×10^{-3}	2×10^{-3}
τ_3 (ps)	1.3	1.4	1.1
τ_{32} (ps)	2.1	2.1	2
τ_{31} (ps)	3.4	4.2	2.4
τ_{21} (ps)	0.5	0.3	0.3
τ_p (ps)	7.38	3.36	3.7
τ_{sp} (ns)	6.06	35.5	7
Threshold current, I_{th} (A)	0.44	1.11	0.91

Table 1. Parameter values for the 5 μm , 9 μm and 103 μm QCLs

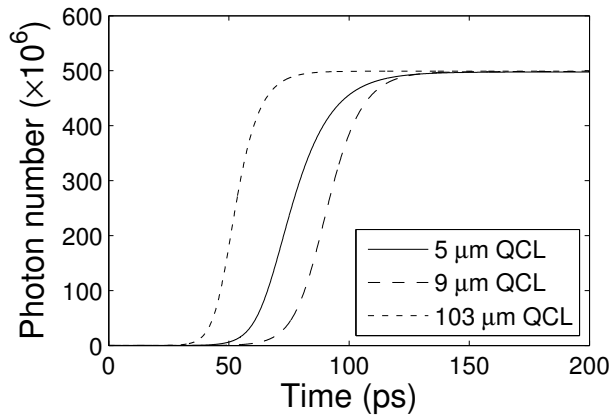


Figure 2. Turn-on response for 5 μm , 9 μm and 103 μm QCLs. Value of injected current has been carefully selected to give the same photon number output. Current is applied at $t = 0$.

from 10% to 90% of its steady value. Here, we have investigated the transient response for the three QCLs for a fixed photon number. This photon number is set at 500×10^6 .

Figure 2 shows 103 μm QCL has the shortest turn-on delay of 43 ps and followed by 5 μm and 9 μm QCLs with turn-on delay of 61 ps and 76 ps respectively. As for rise time, 103 μm QCL has the smallest value of 22 ps and followed by 9 μm and 5 μm QCLs with rise time of 32 ps and 37 ps respectively. To fully understand why the response for the three QCLs differ as such, we observe how a single parameter affect the response. Two parameters chosen in this paper are τ_p and τ_{sp} as these parameters have significant difference between the three QCLs. Only one parameter will have its value changed each time while the rest of the parameters value will remains the same. The results are plotted in Figure 3.

Figure 3 shows that τ_p affects both the turn-on delay and rise time of QCLs. Shorter photon

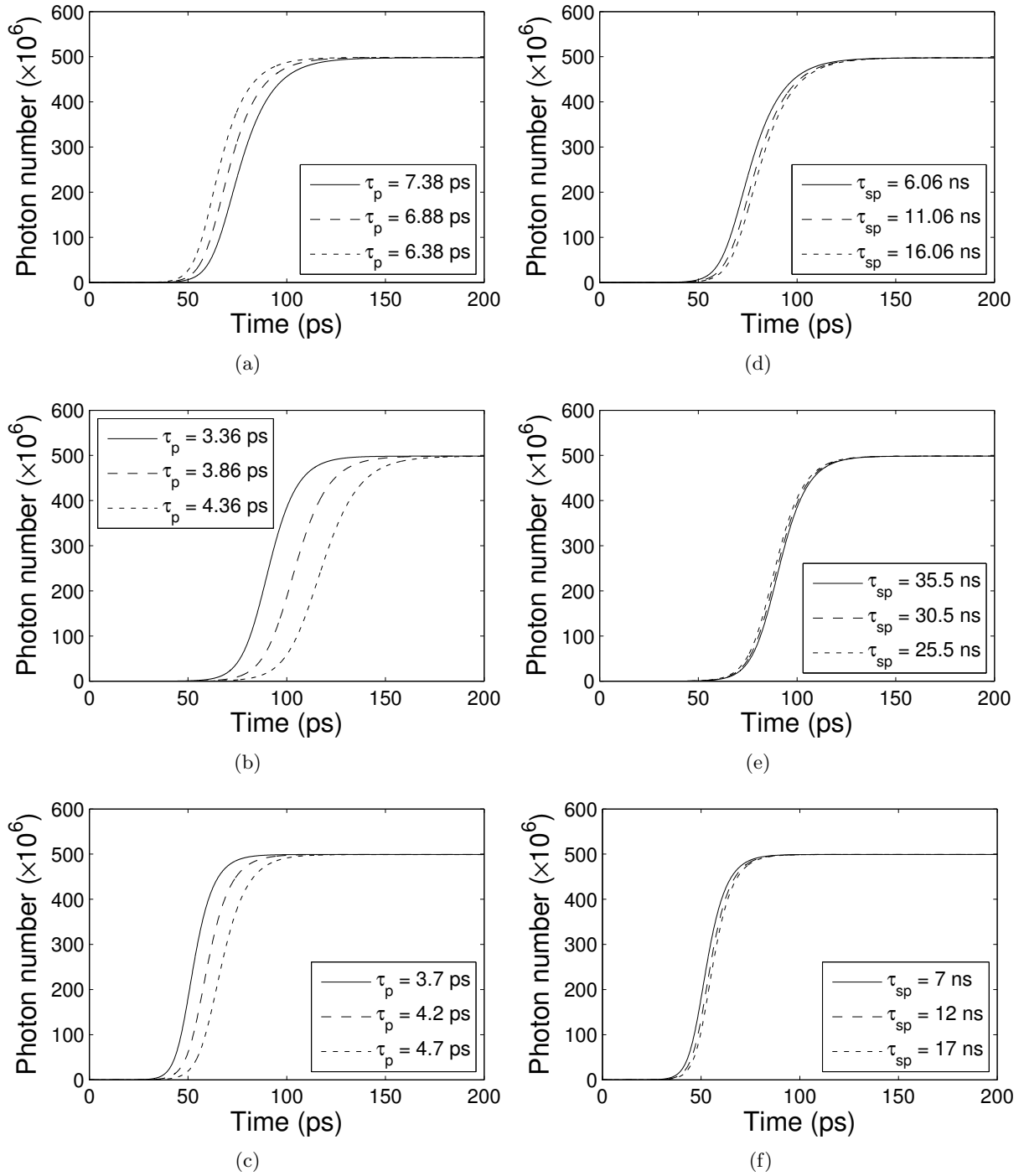


Figure 3. (First column) Turn-on response for (a) 5 μm (b) 9 μm and (c) 103 μm QCL with different value of τ_p . (Second column) Turn-on response for (d) 5 μm (e) 9 μm and (f) 103 μm QCL with different value of τ_{sp} . Value of injected current has been carefully selected to give the same photon number output. Current is applied at $t = 0$.

lifetime contributes to lower turn-on delay and rise time. As for τ_{sp} , this parameter affects the turn-on delay of QCL only. Shorter radiative spontaneous lifetime results in lower turn-on delay. Hence, we can deduce that the 103 μm QCL has the least turn-on delay and rise time

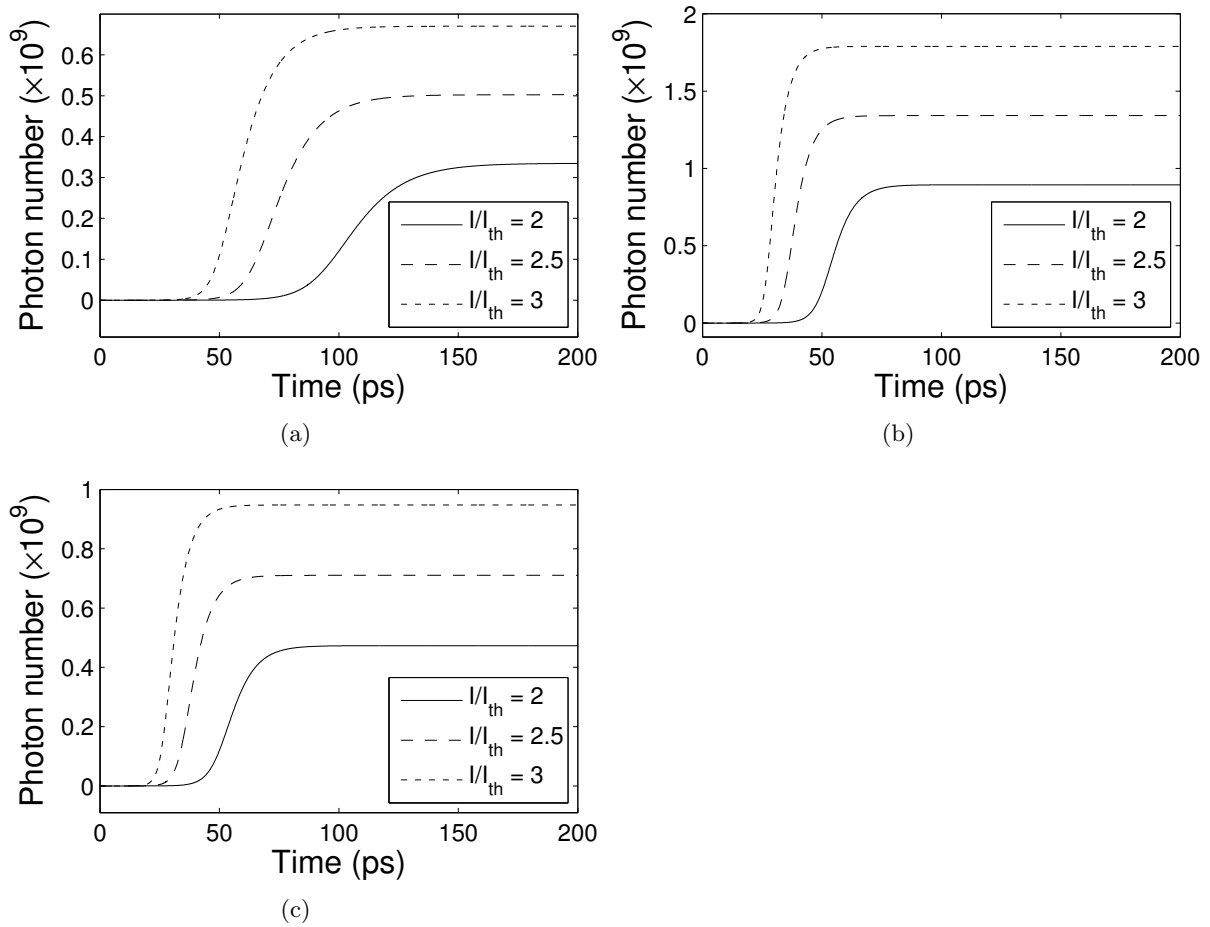


Figure 4. Turn-on response for (a) 5 μm (b) 9 μm and (c) 103 μm QCL with different values of injected current.

partly because it has relatively low values of τ_p and τ_{sp} compared to the 5 μm and 9 μm QCLs. Parameters that affect turn-on delay and rise time are not limited to τ_p and τ_{sp} ; other parameters will also affect the transient response but will not be discussed here.

Now, we investigate the effect of injected current on turn-on delay and rise time. The transient response of different level of injected current for the three QCLs is shown in Figure 4. Turn-on delay decreases with increasing injected current for all three QCLs. This is consistent with the finding of Hamadou et al.[4]. We have also observed that a higher level of injected current results in a lower rise time.

3. Conclusion

In conclusion, we have shown in this paper that the turn-on delay and rise time of quantum cascade lasers at three different wavelengths depends on photon lifetime, radiative spontaneous lifetime and injected current value. Currently, we are working on analytical expression for turn-on delay, rise time and fall time of pulse-modulated QCLs to give a better understanding of how various parameters affect the QCL transient response.

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