

Cross-flow and heat transfer of porous permeable cylinder

I V Morenko¹, V L Fedyaev^{1,2} and E R Galimov²

¹ Institute of Mechanics and Engineering, Kazan Science Center, Russian Academy of Sciences, Russia

² Kazan National Research Technical University named after A N Tupolev, Russia

E-mail: morenko@imm.knc.ru

Abstract. Non-isothermal flow around the porous permeable cylinder of square cross section by a viscous incompressible fluid is considered. The motion and energy equations is solved numerically using the finite volume method. The character of the fluid flow, change of the cylinder drag coefficient and Nusselt number as a function of Reynolds and Darcy numbers is analyzed.

1. Introduction

Porous permeable bodies streamlined by fluid are widely present in nature and engineering particularly in biological filters, cooling systems, heat exchangers, nuclear reactors and dryers.

In virtue of the practical relevance, complexity of hydrodynamic and thermal processes the problems of flowing around the porous permeable bodies are of great interest. First studies of fluid flows through a porous medium were carried out by N.Ye. Zhukovsky [1]. In the 50-80s of the last century flowing around the bodies was studied analytically [2]. Recently, with the development of methods of mathematical modeling and computer technologies the numerical methods are increasingly used [3-7].

In this paper we investigate the non-isothermal flow around the porous permeable cylinder of square cross section by a viscous incompressible fluid at moderate Reynolds numbers $Re=1\div 40$.

2. Governing equations

As a computational domain we choose rectangle with length 0.5 m and width 0.2 m wherein at a distance of 0.1 m from the inlet section at an equal distance from lateral boundaries streamlined cylinder with cross section in the shape of a square with the side $a=0.01$ m is located.

In the Cartesian coordinate x_1Ox_2 system the beginning of which is placed in the center of the cylinder section axis Ox_1 is directed along the incident flow, plane-parallel laminar flow of a viscous incompressible fluid is described by the equations

$$\frac{\partial v_i}{\partial x_i} = 0, \quad (1)$$

$$\frac{1}{\varepsilon} \frac{\partial v_i}{\partial t} + v_j \frac{1}{\varepsilon^2} \frac{\partial v_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \nu \frac{1}{\varepsilon} \frac{\partial^2 v_i}{\partial x_j \partial x_j} - \frac{\nu}{k} v_i, \quad i=1,2. \quad (2)$$



Here t is the time; $v_i = \varepsilon u_i$ is the velocity components of fluid filtration, u_i is the velocity vector components \vec{u} , p is the pressure, ρ is the fluid density, ν is the coefficient of kinematic fluid viscosity, k is the cylinder permeability, the porosity

$$\varepsilon = \begin{cases} 1 & \text{outside the body,} \\ 0 < \varepsilon < 1 & \text{within the body;} \end{cases}$$

summation is performed over the repeated indices.

Energy conservation equation for a fluid has the form

$$\frac{\partial}{\partial t}(\rho E) + \nabla \cdot (\vec{u}(\rho E + p)) = \lambda \Delta T - Q_w, \quad (3)$$

where $E = h - \frac{p}{\rho} + \frac{u^2}{2}$, h is the enthalpy, T is the fluid temperature, λ is the thermal conductivity coefficient of a fluid, Q_w is the capacity of the heat flow to the frame of a permeable cylinder.

Velocity profile, pressure, temperature of the fluid are specified in the inlet section of the computational domain. “Soft” boundary conditions meaning the alignment of both hydrodynamic and thermal characteristics of the fluid flow at a distance from the streamlined cylinder are set at the outlet of the computational domain. On the lateral boundaries – symmetry conditions. At the initial time $t = 0$ fluid instantaneously starts to move.

The problem is solved by the finite volume method.

3. Results

We consider the motion of a fluid with the following parameters (air): density $\rho = 1.225 \text{ kg/m}^3$, dynamic viscosity coefficient $\mu = 1.79 \cdot 10^{-5} \text{ kg/(m}\cdot\text{s)}$, specific heat capacity at constant pressure $c = 1000 \text{ J/(kg}\cdot\text{K)}$, thermal conductivity coefficient $\lambda = 0.0242 \text{ W/(m}\cdot\text{K)}$. The air temperature on inlet $T = T_0 = 300 \text{ K}$, the temperature of the cylinder frame $T_w = 350 \text{ K}$.

When fluid flows around the body the main calculated values are drag coefficient $C_D = \frac{F_{x1}}{0.5 \rho u_\infty^2 a}$,

Nusselt number $Nu = \alpha a / \lambda$, which characterizes the intensity of heat transfer. Here u_∞ is the fluid longitudinal velocity at the inlet of the computational domain, F_{x1} is the projection on axis Ox_1 of the force of hydrodynamic resistance, $\alpha = \frac{\lambda}{T_w - T_\infty} \left(\frac{\partial T}{\partial n} \right)$ is the local coefficient of convective heat exchange, n is the outward normal to the cylinder surface. Average over the cylinder surface Nusselt number \overline{Nu} is also calculated.

During numerical experiments we found that the flowing around the porous cylinder of square cross section at said Reynolds numbers and porosity $\varepsilon = 0.4 \div 0.9$ occurs in the stationary mode, vortex trail does not form. For small numbers of Darcy $Da = k/a^2$ flow around pattern of the permeable cylinder is similar to the flow around pattern of impermeable cylinder (figure 1 a). With the increase of values of Darcy number the fluid passes through the body. In this case when $Da = 10^{-4}$, 10^{-3} the vortex region behind the body is preserved (figure 1 b, c), when $Da = 10^{-2}$ it disappears (figure 1 d).

The drag coefficient of the cylinder decreases with an increase of both Reynolds and Darcy numbers. For calculating its value in the range $5 \leq Re \leq 40$, $10^{-6} \leq Da \leq 10^{-2}$ it is proposed to use the approximation formula

$$C_D \cong 16.0(1 - 0.07 \lg Da) \cdot (1 + 0.06 Re) / Re.$$

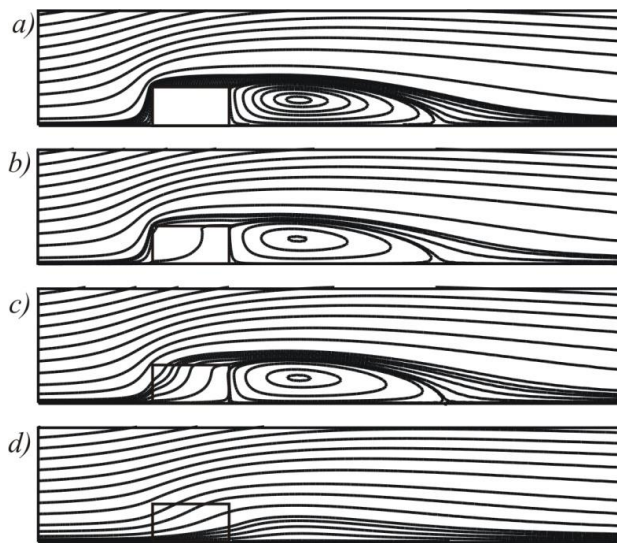


Figure 1. Streamlines on the half of the computational domain when $Re = 40$ and Darcy numbers Da : a) 10^{-6} , b) 10^{-4} , c) 10^{-3} , d) 10^{-2}

Figure 2 shows the variation of Nusselt number at different parts of the cylinder surface. It can be seen that heat transfer is mainly carried out on the windward side of the cylinder on the rear side it is minimal. With the growth of cylinder permeability Nusselt number on its windward side of noticeably increases, but on the lateral and rear sides, conversely, slightly decreases.

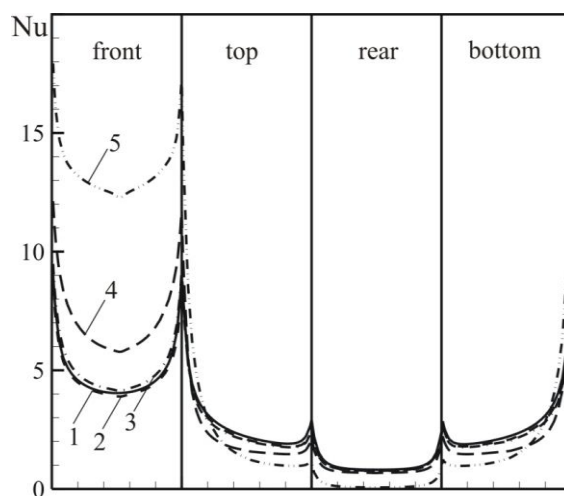


Figure 2. Local Nusselt number when $Re=40$ on the cylinder surface (front, top, rear, bottom sides one after another): 1 – impermeable, 2 – permeable, $Da=10^{-6}$; 3 – $Da=10^{-4}$; 4 – $Da=10^{-3}$; 5 – $Da=10^{-2}$

Comparing the obtained values of the average Nusselt number \overline{Nu}_f on front side of the streamlined cylinder with the data [6, 7], we see that for small permeability ($Da=10^{-6}$, $Da=10^{-4}$) of the cylinder, they are practically the same (figure 3). In case of greater permeability ($Da=10^{-2}$) when $Re>20$ the discrepancy is observed and it increases with the growth of Reynolds number. In our opinion, this discrepancy is due to the fact that in work [6] only the surface of the frame of streamlined body is heated, whereas in this work - the whole frame. Furthermore, takes into account the inertial component depending on the pressure drop when fluid flows through the permeable body which can be better seen at higher Reynolds numbers [6].

In contrast to the front side on the rear side of the cylinder surface heat transfer decreases with an increase of permeability (figure 4). It is interesting that, when the Reynolds numbers increase and the Darcy numbers are small ($Da=10^{-6}$, $Da=10^{-4}$, $Da=10^{-3}$) the average Nusselt number \overline{Nu}_r on this side of the cylinder surface increases, notably the increase is the most intensive at $Re < 5$. In the range $5 \leq Re \leq 40$ dependence of \overline{Nu}_r on Re is almost linear.

In case when Darcy number is relatively high $Da=10^{-2}$ and Re is approximately equal to 5 there is an obviously expressed maximum of dependency of \overline{Nu}_r on Re . Reduction of Nusselt number with the decrease of Reynolds number is quite understandable, whereas deterioration of heat transfer with an increase of Reynolds number when $Re>5$ should be explained. As can be seen from figure 1 *d* the vortex region is not formed behind the cylinder, the fluid flow rate through the cylinder is relatively high in the central part and low at the periphery. Significant unevenness of fluid velocities in cross section of the cylinder transversal to the flow, in our opinion, can be the reason for this unusual behavior of the Nusselt number \overline{Nu}_r as a function of the Reynolds number of the incident flow.

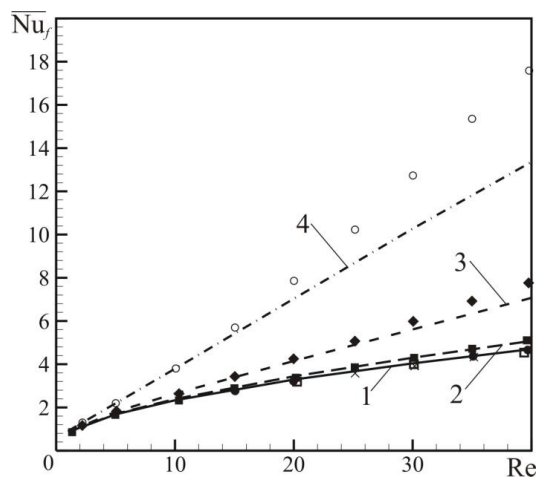


Figure 3. Average Nusselt number \overline{Nu}_f on the front (windward) side of the cylinder surface when $Re=40$ and values of Da : 1 – 10^{-6} ; 2 – 10^{-4} ; 3 – 10^{-3} ; 4 – 10^{-2} . Comparison with the data of other authors: ● – 10^{-6} , ■ – 10^{-4} , ◆ – 10^{-3} , ○ – 10^{-2} , × – impermeable cylinder [6]; □ – 10^{-6} [7]

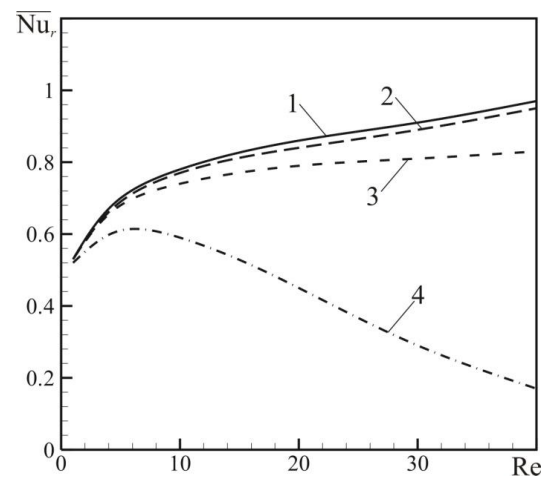


Figure 4. Average Nusselt number \overline{Nu}_r on the rare side of the cylinder surface when $Re=40$ and values of Da : 1 – 10^{-6} ; 2 – 10^{-4} ; 3 – 10^{-3} ; 4 – 10^{-2}

As it appears from the results of calculations on the lateral surfaces of the cylinder average Nusselt number \overline{Nu}_b changes depending on the Reynolds and Darcy numbers as follows

$$\overline{Nu}_b = 0.45(1 - 0.022 \lg Da) \cdot (1 - 0.008 Re) \sqrt{Re}.$$

Average Nusselt number for the entire cylinder surface

$$\overline{Nu} = 1.25 + 0.15(1 + 0.125 \lg Da)(Re^{-5}).$$

When carrying out numerical calculations of flow around the porous permeable cylinder of square section by a viscous incompressible fluid for moderate Reynolds numbers it was determined that with increasing Darcy number a pair of vortex regions behind the body disappears, the drag reduces, the heat transfer increases, mainly on the front surface. As follows from the analysis of the calculated data the approximation dependencies for the drag coefficient of the cylinder, average Nusselt number for the entire surface and Nusselt number for the lateral surface were obtained. It is shown that usage of porous permeable bodies in thermotechnical plants intensifies the heat transfer due to the development of the contact surface; this reduces the hydrodynamic resistance as compared to solid bodies.

References

- [1] Zhukovsky N Ye 1937 *Water seepage through the dam* (Moscow: Complete edition) 363

- [2] Stechkina I B 1979 Resistance of porous cylinders in a viscous fluid flow for low Reynolds numbers *Izv. AS USSR. MZhG* **6** 122–124
- [3] Yu P, Zeng Y, Lee T S, Chen X B and Low H T 2011 Steady flow around and through a permeable circular cylinder *Computers & Fluids* **42** 1–12
- [4] Bhattacharyya S, Dhinakaran S and Khalili A 2006 Fluid motion around and through a porous cylinder *Chemical Engineering Science* **61** 4451–4461
- [5] Jue T 2004 Numerical analysis of vortex shedding behind a porous square cylinder *Int J Numer Methods Heat Fluid Flow* **14**(5) 649–63
- [6] Dhinakaran S and Ponmozhi J 2011 Heat transfer from a permeable square cylinder to a flowing fluid *Energy Conversion and Management* **52** 2170–2182
- [7] Sharma A and Eswaran V 2004 Heat and fluid flow across a square cylinder in the two-dimensional laminar flow regime *Numer. Heat Transfer Part A* **45** 247–269