# Research on optimal driver steering model based on Multi-Point preview

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**Abstract.** In this paper, multi-point preview control algorithm is applied to driver steering control model. This paper builds multi-point preview road model in the form of state shift register. Based on the linear quadratic regulator (LQR) optimal control theory it optimizes driver steering control model with multi-point preview. Meanwhile, in the Matlab \ Simulink environment, based on vehicle system dynamics and optimal control theory, different preview points and weighted coefficients are simulated to study the influence of driver steering model. The simulation results show that the multi-point preview control mode has excellent driving performance. And in this paper, the main parameters affecting the preview control algorithm such as speed, preview weighted coefficients and the number of preview points and so on are discussed.

## 1. Introduction

The driver steering model describes the driver's steering behaviour characteristics in the form of mathematical model. It is good for the study of driver steering control system and real vehicle test validation, analysis and aided design. Therefore, the driver steering model research has received extensive attention of the researchers [1-2]. With the deepening of the study there has been a large number of different control methods of the driver steering model [3-6]. At present, the most widely used model is the optimal preview model proposed by MacAdam [7], and the model optimizes the input with lateral deviations. The preview optimal curvature model proposed by Academician Guo, which establishes the relationship between vehicle steering characteristic parameters and driver characteristic parameters [8]. And Sharp optimizes both the lateral deviation and the yaw angle deviation [8]. Then, for the shortcomings of the specific implementation process of the steering angle, Cole et al. proposed a driver steering control model based on Neuromuscular System (NMS) [9].

This paper builds multi-point preview road model in the form of state shift register. Based on the linear quadratic (LQR) optimal control theory it optimizes driver steering control model with multi-point preview. Meanwhile, this paper chooses different vehicle speed, weighted coefficients and preview points' number to simulate, respectively, to study the driver's steering behaviour characteristic, which provides experience and reference for better establishing driver steering model.

## 2. Single Track Vehicle Model

Referring to single track vehicle model shown in figure 1 and ignoring the influence of the nonlinearity of the suspension and tire this paper directly uses the wheel steering angle as input. Select state variables: vehicle lateral speed $v_y$, yaw angular speed $\omega$, lateral displacement $y$, and yaw angle $\psi$. The following four-dimensional linear vehicle model is established, as shown in Equation (1) and the specific parameter values are shown in table 1.

$$\begin{bmatrix} \dot{v}_y \\ \dot{\omega} \\ \dot{y} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} \frac{C_f + C_r}{v_x m} & \frac{l_f C_f - l_r C_r}{v_x m} - v_x & 0 & 0 \\ \frac{l_f C_f - l_r C_r}{v_x I_{zz}} & \frac{l_f^2 C_f + l_r^2 C_r}{v_x I_{zz}} & 0 & 0 \\ 1 & 0 & 0 & v_x \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_y \\ \omega \\ y \\ \psi \end{bmatrix} + \begin{bmatrix} \frac{-C_f}{m n_{rsw}} \\ \frac{-l_f C_f}{I_{zz} n_{rsw}} \\ 0 \\ 0 \end{bmatrix} \theta_{sw} \tag{1}$$

Where $\theta_{sw} = n_{rsw}\delta$, and $\theta_{sw}$ is the wheel steering angle.

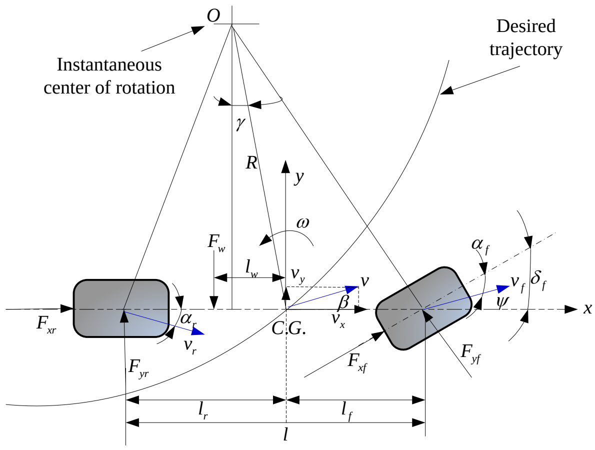



**Figure 1.** Force analysis structure diagram of vehicle.

The above state-space Equation (1) can be converted into the following standard form

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned} \tag{2}$$

Assume that the input is the zero-order hold state and transfer it into a discrete system by the function c2d in Matlab.

$$\begin{aligned} x(k+1) &= A_d x(k) + B_d u(k) \\ y(k) &= C_d x(k) + D_d u(k) \end{aligned} \tag{3}$$

Where $k$ denotes time stride, $\boldsymbol{x(k)} = [v_y(k) \quad \omega(k) \quad y(k) \quad \psi(k)]$ .

**Table 1.** Single track vehicle model parameters.

| symbol | name | value | unit |
|---|---|---|---|
| $I_{zz}$ | rotational inertia | 2550 | kgm$^2$ |
| $M$ | vehicle weight | 1673 | kg |
| $l_r$ | rear wheelbase | 1.73 | m |
| $l_f$ | front wheelbase | 0.913 | m |
| $C_f$ | front wheel cornering stiffness | -2*88310 | N/rad |
| $C_r$ | rear wheel cornering stiffness | -2*64076 | N/rad |
| $n_{rsw}$ | gear ratio | 16 | - |

## 3. Multi-Point Preview Model for Road

### *3.1 Road Model*

Taking the center line of the test road as the ideal road track that drivers desire to track, the concrete ideal road parameter settings are shown in table 2. Double change road is shown in figure 2.

**Table 2.** Ideal road curve setting.

| Longitudinal displacement (x/m) | lateral displacement (y/m) |
|---|---|
| <321 | 0 |
| 363~398 | 3.5 |
| >433 | 0 |

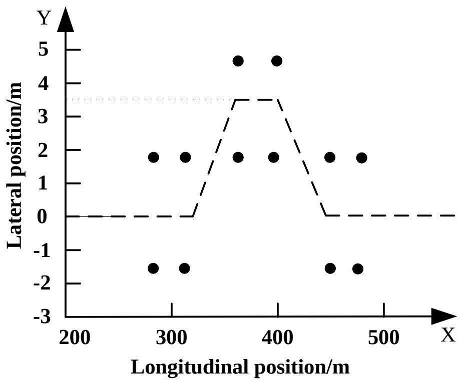



**Figure 2.** Two-shift road setting.

### *3.2 Multi-Point Preview Model for Road*

When the vehicle is moving, the driver firstly obtains the front road information through the preview, and then select a plurality of road model preview points in the road model as the pre-road input in the case of suitable road condition.

When establishing a road model, the linear form of road model is converted into a discrete form by sampling. Its concrete conversion process is similar to "queue": choose $N_p + 1$ discrete points, and then take them as system state variables. New discrete points continue to enter the system, first in first out, and repeat it in this way until all road sampling points gradually entering into the system. As time goes by, these points will be near the vehicle gradually (shown below)

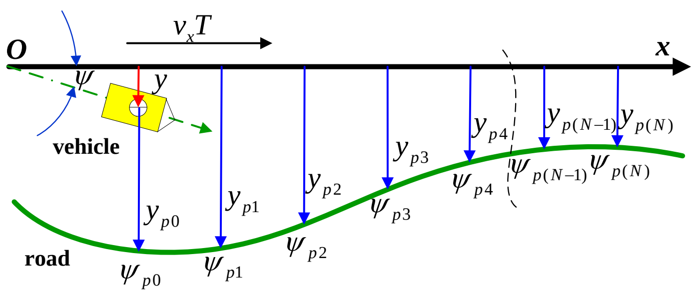



**Figure 3.** Vehicle and preview road structure chart in geodetic coordinate system.

Where $v_x$ is vehicle speed, and T is the simulation stride. $y_{p0}$, $y_{p1}$, $y_{p2}$ … $y_{p(N)}$ denotes vehicle lateral displacements, and $\psi_{pi}$ denotes road heading angle. Assume that the road ahead is divided into $N_p + 1$ equidistant points, then this dynamic road model can be represented as a mathematical model as follows

$$y(k) = Dy(k) + Ey_p(k) \tag{4}$$

Where $\boldsymbol{y_p(k)} = [y_{p0}(k) \quad y_{p1}(k) \quad y_{p2}(k) \quad ...... \quad y_{p(N-1)}(k) \quad y_{p(N)}(k)]^T$ denotes road input. The road model system matrix defined as

$$D = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix} \quad E = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \tag{5}$$

## 4. Control Algorithm

### *4.1. Control Method*

Using LQR theory, taking full account of road information and establishing optimal-preview driver model can integrate vehicle and road into a whole. The "road-vehicle" system is shown in figure 3. It can be written as a form of Equation (6)

$$\begin{bmatrix} x(k+1) \\ y_p(k+1) \end{bmatrix} = \begin{bmatrix} A_d & 0 \\ 0 & D \end{bmatrix} \begin{bmatrix} x(k) \\ y_p(k) \end{bmatrix} + \begin{bmatrix} 0 \\ E \end{bmatrix} y_{p(N+1)}(k) + \begin{bmatrix} B_d \\ 0 \end{bmatrix} \theta_{sw}(k) \tag{6}$$

According to the loss function Equation (7), this paper optimizes the input of the system by using the linear quadratic optimal control theory.

$$J = \sum_{k=0}^{\infty} \begin{pmatrix} q_y (y(k) - y_{p0}(k))^2 + q_\Psi (\psi(k) \\ -\psi_{p0}(k))^2 + R_2 (\theta_{sw}(k))^2 \end{pmatrix} \tag{7}$$

Weighted matrix defined as

$$R_1 = C^T QC \,, \; R_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 1 & 1/v_x T & -1/v_x T & 0 & 0 & \dots & 0 \end{bmatrix}, \; Q = \begin{bmatrix} q_y & 0 \\ 0 & q_\Psi \end{bmatrix}$$

Where $q_y$ and $q_\Psi$ are lateral position and heading Angle error loss function weighted coefficient, respectively. And the two together generate the weighted matrix $R_1$. $R_2$ is a scalar that controls the weight of the input.

By employing DLQR function in Matlab, considering $R_1$, $R_2$ and "road-vehicle" model, the state feedback gain matrix $K_p$ will be obtained when the loss function $J$ reaches the minimum.

$$\boldsymbol{K_p} = [k_1 \quad k_2 \quad k_3 \quad k_4 \quad k_{p1} \quad k_{p2} \quad \cdots \quad k_{pn}]$$

Where $k_1$ to $k_4$ denote state gains ($\begin{bmatrix} v_y(k) & w(k) & y(k) & \psi(k) \end{bmatrix}^T$) associated with vehicle status, respectively. Then $k_{p1}$ to $k_{pn}$ denote preview gains associated with road information, respectively. Thus, producing the optimal control input

$$\hat{\theta}_{sw}(k) = -\boldsymbol{K_p} \begin{bmatrix} x(k) & y_p(k) \end{bmatrix}^T$$

### *4.2. Vehicle Coordinate System Transformation*

In the actual case, the information used by the driver when making decisions is observed in the vehicle coordinate system, as shown in figure 4. Then the road information obtained by the driver is the vehicle coordinate system observation values: $\left[y_{r0}, y_{r1}, y_{r2}, \cdots, y_{r(N)}\right]$, namely, the driver uses the relative error between vehicle and the preview road, rather than the observation values in the geodetic coordinate system $\left[y_{p0}, y_{p1}, y_{p2}, \cdots, y_{p(N)}\right]$.

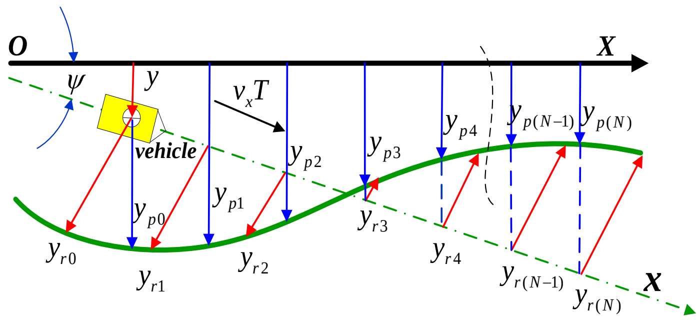



**Figure 4.** Vehicle and the preview road diagram in the vehicle coordinate system.

Assuming that yaw angle is very small, in the geodetic coordinate system, vehicle lateral displacement defined as

$$y_{pn}(k) = y_{rn}(k) + y(k) + nv_x T\psi(k) \tag{8}$$

The optimal steering angle in the geodetic coordinate system is

$$\begin{aligned}\hat{\theta}_{sw}(k) = k_1 v_y(k) + k_2 w(k) + k_3 y(k) \\ + k_4 \Psi(k) + \sum_{n=0}^{N} k_{pn} y_{pn}(k)\end{aligned} \tag{9}$$

The optimal steering angle in the vehicle coordinate system is

$$\hat{\theta}_{sw}(k) = k_1 v_y(k) + k_2 w(k) + \sum_{n=0}^{N} k_{pn} y_{rn}(k) \tag{10}$$

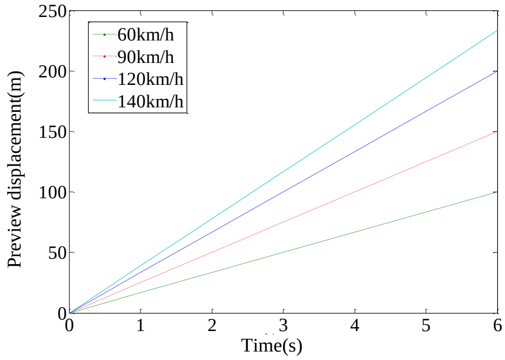
The optimal steering angles in the two coordinate systems are variable, so Equation (9) and Equation (10) must be equal.

$$k_3 = -\sum_{n=0}^{N} k_{pn}, \quad k_4 = -\sum_{n=0}^{N} k_{pn} n v_x T \tag{11}$$

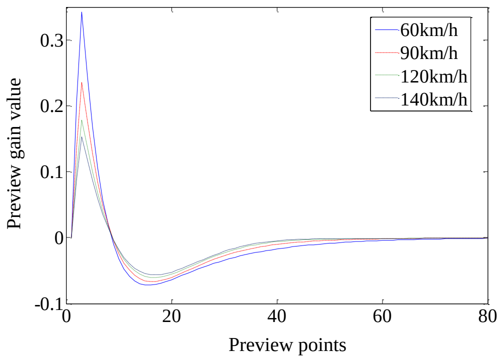
## 5. Simulation Results and Analysis

The multi-point preview control algorithm is related to the parameters such as vehicle preview distance, the number of preview point and weighted coefficient, etc. In order to study the validity of the multi-point preview algorithm, in the Matlab / Simulink software environment, this paper simulates and analyses different preview distance, the number of preview point and preview weighted coefficient, respectively.

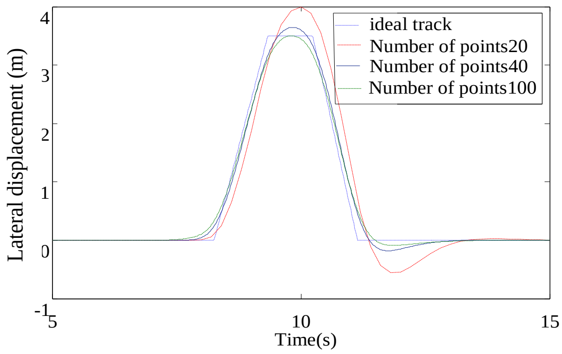
### *5.1. Influence of Preview Distance and the Number of Preview Point on Steering Performance*




(a)




(b)

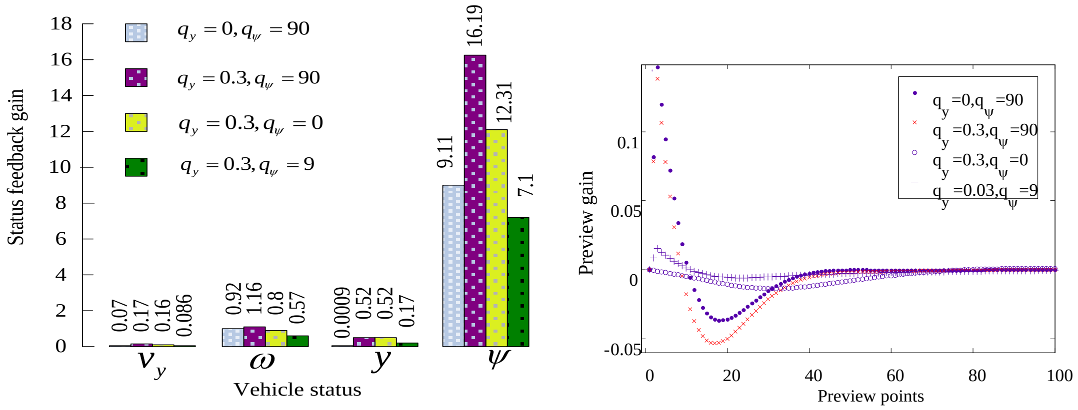



(c)

**Figure 5.** Influence of preview distance and the number of preview point on steering performance. (a) Relationship between preview distance and speed; (b) Relationship between preview gain and speed; (c) The movement track of different preview points, speed is 140 km/h.

Figure 5(a) & (b) are the simulation results of different speeds when the number of preview points is invariant. When the preview time is constant, as the speed increases, the preview distance increases. And the farther the driver preview, the closer the preview gain value to zero. When the distance exceeds the driver's physiological limits, then the preview feedback gain value is zero, and the preview point in a far distance has no effect on the steering performance. It can also be seen from figure 5(c) that in the case of multi-point preview, when the number of preview points is less or the preview distance is too close, the prediction effect of the trajectory is deteriorated, which results in the decrease of tracking accuracy. The more the number of points, the better tracking result is. Therefore, in essence, the preview part is associated with the whole system, and the driver can control the vehicle only when the driver is within the physiological limits. Otherwise, the vehicle will be out of control.

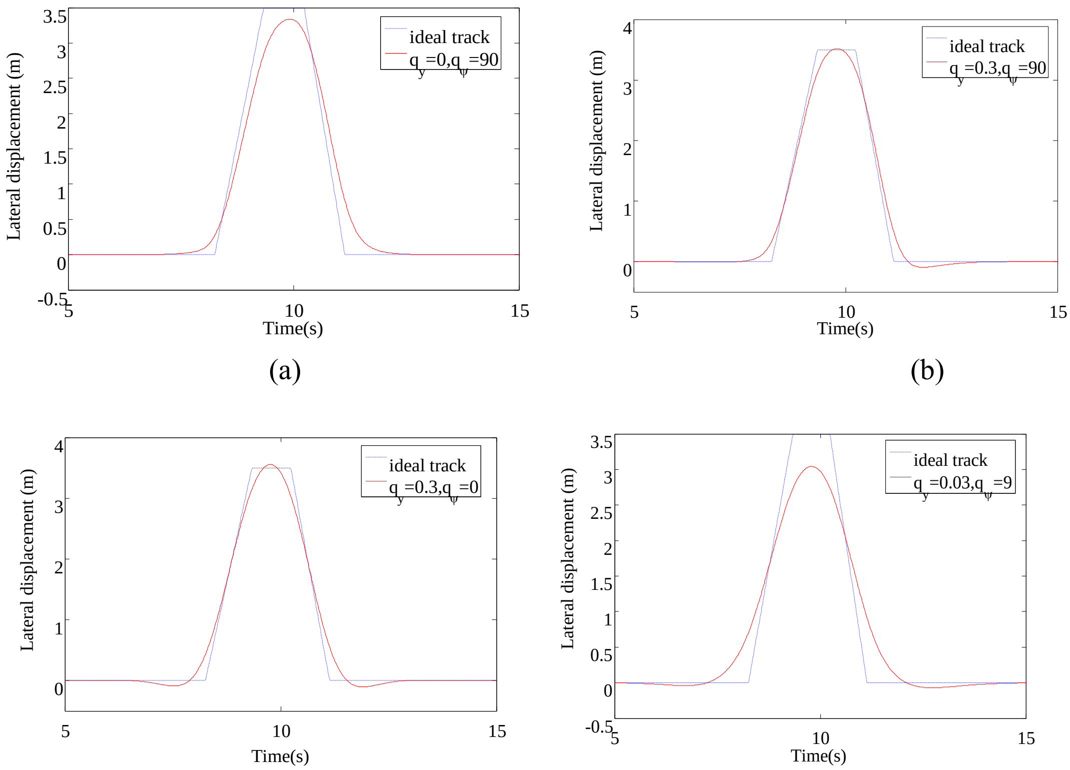
### *5.2. Influence of Different Weighted Coefficients on Steering Performance*




(a) (b)

**Figure 6.** Status and preview gain. (a) Status gain; (b) Preview gain.

Figure 6 shows the control trajectory following characteristics of different weighted coefficients and the gain. With the change of the weighted coefficients, both the trajectory following characteristic and the gain values all change. In the status gain values, the yaw rate changes greatly, which has a great influence on the driver steering control, namely, the driver has a greater dependence on the yaw rate. At the same time, obviously, the preview is more dependent on weight. As the preview distance increases, the preview gain gradually approaches zero, and the preview gain at infinity is zero, which means that the point at which the driver's preview distance is closer has an effect on the driver steering control. However, with the increase of the preview distance, the influence of the far points on the steering control is getting smaller and smaller.




(c) (d)

**Figure 7.** (a)~ (d) The control trajectory following characteristics of different weighted coefficients. (a) Controller 1; (b) Controller 2; (c) Controller 3; (d) Controller 4.

The trajectory following characteristics of each controller are shown in figure 7. The controller 1 shows that when the lateral position deviation is very small and the heading angle error is very large the road trajectory following characteristic is remarkably poor. As the weight of the lateral position error increases, the trajectory following control characteristics gradually improve, as shown by the controller 2. When the controller 3 shows that the heading angle error is small, the driver turns to a certain amount of rotation in the reverse direction, and the reverse steering operation is related to the vibration at the time of steering input by the driver. When the lateral position error and the heading angle error weight are small (controller 4), the steering shear angle often occurs. For different weight conditions, there are different trajectory following characteristics. Even if different drivers in the same road conditions, there will be a different trajectory following result, so it's necessary to seek the best control strategy to the driver steering control model design.

## 6. Conclusion

Closed-loop simulation results show that the multi-point preview of the driver steering model has a good trajectory following effect. The optimal driver steering model can be obtained by selecting different weighted coefficients and the number of preview points, which can be used as a reference for the future research of various driver steering models.

## 7. Acknowledgments

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