

A Study of Residual Amplitude Modulation Suppression in Injection Locked Quantum Cascade Lasers Based on a Simplified Rate Equation Model

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Abstract. Using results that come out of a simplified rate equation model, the suppression of residual amplitude modulation in injection locked quantum cascade lasers with the master laser modulated by its drive current is investigated. Quasi-static and dynamic expressions for intensity modulation are used. The suppression peaks at a specific value of the injection ratio for a given detuning and linewidth enhancement factor. The intensity modulation suppression remains constant over a range of frequencies. The effects of injection ratio, detuning, coupling efficiency and linewidth enhancement factor are considered.

1. Introduction

Quantum cascade lasers (QCLs) are good candidates for wave modulation spectroscopy (WMS) because they are tunable and can operate at room temperature (unlike lead-salt based lasers which are commonly used in WMS). A simple way to achieve frequency modulation of the output from a QCL is to modulate the drive current (direct modulation)[1]. However along with the direct frequency modulation, a significant amount of amplitude modulation, known as residual amplitude modulation (RAM)[2,3] occurs which adversely affects the sensitivity of WMS. It is possible to suppress RAM using a master-slave QCL system with direct modulation of the master QCL.

Here, results from a model based on the work of Webb and Haldar[4], extended to include injected terms are used to assess the suppression of RAM in injection locked QCLs.

2. Rate Equations and Parameters for Injection Locked QCLs

Using the simplified model of an isolated QCL[4,5], single mode rate equations for an injection locked slave QCL, ignoring the coupling of the spontaneous emission to the lasing mode, can be written as:

$$\frac{dN_3}{dt} = \frac{I}{e} - \frac{N_3}{\tau_3} - G(N_3 - N_2)P \quad (1)$$

$$\frac{dN_2}{dt} = \frac{N_3}{\tau_{32}} - \frac{N_2}{\tau_{21}} + G(N_3 - N_2)P \quad (2)$$



$$\frac{dP}{dt} = \left[NG(N_3 - N_2) - \frac{1}{\tau_p} \right] P + 2k_c \sqrt{PP_{\text{inj}}} \cos(\phi - \phi_{\text{inj}}) \quad (3)$$

$$\frac{d\phi}{dt} = \frac{\alpha}{2} \left[NG(N_3 - N_2) - \frac{1}{\tau_p} \right] - \Delta\omega - k_c \sqrt{\frac{P_{\text{inj}}}{P}} \sin(\phi - \phi_{\text{inj}}) \quad (4)$$

where N_i is the electron number in level i ; I is the injected current; G is the gain coefficient per stage, which is the product of the confinement factor and the gain due to stimulated emission; τ_{jk} , τ_{32} , and τ_{21} is the lifetime representing a transition from level j to k ; τ_p is the photon lifetime; P is the photon number in the cavity and ϕ is the corresponding phase; N is the number of stages; k_c is the coupling efficiency; $\Delta\omega = \omega_{\text{master}} - \omega_{\text{slave}}$ is the detuning frequency; P_{inj} is the injected photon number; ϕ_{inj} is the injected phase; α is the linewidth enhancement factor of the slave laser and t is time. τ_3 is given by $1/\tau_3 = 1/\tau_{31} + 1/\tau_{32}$.

3. Steady state quantities and static locking range

Steady state quantities can be found by solving Eqs. (1) to (4) with the time derivatives set to zero. Such quantities will be denoted by adding a subscript of 0 (e.g. the steady state electron number in level 1 is N_{10}).

It can be shown from the above model that the static locking range is

$$-k_c \sqrt{(1 + \alpha^2)R_{\text{inj}}} < \Delta\omega < k_c \sqrt{R_{\text{inj}}} \quad (5)$$

where

$$R_{\text{inj}} = \frac{P_{\text{inj}0}}{P_0} \quad (6)$$

is the injection rate under steady conditions.

4. Suppression of Intensity Modulation

Let $m = \Delta P_{\text{inj}0}/P_{\text{inj}0}$ and $M = \Delta P_0/P_0$, represent the intensity modulation (IM) indices of the master and slave laser respectively. Then an IM suppression quantity can be defined by

$$\frac{M}{m} = \left(\frac{P_{\text{inj}0}}{P_0} \right) \left(\frac{dP_0}{dP_{\text{inj}0}} \right) = R_{\text{inj}} \left(\frac{dP_0}{dP_{\text{inj}0}} \right) \quad (7)$$

By differentiating the appropriate quantities it can be shown that

$$\frac{M}{m} = R_{\text{inj}} \left[R_{\text{inj}} + \frac{I_{\text{thf}} \tau_p N^2 G^2 \Delta(N_{d0})^2 P_0 \sqrt{(1 + \alpha^2)k_c^2 R_{\text{inj}} - \Delta\omega^2}}{I_0 k_c^2 K} \right]^{-1} \quad (8)$$

with

$$P_0 = K \left(\frac{I_0}{I_{\text{thf}} \tau_p N G \Delta N_{d0}} - 1 \right)$$

$$\Delta N_{d0} = N_{30} - N_{20} = \frac{I_0 \tau_3 (1 - \tau_{21}/\tau_{32})}{e (1 + P_0/K)}$$

and $K = [\tau_3 (1 + \tau_{21}/\tau_{31}) G]^{-1}$.

5. Frequency Response of IM Suppression

For a sinusoidal current of angular frequency ω_m superimposed on the bias current of the master laser, a dynamic analysis leads to the frequency response of the IM suppression ratio being given by

$$\left| \frac{M}{m} \right| = \left| \frac{D_{IM}}{P_0 \Lambda(j\omega_m)} \right| \quad (9)$$

where $j = \sqrt{-1}$,

$$\Lambda(s) = s^4 + d_3 s^3 + d_2 s^2 + d_1 s + d_0$$

with coefficients given by

$$\begin{aligned} d_3 &= \frac{1}{\tau_3} + \frac{1}{\tau_{21}} + 2GP_0 - \left(NG\Delta N_{d0} - \frac{1}{\tau_p} \right) \\ d_2 &= -k_c \left(NG\Delta N_{d0} - \frac{1}{\tau_p} \right) \sqrt{R_{inj}} \cos(\Delta\phi_0) - \left(\frac{1}{\tau_3} + \frac{1}{\tau_{21}} \right) \left(NG\Delta N_{d0} - \frac{1}{\tau_p} \right) \\ &\quad + \frac{2GP_0}{\tau_p} + \frac{1}{\tau_3\tau_{21}} + \left(\frac{1}{\tau_{31}} + \frac{1}{\tau_{21}} \right) GP_0 - k_c^2 R_{inj} (\cos^2(\Delta\phi_0) - \sin^2(\Delta\phi_0)) \\ d_1 &= -k_c \left(\frac{1}{\tau_3} + \frac{1}{\tau_{21}} \right) \left(NG\Delta N_{d0} - \frac{1}{\tau_p} \right) \sqrt{R_{inj}} \cos(\Delta\phi_0) + \frac{2k_c GP_0 \sqrt{R_{inj}} \cos(\Delta\phi_0)}{\tau_p} \\ &\quad - 2k_c^2 GP_0 R_{inj} (\cos^2(\Delta\phi_0) - \sin^2(\Delta\phi_0)) - \frac{NG\Delta N_{d0} - 1/\tau_p}{\tau_3\tau_{21}} + \left(\frac{1}{\tau_{31}} + \frac{1}{\tau_{21}} \right) \frac{GP_0}{\tau_p} \\ &\quad - 2\alpha k_c NG^2 \Delta N_{d0} P_0 \sqrt{R_{inj}} \sin(\Delta\phi_0) - \left(\frac{1}{\tau_3} + \frac{1}{\tau_{21}} \right) k_c^2 R_{inj} (\cos^2(\Delta\phi_0) - \sin^2(\Delta\phi_0)) \\ d_0 &= -\frac{k_c (NG\Delta N_{d0} - 1/\tau_p) \sqrt{R_{inj}} \cos(\Delta\phi_0)}{\tau_3\tau_{21}} + \left(\frac{1}{\tau_{31}} + \frac{1}{\tau_{21}} \right) \frac{k_c GP_0 \sqrt{R_{inj}} \cos(\Delta\phi_0)}{\tau_p} \\ &\quad - \left(\frac{1}{\tau_{31}} + \frac{1}{\tau_{21}} \right) \alpha k_c NG^2 \Delta N_{d0} P_0 \sqrt{R_{inj}} \sin(\Delta\phi_0) - \frac{k_c^2 R_{inj}}{\tau_3\tau_{21}} (\cos^2(\Delta\phi_0) - \sin^2(\Delta\phi_0)) \\ &\quad - \left(\frac{1}{\tau_{31}} + \frac{1}{\tau_{21}} \right) k_c^2 GP_0 R_{inj} (\cos^2(\Delta\phi_0) - \sin^2(\Delta\phi_0)) \end{aligned}$$

and

$$\begin{aligned} D_{IM} &= \left[k_c \sqrt{R_{inj}} P_0 \cos(\Delta\phi_0) \left(j\omega_m + k_c \sqrt{R_{inj}} \cos(\Delta\phi_0) \right) + k_c \sqrt{R_{inj}} P_0 \sin(\Delta\phi_0) \right. \\ &\quad \times \left. \left(\alpha' j\omega_m + k_c \sqrt{R_{inj}} \sin(\Delta\phi_0) \right) \right] \\ &\quad \times \left[-\omega_m^2 + (\tau_3^{-1} + \tau_{21}^{-1} + 2GP_0) j\omega_m + (\tau_{31}^{-1} + \tau_{21}^{-1}) GP_0 + (\tau_3\tau_{21})^{-1} \right] \end{aligned}$$

in which α' is the linewidth enhancement factor of the master QCL.

6. IM Suppression Plots and Discussion

In this section some plots of the IM suppression as measured through the suppression ratio in Eqs. (8) and (9) will be given and discussed.

The parameter values used are given in Table 1. There are three other required parameters: the coupling efficiency, k_c and the linewidth enhancement factors α and α' . Following Ref. [5] $k_c = v_g(1 - R)/(2L\sqrt{R}) = 27.7 \text{ ns}^{-1}$ (using the values in Table 1). Note that the formula for k_c is derived for a plane wave in a Fabry-Perot cavity with a beam cross-section that matches the

Table 1. Typical parameters of for a slave QCL[7]

Parameter	Value
Wavelength, λ	$8\mu\text{m}$
Temperature, T	10K
Cavity width, W	$14\mu\text{m}$
Cavity length, L	2.25mm
Facet power reflection coefficient, R	0.28
Length of a single gain stage, L_p	44.3nm
Number of gain stages, N	30
Group velocity, v_g	$(3/3.28) \times 10^8 \text{ms}^{-1}$
Mirror loss coefficient, α_m	5.6cm^{-1}
Internal loss coefficient, α_i	24cm^{-1}
Confinement factor per gain stage, Γ	0.016
Gain coefficient per stage, G	$3.4 \times 10^3 \text{s}^{-1}$
τ_{31}	3.6ps
τ_{32}	3.1ps
τ_{21}	0.3ps
τ_p	3.69ps
Threshold current, I_{thf}	0.28A

active region of the slave laser. This cross-section is obtained by transforming the waist of the Gaussian beam of the master laser so that it equals that of the slave laser.

For the linewidth enhancement factors we set $\alpha = \alpha' = 0.3$ [6].

The detuning frequency and injection ratio values in the work here are chosen such that the IM suppression study is carried out within the locking range given by expression (5).

In what follows the IM suppression is plotted in decibels and is defined by $-20 \log(M/m)$; with this definition, large values imply better IM suppression of the slave laser light.

6.1. Quasi-static IM suppression

Figs. 1 to 4 show how IM suppression varies with injection ratio R_{inj} for different parameter values. In all cases a single maximum is evident so that there is a particular injection ratio value that optimizes the suppression. In Fig. 1 it can be seen that overall, the IM suppression curve shifts in the direction of higher injection ratio values as the coupling efficiency decreases. Fig. 2 compares IM suppression plots at different injected currents. In both figures, a suppression above 40 dB is apparent, consistent with experimental observations[6]. Figs. 3 and 4 show IM suppression versus injection ratio for different detuning frequencies and linewidth enhancement factors respectively. Suppression increases as detuning frequency decreases, and also increases with increasing linewidth enhancement. The suppression peak moves to higher injection ratios when the detuning frequency increases and moves in the same direction as the linewidth enhancement factor decreases.

At the edge of the static locking range, IM suppression, as is evident from (8), is 0 dB ($M/m = 1$). For all of the curves discussed so far, the suppression decreases monotonically after the peak value is reached—the same behaviour as is observed in injection-locked interband lasers[8].

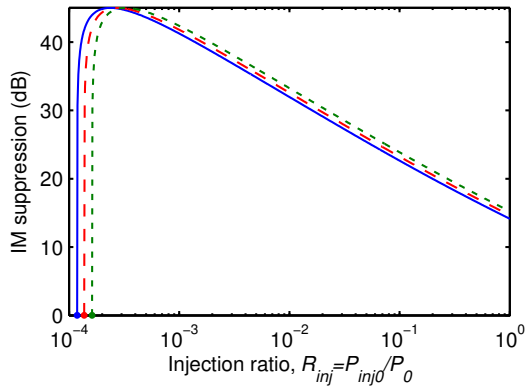


Figure 1. IM suppression against injection ratio for $k_c = 27.7\text{ns}^{-1}$ (solid line), 25.7ns^{-1} (dashed-line, long dashes) and 23.7ns^{-1} (dashed-line, short dashes). Detuning frequency, injected current and linewidth enhancement factor are set at -50MHz , $1.6I_{\text{thf}}$ and 0.3 respectively. IM suppression is given by $-20\log(M/m)$ (dB).

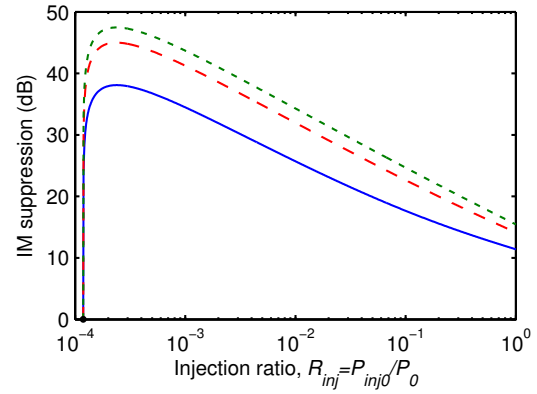


Figure 2. IM suppression against injection ratio for $I_0 = 1.2I_{\text{thf}}$ (solid line), $1.6I_{\text{thf}}$ (dashed-line, long dashes) and $2I_{\text{thf}}$ (dashed-line, short dashes). Detuning frequency and linewidth enhancement factor are set at -50MHz and 0.3 respectively.

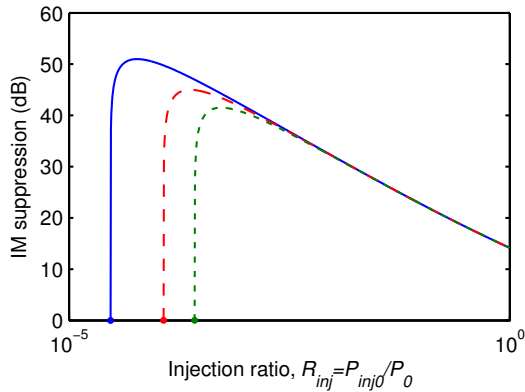


Figure 3. IM suppression against injection ratio for $\Delta f = -25\text{MHz}$ (solid line), -50MHz (dashed-line, long dashes) and -75MHz (dashed-line, short dashes). Injected current and linewidth enhancement factor are set at $1.6I_{\text{thf}}$ and 0.3 .

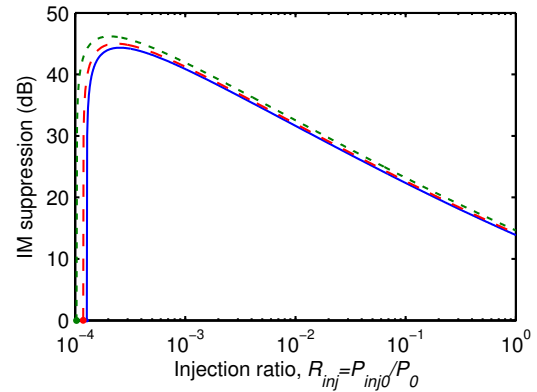


Figure 4. IM suppression against injection ratio for $\alpha = 0.1$ (solid line), 0.3 (dashed-line, long dashes) and 0.5 (dashed-line, short dashes). Detuning frequency and injected current are set at -50MHz and $1.6I_{\text{thf}}$ respectively.

6.2. Frequency response of IM suppression

The variation of IM suppression against modulating frequency is plotted in Fig. 5 for different values of injection ratio, and in 6 for different values of injection current. As the modulating frequency approaches zero, IM suppression approaches its quasi-static value. However, unlike interband lasers in which the IM suppression decreases at high modulating frequencies, the suppression observed here does not vary greatly with modulating frequency. This is because QCLs are overdamped systems, whereas interband lasers are underdamped.

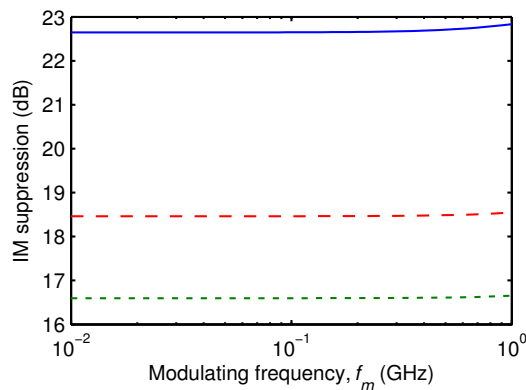


Figure 5. IM suppression against modulating frequency for injection ratio of 0.1 (solid line), 0.3 (dashed-line, long dashes) and 0.5 (dashed-line, short dashes). Detuning frequency, injected current and linewidth enhancement factor for both master and slave QCLs are set at -50MHz , $1.6I_{\text{thf}}$ and 0.3 respectively. IM suppression is given by $-20 \log(M/m)(\text{dB})$.

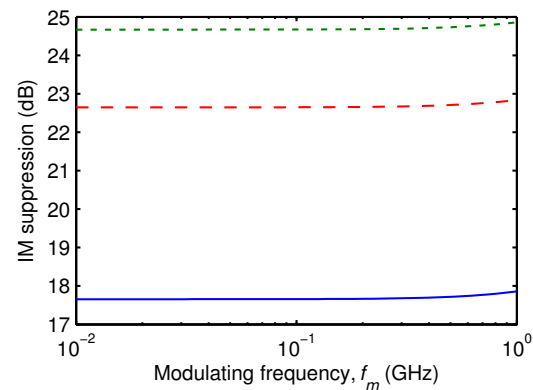


Figure 6. IM suppression against modulating frequency for $I_0 = 1.2I_{\text{thf}}$ (solid line), $1.6I_{\text{thf}}$ (dashed-line, long dashes) and $2I_{\text{thf}}$ (dashed-line, short dashes). Detuning frequency, injection ratio and linewidth enhancement factor for both master and slave QCLs are set at -50MHz , 0.1 and 0.3 respectively.

7. Conclusion

In this paper, expressions derived from a relatively simple two-level model have been used to study IM suppression for an injection locked slave QCL. A high degree of suppression is favourable for applications such as wave modulation spectroscopy. It has been demonstrated that maximum IM suppression occurs at a specific injection ratio for a given detuning frequency value and linewidth enhancement factor. Large injected currents, small detuning frequencies, and large linewidth enhancement factors increase the suppression. The IM suppression peaks at a lower injection ratios as the detuning frequency decreases and as linewidth enhancement factor increases. IM suppression remains almost constant throughout the range of modulating frequencies considered.

References

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