

A channel estimation scheme for MIMO-OFDM systems

Chunlong He, Chu Tian, Xingquan Li, Ce Zhang, Shiqi Zhang, and Chaowen Liu

College of Information Engineering, Shenzhen University, Shenzhen, China,
{chunlonghe@163.com}

Abstract. In view of the contradiction of the time-domain least squares (LS) channel estimation performance and the practical realization complexity, a reduced complexity channel estimation method for multiple input multiple output-orthogonal frequency division multiplexing (MIMO-OFDM) based on pilot is obtained. This approach can transform the complexity of MIMO-OFDM channel estimation problem into a simple single input single output-orthogonal frequency division multiplexing (SISO-OFDM) channel estimation problem and therefore there is no need for large matrix pseudo-inverse, which greatly reduces the complexity of algorithms. Simulation results show that the bit error rate (BER) performance of the obtained method with time orthogonal training sequences and linear minimum mean square error (LMMSE) criteria is better than that of time-domain LS estimator and nearly optimal performance.

1. Introduction

Multiple input multiple output (MIMO) systems have been discussed indispensable for the transmission improvement capacity. Orthogonal frequency division multiplexing (OFDM) has been considered as an important technique in the broadband wireless communication systems. Their combination, MIMO-OFDM technology, has attracted great interests in these recent years for enhancing the capacity, energy efficiency and improving the link reliability for the future fourth generation (4G) mobile communication systems [1-3]. However, in order to get the promised theoretical capacity and to achieve maximum gain of diversity, MIMO-OFDM systems require accurate channel state information (CSI) at the receiver [4]. The knowledge of the CSI at the receiver is crucial for coherent detection and space-time decoding. In OFDM systems, channel estimation has been successfully used to improve the system performance [5-6]. It has been demonstrated that coherent demodulation can be used instead of differential de-modulation a 3-4dB gain of signal-to-noise ratio [7-8]. However, the method of channel estimation in OFDM can't be used to MIMO-OFDM systems directly because the antenna interfere with each other. There are some literatures on channel estimation for MIMO-OFDM systems. The minimum mean square error (MMSE) channel estimators in [5] and the time-domain Least square (LS) channel estimators in [6] are presented, both of which have good performance at the cost of high complexity of calculating the pseudo-inversion of the matrix. Hence, several reduced complexity channel estimation methods are proposed in [9-14], whereas introducing performance degradation and limitations of application.

In this paper, we obtain a novel reduced complexity channel estimation method for MIMO-OFDM systems which is applicable to the slow-fading channel. This approach can transform the complexity of MIMO-OFDM channel estimation problem into a simple SISO-OFDM channel estimation problem and therefore there is no need for large matrix pseudo-inverse. As demonstrated by computer simulations, the proposed method provides better performance and lower complexity.

The rest of this paper is organized as follows. The system model is presented in the second section. Section III introduces the time-domain LS channel estimator. A novel reduced complexity channel



estimation method is derived In Section IV. In Section V, the simulation environment and results are described. Section VI concludes the paper.

Notation: In this paper, Bold upper and low letters denote matrices and column vectors, respectively; $(\cdot)^T$, $(\cdot)^*$, $(\cdot)^H$ denote transpose, conjugate, and Hermitian transpose, respectively; $(\cdot)^+$ and $|\cdot|$ denote matrix pseudo-inversion and absolute value, respectively.

2. System Model

A MIMO-OFDM system with two transmitter and two receiver antennas is shown in Fig.1 [8]. At a transmission time n , a binary data block $\{\mathbf{b}[n, k]: k = 0, 1 \dots K-1\}$ is coded into two different signals, $\{\mathbf{t}_i[n, k]: k = 0, 1 \dots K-1, i = 1, 2\}$, where K , k , and i are respectively the number of sub-channels of the OFDM systems, sub-channel index, and antenna index. Each of these signals forms an OFDM block. At the receiver, the discrete Fourier transform (DFT) of the received signal at each receiver antenna is the superposition of two distorted transmitted signals. The received signal at the j -th receiver antenna can be expressed as in [8]

$$\mathbf{r}_j[n, k] = \sum_{i=1}^2 \mathbf{H}_{ij}[n, k] \mathbf{t}_i[n, k] + \mathbf{w}_j[n, k] \quad (1)$$

where $\mathbf{H}_{ij}[n, k]$ is the channel frequency response for the k -th sub-channel at the time n , corresponding to the i -th transmitter antenna and j -th receiver antenna; $\mathbf{w}_j[n, k]$ is the additive complex Gaussian noise, with zero mean and variance δ_n^2 . The noise is uncorrelated for different n , k , or j [8].

The channel is assumed to be a wide-sense stationary (WSS) channel model and unchanged over the duration of one OFDM symbol. The LS estimator for the current OFDM symbol is not related to the previous or the following OFDM symbols. The goal of MIMO-OFDM demodulation is to recover the transmitted signal from the received signals. From (1) we can obtain the frequency response at the k -th tone of the n th block corresponding to the i -th transmitter antenna can be shown as [8]

$$\mathbf{H}[n, k] = \sum_{l=0}^{K_0-1} \mathbf{h}[n, k] \mathbf{W}_K^{kl} \quad (2)$$

where K_0 is the number of multi-path, $\mathbf{W}_K = \exp(-j(2\pi / K))$.

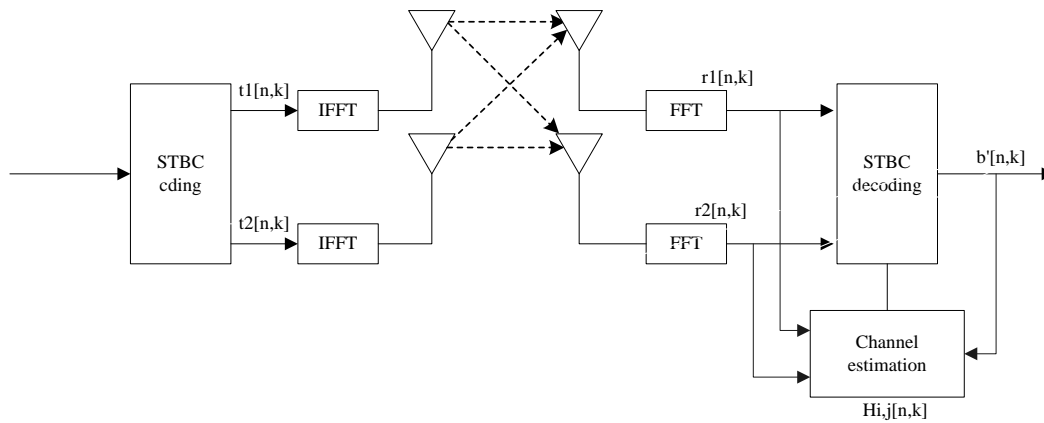


Figure 1. Block diagram of a STBC-OFDM system with two transmitter and two receiver antennas

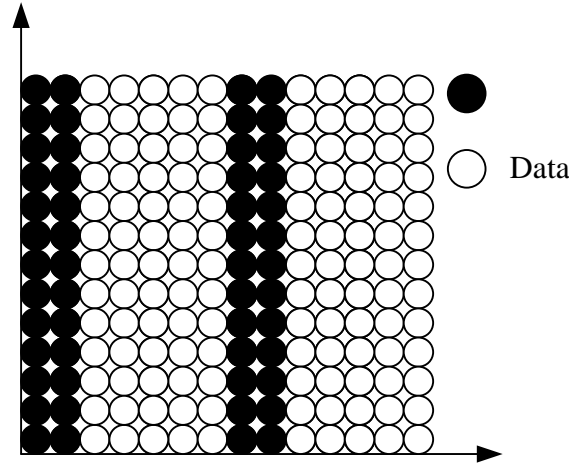


Figure 2. Block-type pilot sub-carrier arrangement

3. The Time-Domain Ls Channel Estimator

For an MIMO-OFDM systems, the received signal is overlapped signals transmitted from different transmit antennas, which makes the channel estimation difficult. Fortunately, channel parameters for different sub-carriers of each channel are correlated and a channel estimator has been developed in [8] based on this correlation. From (2), we only need to estimate $\mathbf{h}_i[n, k]$, then $\mathbf{H}_{ij}[n, k]$ can be obtained by FFT. If the transmitted signals $\{\mathbf{t}_i[n, k]: k = 0, 1 \dots K-1, i = 1, 2\}$ are known, the temporal estimation of $\mathbf{h}_i[n, k]$, can be found by [8]

$$\begin{bmatrix} \mathbf{h}_1(n) \\ \mathbf{h}_2(n) \end{bmatrix} = \begin{bmatrix} \mathbf{Q}_{11}(n) & \mathbf{Q}_{21}(n) \\ \mathbf{Q}_{12}(n) & \mathbf{Q}_{22}(n) \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{p}_1(n) \\ \mathbf{p}_2(n) \end{bmatrix} \quad (3)$$

Where,

$$\mathbf{p}_j[n, l] = \sum_{k=0}^{K-1} \mathbf{r}[n, k] \mathbf{t}_j^*[n, k] \mathbf{W}_K^{-kl} \quad (4)$$

$$\mathbf{q}_{ij}[n, l] = \sum_{k=0}^{K-1} \mathbf{t}_i[n, k] \mathbf{t}_j^*[n, k] \mathbf{W}_K^{-kl} \quad (5)$$

$$\mathbf{Q}_{ij}[n] = \begin{pmatrix} \mathbf{q}_{ij}[n, 0] & \mathbf{q}_{ij}[n, -1] & \dots & \mathbf{q}_{ij}[n, -K_0 + 1] \\ \mathbf{q}_{ij}[n, 1] & \mathbf{q}_{ij}[n, 0] & \dots & \mathbf{q}_{ij}[n, -K_0 + 2] \\ \dots & \dots & \dots & \dots \\ \mathbf{q}_{ij}[n, K_0 - 1] & \mathbf{q}_{ij}[n, K_0 - 2] & \dots & \mathbf{q}_{ij}[n, 0] \end{pmatrix} \quad (6)$$

$$\mathbf{p}_i[n] = (\mathbf{p}_i[n, 0], \mathbf{p}_i[n, 1], \dots, \mathbf{p}_i[n, K_0 - 1])^T \quad (7)$$

From (6), we can see that $2K_0 \times 2K_0$ matrix inversion is required to get the temporal estimation of $\mathbf{h}_i[n, k]$, which requires intensive computations. Next we will propose a novel method to reduce the computation complexity, with this method no matrix inverse is required.

4. The Novel Reduced Complexity Channel Estimation Method

Assuming that the channel is quasi-fixed over two consecutive OFDM symbols and every two neighboring pilots are grouped as one cluster, and these clustered pilot groups are equi-spaced as show in Fig. 2. The pilot signal is assigned to a particular OFDM block at each transmit antennas, which is sent periodically in time-domain. Let's the first cluster pilot OFDM for example, letting

$$\mathbf{H}_{11}[1] = \mathbf{H}_{11}[2]$$

$$\mathbf{H}_{21}[1] = \mathbf{H}_{21}[2]$$

From (1), the received signals can be expressed as

$$\mathbf{r}_1[1] = \mathbf{H}_{11}[1]\mathbf{t}_1[1] + \mathbf{H}_{21}[1]\mathbf{t}_2[1] + \mathbf{w}_1[1] \quad (8)$$

$$\mathbf{r}_1[2] = \mathbf{H}_{11}[2]\mathbf{t}_1[2] + \mathbf{H}_{21}[2]\mathbf{t}_2[2] + \mathbf{w}_1[2] \quad (9)$$

From (8) and (9), we have

$$\begin{aligned} \mathbf{Z}_{11} &= \frac{\mathbf{r}_1[1]}{\mathbf{t}_2[1]} - \frac{\mathbf{r}_1[2]}{\mathbf{t}_2[2]} \\ &= \frac{\mathbf{H}_{11}[1]\mathbf{t}_1[1] + \mathbf{H}_{21}[1]\mathbf{t}_2[1] + \mathbf{w}_1[1]}{\mathbf{t}_2[1]} - \frac{\mathbf{H}_{11}[2]\mathbf{t}_1[2] + \mathbf{H}_{21}[2]\mathbf{t}_2[2] + \mathbf{w}_1[2]}{\mathbf{t}_2[2]} \\ &= \left(\frac{\mathbf{H}_{11}[1]\mathbf{t}_1[1]}{\mathbf{t}_2[1]} - \frac{\mathbf{H}_{11}[2]\mathbf{t}_1[2]}{\mathbf{t}_2[2]} \right) + \left(\frac{\mathbf{w}_1[1]}{\mathbf{t}_2[1]} - \frac{\mathbf{w}_1[2]}{\mathbf{t}_2[2]} \right) \\ &= \mathbf{H}_{11}[1] \left(\frac{\mathbf{t}_1[1]}{\mathbf{t}_2[1]} - \frac{\mathbf{t}_1[2]}{\mathbf{t}_2[2]} \right) + \left(\frac{\mathbf{w}_1[1]}{\mathbf{t}_2[1]} - \frac{\mathbf{w}_1[2]}{\mathbf{t}_2[2]} \right) \\ &= \mathbf{H}_{11}[1]\mathbf{T}1 + \mathbf{W}1 \end{aligned} \quad (10)$$

Where

$$\mathbf{T}1 = \frac{\mathbf{t}_1[1]}{\mathbf{t}_2[1]} - \frac{\mathbf{t}_1[2]}{\mathbf{t}_2[2]} \quad (11)$$

$$\mathbf{W}1 = \frac{\mathbf{w}_1[1]}{\mathbf{t}_2[1]} - \frac{\mathbf{w}_1[2]}{\mathbf{t}_2[2]} \quad (12)$$

Similarly

$$\mathbf{Z}_{21} = \frac{\mathbf{r}_1[1]}{\mathbf{t}_1[1]} - \frac{\mathbf{r}_1[2]}{\mathbf{t}_1[2]} = \mathbf{H}_{21}[1]\mathbf{T}2 + \mathbf{W}2 \quad (13)$$

$$\mathbf{Z}_{12} = \mathbf{H}_{12}[1]\mathbf{T}1 + \mathbf{W}1 \quad (14)$$

$$\mathbf{Z}_{22} = \mathbf{H}_{22}[1]\mathbf{T}2 + \mathbf{W}2 \quad (15)$$

Where

$$\mathbf{T}2 = \frac{\mathbf{t}_2[1]}{\mathbf{t}_1[1]} - \frac{\mathbf{t}_2[2]}{\mathbf{t}_1[2]} \quad (16)$$

$$\mathbf{W}2 = \frac{\mathbf{w}_1[1]}{\mathbf{t}_2[1]} - \frac{\mathbf{w}_1[2]}{\mathbf{t}_2[2]} \quad (17)$$

So the channel responses corresponding to different transmit antennas are decoupled. And the complexity of MIMO-OFDM channel estimation problem transformed into a simple SISO-OFDM

channel estimation problem. The LS [14-15] estimators for pilot signals channel responses are given by

$$\hat{\mathbf{H}}_{11LS} = \mathbf{T}_1^{-1} \mathbf{Z}_{11} = \left(\frac{\mathbf{Z}_{11}[1,0]}{\mathbf{T}_1[1,0]}, \frac{\mathbf{Z}_{11}[1,1]}{\mathbf{T}_1[1,1]} \dots \frac{\mathbf{Z}_{11}[1,K-1]}{\mathbf{T}_1[1,K-1]} \right) \quad (18)$$

$$\hat{\mathbf{H}}_{21LS} = \mathbf{T}_2^{-1} \mathbf{Z}_{21} = \left(\frac{\mathbf{Z}_{21}[1,0]}{\mathbf{T}_2[1,0]}, \frac{\mathbf{Z}_{21}[1,1]}{\mathbf{T}_2[1,1]} \dots \frac{\mathbf{Z}_{21}[1,K-1]}{\mathbf{T}_2[1,K-1]} \right) \quad (19)$$

$$\hat{\mathbf{H}}_{12LS} = \mathbf{T}_1^{-1} \mathbf{Z}_{12} = \left(\frac{\mathbf{Z}_{12}[1,0]}{\mathbf{T}_1[1,0]}, \frac{\mathbf{Z}_{12}[1,1]}{\mathbf{T}_1[1,1]} \dots \frac{\mathbf{Z}_{12}[1,K-1]}{\mathbf{T}_1[1,K-1]} \right) \quad (20)$$

$$\hat{\mathbf{H}}_{22LS} = \mathbf{T}_2^{-1} \mathbf{Z}_{22} = \left(\frac{\mathbf{Z}_{22}[1,0]}{\mathbf{T}_2[1,0]}, \frac{\mathbf{Z}_{22}[1,1]}{\mathbf{T}_2[1,1]} \dots \frac{\mathbf{Z}_{22}[1,K-1]}{\mathbf{T}_2[1,K-1]} \right) \quad (21)$$

And the mathematical representation for LMMSE [15] estimators of pilot signals channel responses as follows:

$$\hat{\mathbf{H}}_{11LMMSE} = R_{hh} [R_{hh} + \sigma_n^2 (\mathbf{X}_1^H \mathbf{X}_1^{-1})]^{-1} \hat{\mathbf{H}}_{11LS} \quad (22)$$

$$\hat{\mathbf{H}}_{21LMMSE} = R_{hh} [R_{hh} + \sigma_n^2 (\mathbf{X}_2^H \mathbf{X}_2^{-1})]^{-1} \hat{\mathbf{H}}_{21LS} \quad (23)$$

$$\hat{\mathbf{H}}_{12LMMSE} = R_{hh} [R_{hh} + \sigma_n^2 (\mathbf{X}_1^H \mathbf{X}_1^{-1})]^{-1} \hat{\mathbf{H}}_{12LS} \quad (24)$$

$$\hat{\mathbf{H}}_{22LMMSE} = R_{hh} [R_{hh} + \sigma_n^2 (\mathbf{X}_2^H \mathbf{X}_2^{-1})]^{-1} \hat{\mathbf{H}}_{22LS} \quad (25)$$

Where,

$$\mathbf{X}_1 = \text{diag}(\mathbf{T}_1[1,0], \mathbf{T}_1[1,1], \dots, \mathbf{T}_1[1,K-1]) \quad \mathbf{X}_2 = \text{diag}(\mathbf{T}_2[1,0], \mathbf{T}_2[1,1], \dots, \mathbf{T}_2[1,K-1])$$

R_{hh} is the auto-covariance matrix of \mathbf{h} . Then, the data sub-carriers can be estimated by linear interpolating neighboring pilot sub-channels.

5. Simulation Results

For the simulations, two transmit and two receive antennas MIMO-OFDM system with 64 sub-carriers and a cyclic extended guard interval of length 16 is considered. The channel bandwidth $B=20$ MHz, the sampling rate $f_s=20$ MHz, which is shown in Table I and the carrier frequency $f_c=3.5$ GHz are selected similar to the HiperLAN/2 standard [16]. The channel is modeld as Rayleigh fading with parameters shown in Table II. QPSK is used for the bit error rate (BER) simulations, and the time-domain channel response length K_0 is equal to 11 samples. The BER performance is measured by averaging over 10000 OFDM blocks and perfect synchronization between the transmitter and the receiver is assumed.

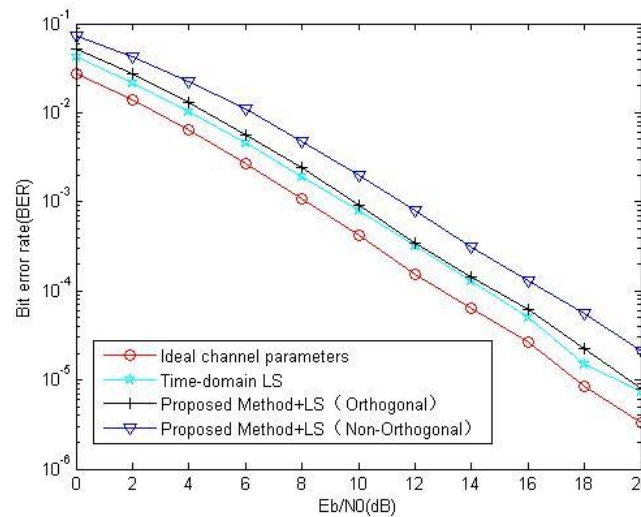
Table 1. Simulation Parameters

Parameters	Specifications
FFT Length	64
Sub-carriers	64
Guard Interval	16
Pilot Interval	4
Modulation Level	QPSK
Channel Model	Rayleigh Fading

Table 2. Rayleigh Fading Parameters

Path Number	Average Power(dB)	Delay(ns)
1	0	0
2	-6	100
3	-12	200
4	-18	300
5	-24	400
6	-30	500

The BER performance curves versus E_b/N_0 for the time-domain LS estimation, the proposed estimation method and the ideal estimation in slow Rayleigh fading channel are shown in Fig.3-4. Fig.3 shows that the time-domain LS estimation exhibits 2~2.5dB BER gain compared with the obtained method under the LS criteria when the pilot sequences are non-orthogonal. When the pilot sequences are orthogonal, the obtained method has the same good performance with the time-domain LS estimation. Fig.4 shows that the performance of the obtained method under the LMMSE criteria is 1~1.5dB better than that of the time-domain LS estimation and nearly optimal performance when the pilot sequences is orthogonal. So we can get the conclusion that the obtained method with time orthogonal training sequences and LMMSE criteria has better estimation performance compared with the time-domain LS estimator. The calculation complexity of the obtained method is significant reduced and it is easy to implement in practical systems.

**Figure 3.** The BER performance versus E_b/N_0 with LS criteria (vehicle speed= 15 km/h)

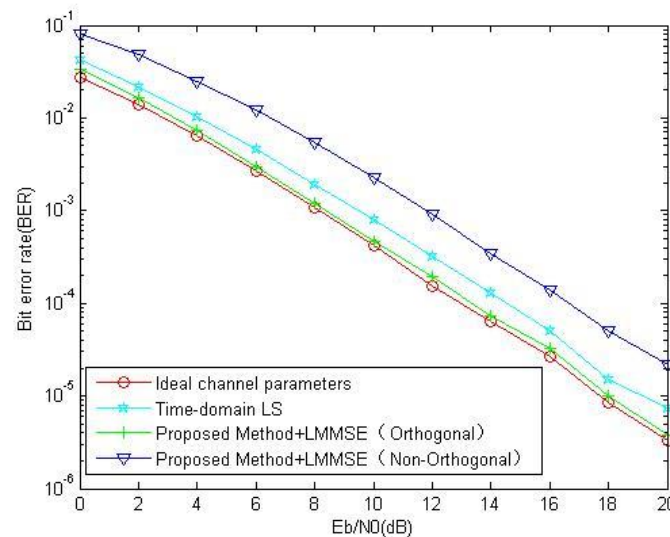


Figure 4. The BER performance versus E_b/N_0 with LMMSE criteria (vehicle speed= 15 km/h)

6. Conclusions

In this paper, a reduced complexity channel estimation method is obtained for MIMO-OFDM system. Since the matrix inversion is avoided, significant complexity reduction is obtained while achieving an accurate frequency domain channel estimates. Compared with the time-domain LS estimator, better performance and low complexity can be obtained.

7. Acknowledgment

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8. References

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